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THE MATHEMATICAL REPOSITORY.

VOL. III.

CONTAINING
Analytical Solutions
OF
A great Number of the moſt difficult
PROBLEMS,
RELATING TO

ANNUITIES,
REVERSIONS,
SURVIVORSHIPS,
INSURANCES, and
LEASES dependent on Lives;

In which it has been endeavoured to exhauſt the SUBJECT.

By JAMES DODSON, F. R. S.
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MDCCLV.

To the Right Honourable

George Earl of Macclesfield,

PRESIDENT,

The COUNCIL,

And the reſt of the Fellows of the

ROYAL SOCIETY,

This Work is, with the greateſt Deference,
inſcribed, by

Their moſt obliged, and

Moſt humble Servant,

JAMES DODSON.

GE III.
BR. MUS.

THE PREFACE.

THE ſecond Volume of this work being cloſed, as it were, with the beginning of a new ſubject; the reader might readily conjecture, that it was deſigned to proſecute it in a ſubſequent volume; but the author, at that time, was dubious whether it could be done, conſiſtent with the reſtriction, which he had laid himſelf under, of introducing no proceſs, that could not be underſtood by thoſe, whoſe knowledge extended only to common algebra: he ſaw, that the doctrine of fluxions had been uſed, by authors of the greateſt reputation, in the ſolution of ſome of the moſt ſimple of the queſtions, which remained to be conſidered; and was not ſanguine enough to conceive, that he ſhould ſucceed, not only in them, but alſo in the more complex, upon ſimpler principles: he ſaw, nevertheleſs, the great advantage, that would reſult from the ſolution of thoſe queſtions, by a proceſs purely arithmetical, and therefore determined to attempt it; the attempt has ſucceeded, even beyond his hopes, and he now ſubmits to the approbation or cenſure of the public, a third volume, every queſtion of which is either directly or remotely applicable to, probabilities, expectations, annuities, reverſions, or inſurances, dependent on lives or ſurvivorſhips.

The principle, by which he has been enabled to perform this, was uſed in the ſecond volume (page 341) in the inveſtigation of the expectation of a ſingle life; and its application, to an annuity, is equally natural, and eaſy; thus, conceive a perſon of ſuch an age, that it cannot be reaſonably expected, that he ſhould live beyond the end of one year, ſuppoſe him poſſeſſed of an annuity of 1£. (or any other ſum) with condition, that ſuch part of that annual payment is to be made, to his heir, as ſhall be proportional to the time elapſed, between the beginning of the year and the time of his deceaſe: then the expectation of the ſum, to be ſo received by the heir, (being valued at the beginning of the year) will be half the annual payment, the intereſt of money not being here conſidered: the uſe of this principle is alſo extended to ſurvivorſhips, with ſucceſs; as will appear in the following ſheets.

The ſolutions of the more complex queſtions, relating both to annuities and ſurvivorſhips, would, however, have been inſupportably tedious, if the method of approximation, uſed in the ſecond volume, had not been again admitted; and it was ſo, with the greater alacrity, ty, becauſe the reſulting expreſſions are more manageable, than thoſe in that volume.

The truly ingenious Mr. *Thomas Simpſon*, in his ſelect Exerciſes, pages 322 and 323; ſpeaking of thoſe Queſtions, which relate to ſurvivorſhips, has the following paſſage. "I "have, already, been very particular on theſe "Kinds of Problems, and the more ſo, as "there is no Method yet publiſhed (that I "know off) by which They can be rightly "determined. 'Tis true the Manner of pro"ceeding by firſt finding the Probability of "Survivorſhip (which Method is uſed in my "former Work, and which a celebrated Au"thor on this Subject has largely inſiſted on, "in three ſucceſſive Editions) may be applied "to good Advantage, when the given Ages "are nearly equal: But then, it is certain, "that this is not a genuine Way of going "to Work; and that the Concluſions hence "derived are, at the beſt, but near Approx"imations. The Rate of Intereſt that Money "bears muſt be compounded with the Proba"bility of Survivorſhip, and the Expectation "on each particular Year muſt be determin"ed, in order to have a true Solution."

The author, of this work, had been of the ſame opinion with Mr. *Simpſon*, long before he was confirmed therein, by reading the above paſſage; and it is a great ſatisfaction to him, that he has been (almoſt every where) able to adhere ſtrictly thereto, in his ſolutions.

The caſes, relating to the order of ſurvivorſhip, have been hitherto exemplified no farther, than where three lives were concerned, but are here extended to four; this, it is true, takes up a great deal of room, but then, it will be a great eaſe, when it ſhall be wanted, to find every poſſible caſe inveſtigated; and the proceſſes will be of ſervice, as more examples of the manner of algebraic computation.

The ſubjects of copyholds, and leaſes of lands, for one, two, or three lives are alſo here diſcuſſed, in all the varieties that have occured, either in the author's reading, practice, or imagination; and it is hoped, that ſome new light is thrown, on thoſe ſubjects.

'Tis true, that as theſe do not appear till near the cloſe of the volume, the author has been obliged (in order to prevent the ſwelling thereof, beyond a proper ſize) to abbreviate the inveſtigations, by leaving out ſome intermediate ſteps; but as it may be reaſonably ſuppoſed, that the reader will be then ſufficiently maſter of the ſubject, to ſupply that defect, 'tis hoped, that it will be excuſed; eſpecially, as the ſo doing will be a good teſt of his ability, in ſuch kind of computation: 'tis for the ſame reaſons, that rules, in words at length, have been omitted throughout the whole volume.

The author thinks, that little need be here ſaid concerning inſurances on lives; as it is a ſubject not before handled, and will ſhew its own uſe.

There are inſerted, at the end of the volume, a table of the values of annuities for ſingle lives, ſecured by land; and a table of the differences, between the values of annuities for ſingle lives (ſecured by land) and the quotients ariſing from the diviſions of the complements of thoſe lives, by 6 times the rate of intereſt; which differences occur in moſt of the ſolutions, contained in this volume.

It will appear, by the under written corrections, which the author thinks himſelf obliged to make, of errors committed in the ſecond volume, that he has too much occaſion to fear, that ſome ſlips have eſcaped him in this; eſpecially in thoſe parts thereof, wherein he has ventured beyond the beaten path: he begs leave, therefore, to beſpeak the candour of the judicious readers, profeſſing himſelf obliged to thoſe gentlemen that have, or ſhall, point out his miſtakes; and promiſing, publicly, to communicate and amend thoſe, which he ſhall be informed of, as he now does them which are below enumerated.

In the expreſſion given toward the bottom of page 151, Vol. II. the quantities, α and β, are ſeverally multiplied into r; but the multiplication was in the ſubſequent work of the original calculation, inadvertently, omitted:

The tables of obſervations (pages 158 and 160) ſhould, each of them, have been extended one line farther, in the manner following; viz. under the laſt printed lines of each, let lines be drawn, to denote the beginning of a new interval; then in table page 158, place 95, in the firſt column; 0, in the ſecond; 0, in the third; and + 1, in the fourth: and, in the table page 160, place 91, in the firſt column; 0, in the ſecond; 0, in the third; and + 1, in the fourth.

It happens, that theſe omiſſions affect the examples, given in pages 162 and 163, only in the ſecond decimal place; by which means they were not diſcovered, before the publication of that volume; the firſt was ſince ſeen, by the author, on reperforming the operation, with a gentleman who choſe to ſtudy the ſubject: and the latter appeared, on its being diſcovered by another gentleman, that the values of elder lives were not given truly, by the rule: the neceſſary corrections follow,

In the 1ſt expreſſion p. 152 for $\frac{\alpha}{r^s}$ and $\frac{\beta}{r^{s+t}}$

read $\frac{r\alpha}{r^s}$ and $\frac{r\beta}{r^{s+t}}$;

the ſecond ſhould ſtand, as below,

$$\frac{PPr}{a} \times \left\{ \begin{array}{l} \frac{a}{Pr} - \frac{a-b}{\cdot s} + \frac{\alpha}{r^s} + \frac{\beta}{r^{s+}} \\ - \frac{d}{rPr^{s+t+v}} + \frac{c-d}{vr^{s+t+v}} \end{array} \right.$$

I

In the ſame page, line the 12th from the bottom, for *at the end*, read, *next following the end*.

The expreſſion at the top of page 153 ſhould ſtand thus,

$$\frac{PPr}{a}\times\left\{\begin{array}{l}\frac{a}{Pr}-\frac{a-b}{s}-\frac{b}{rPr^{s+t\,\&c.\,+\kappa}}+\frac{g-b}{\kappa r^{s+t\,\&c.\,+\kappa}}\\+\frac{\alpha}{r^{s}}+\frac{\beta}{r^{s+t}}+\frac{\gamma}{r^{s+t+v}}+\frac{\delta}{r^{s+t+v+w}}\end{array}\right.$$

$$(m-1).$$

And thoſe, in pages 154 and 155, will be rectified, by writing $\frac{PPr}{a}$ for $\frac{PP}{a}$; $\frac{a}{Pr}$ for $\frac{a}{P}$; $\frac{a-b}{s}$ for $\frac{\overline{a-b}\times r}{s}$; and D for Dr.

In the rule, in words at length, page 161, let the 5th and 6th paragraphs ſtand thus.

Multiply the number which ſtands in the ſecond column of the table of obſervations, on a line with the given age, by the intereſt of one pound, and divide the product by the rate; placing the quotient under the ſign +: alſo multiply this quotient, by the intereſt of 1£. reſerving the product for future uſe.

Place the number which ſtands in the third column of the table of obſervations, on a line with the given age, under the ſign —.

In the operation of the example, page 162, line 12, after 89, inſert 95; line 15, after 79, inſert 85; and line 19 after $\overset{+}{1}$, inſert $\overset{+}{1}$.

That part of the example contained in page 163 ſhould ſtand thus

Now if 524, the number of perſons living at 10 years of age, be multiplied by 0,04, the intereſt of 1*l.* it produces 20,96; which being divided by (1+0,04=) 1,04 the rate, quotes 20,1538 which is placed under + : and this quotient, 20,1538 multiplied by 0,04, produces 0,80615, the diviſor afterwards uſed.

Years	+	Years	—
2	0,9246	10	0,6756
33	0,2741	14	0,5775
37	0,2343	19	0,4746
44	0,1780	29	0,3207
51	0,1353		7,0000
60	0,0951		
64	0,0813		9,0484
66	0,0751		
75	0,0528		
79	0,0451		
85	0,0357		
	20,1538		
ſum +	22,2852		
ſum —	9,0484		

Alſo 7, the number of perſons dying between the ages 10 and 11, is placed under —.

0,80615)13,2368(16,420

The manner of correcting the other example is ſo evident, from this, that it need not be inſerted, the true reſult being 17,788.

Bell Dock, Wapping,
Jan. 23d. 1755.

THE

MATHEMATICAL

REPOSITORY.

QUESTION I.

TO find the value of an annuity, ſecured by a grant of lands, which is to continue during the life of a perſon of a given age? *

An annuity for life, ſecured by a grant of lands, differs from that kind of annuity for life, whoſe value was computed in Queſt. 56. *Vol.* 2, *in this; that here, if the annuitant dies at any time, between the ſtated times of the payment of the annuity, his heirs are to receive ſuch a ſum, as will be proportional to the time elapſed, between the laſt*

---

* Note, all the ſolutions relating to annuities, reverſions, ſurvivorſhips, &c. in this volume, are computed upon the ſuppoſition that the decrements of life are equal; and may be adapted to any table of obſervations, by the application of the 104th queſtion of the ſecond Volume; alſo, in thoſe examples where no rate of intereſt is mentioned, the reader is deſired to compute at 4 per cent; again where the ſymbols t, m, n, and v, are uſed as the complements of life, t is always the greateſt, m greater than n; and n greater than v; laſtly $\mathfrak{F}$ always denotes the value of that ſingle life, whoſe complement is t; and $\mathfrak{M}$, $\mathfrak{N}$, $\mathfrak{V}$, thoſe of ſingle lives whoſe complements are ſeverally m, n, and v.

time of payment and the time of death; whereas, in the former caſe, if the annuitant dies on the day preceding the time of payment, or ſooner, his heirs cannot claim any part of ſuch payment.

SOLUTION.

If n denote the complement of life, then (by arguing as in the queſtion above quoted) the reſpective probabilities of receiving the whole payment of the annuity, at the end of the firſt, ſecond, third, &c. year, are $\frac{n-1}{n}$, $\frac{n-2}{n}$, $\frac{n-3}{n}$, &c. beſides which, in this caſe, the annuitant has the expectation of receiving ſuch part of the annuity, as ſhall be proportional to ſo much of the year, as may be elapſed before his deceaſe, if he dies before the whole becomes due.

Now, it is an equal chance, whether the annuitant dies in the firſt, or ſecond half, of any one year; and conſequently, whether his heirs receive leſs or more than half the annual payment, growing in the year of his deceaſe; they may therefore be eſteemed, to have an expectation of half that payment, and we may (as in queſtion 105, Vol. 2.) encreaſe each of the above probabilities by, $\frac{1}{2n}$, the probability of the annuitant's living half of that year in which he dies; by which means, the reſpective value of the firſt, ſecond, third, &c. payment of the annuity will become $\Big(\frac{n-1}{n}+\frac{1}{2n}$, $\frac{n-2}{n}+\frac{1}{2n}$, $\frac{n-3}{n}+\frac{1}{2n}$, &c. or$\Big)$ $\frac{2n-1}{2n}$, $\frac{2n-3}{2n}$, $\frac{2n-5}{2n}$, &c. the ſame with the expectations of life, for thoſe years.

But the intereſt of money being conſidered, the preſent values of thoſe payments will become $\Big(\frac{n-1}{nr}+\frac{1}{2nr}$,

$\frac{1}{2nr}$, $\frac{n-2}{nr^2}+\frac{1}{2nr^2}$, $\frac{n-3}{nr^3}+\frac{1}{2nr^3}$, &c. or) $\frac{2n-1}{2nr}$, $\frac{2n-3}{2nr^2}$, $\frac{2n-5}{2nr^3}$, &c. And conſequently, the required value of the annuity will be repreſented by $\frac{2n-1}{2nr}+\frac{2n-3}{2nr^2}+\frac{2n-5}{2nr^3}(n)$.

Now this differs from, $\frac{n-1}{nr}+\frac{n-2}{nr^2}+\frac{n-3}{nr^3}(n)$, the value of an annuity for a ſingle life upon the other ſuppoſition (ſee queſt. 56. Vol. 2.) by $\frac{1}{2nr}+\frac{1}{2nr^2}+\frac{1}{2nr^3}(n)$; which (by queſt. 15. Vol. 2.) is equal to $\left(\frac{1}{2n}\times\frac{1-p}{r-1}=\right)\frac{1-p}{2n}P$, where p denotes the preſent worth of one pound, due at the end of n years; (ſee table pag. 166. Vol. 2.) and P is the value of the perpetuity (ſee table pag. 77. Vol. 2.) $=\frac{1}{r-1}$; or the whole quantity, $\frac{1-p}{2n}P$, denotes the quotient ariſing, by dividing the preſent value of an annuity certain, for the complement of life, by twice that complement.

If therefore, to $P-\frac{1-p}{n}Q$ (the value of the annuity found by queſt. 56, Vol. 2.) we add $\frac{1-p}{2n}P$ (the quotient above deſcribed) the ſum, $P+\frac{1-p}{2n}P-\frac{1-p}{n}Q$, will be the anſwer; where Q denotes $\left(\frac{r}{\overline{r-1}^2}\text{ or}\right)rPP$.

If a table of the values of annuities on lives, according to the ſolution, in queſtion 56, Vol. 2. be at hand; then (calling the value of a life, found by thoſe tables,

N; and the value required, $\mathfrak{P}$) then $\mathfrak{P} = N + \frac{1-p}{2n}P$; upon which principle the following tables are calculated; and may be compared with thoſe, given in the ſecond volume.

Scholium. The above rule (altho deduced without the aſſiſtance of Fluxions) will, upon the following trial, be found to agree (almoſt exactly) with one, deduced for this purpoſe, by Mr. *De Moivre*, from a fluxional proceſs, in No. 473, of the philoſophical tranſactions; which rule is preparatory to that general one, for finding the value of a life from a given table of obſervations, mentioned in page 18 of the preface to the ſecond volume.

Mr. *De Moivre*'s rule is expreſſed by $\frac{1}{r-1} - \frac{P}{an}$; where $\frac{1}{r-1}$ ſignifies the ſame with the perpetuity (above denoted by P); a ſignifies the hyperbolic Logarithm of the quantity r (the amount of 1 pound and its intereſt for one year); and P denotes the amount of an annuity, certain, of one pound, to continue during the complement of life; which complement is there, as well as here, expreſſed by n.

This rule, therefore, reduced to the notation, uſed in this work, will become $P - \frac{\overline{1-p}\times P}{an}$; for $\left(\frac{1-p}{r-1}\text{ or}\right)\overline{1-p}\times P$ is the value of an annuity of 1 £. for n years certain, by queſt. 15. Vol. 2.

Now, if the reſult above found be truly equal to that of Mr. *De Moivre*; Then,

$$P + \frac{1-p}{2n}P - \frac{1-p}{n}rPP = P - \frac{1-p}{an}P,$$

$$\text{Or}\quad \frac{1-p}{2n}P - \frac{1-p}{n}rPP = -\frac{1-p}{an}P;$$

That is $\left(\text{dividing the laſt equation by } \frac{1-p}{n}P\right)$

$\frac{1}{2} - rP = -\frac{1}{a}$

Or $\left(\text{becauſe } P = \frac{1}{r-1}\right) \frac{1}{2} - \frac{r}{r-1} = -\frac{1}{a}$

Whence $\frac{1}{a} = \left(\frac{r}{r-1} - \frac{1}{2} =\right) \frac{r+1}{\overline{r-1}\times 2}$

Conſequently $a = \frac{\overline{r-1}\times 2}{r+1}$.

But (by queſt. 195, part 2. Vol. 1.) the hyperbolic logarithm of any number *r* will be repreſented by the ſeries:

$$2\times\overline{\frac{r-1}{r+1}+\frac{1}{3}\times\left.\frac{r-1}{r+1}\right|^{3}\ \&c.}$$

of which the above compariſon gives the firſt term; which in this caſe is exact enough, as will appear by the following calculation.

As the intereſt of money has not, for many years, exceeded 5 per cent. and is continually decreaſing; *r* will not, in any caſe, be expounded by a number greater than (1,05 or) $\frac{21}{20}$; whence $r+1=\frac{41}{20}$, & $r-1=\frac{1}{20}$; Therefore $\frac{r-1}{r+1}=\frac{1}{41}$; and the above ſeries, being expreſſed in numbers, will be

$$\frac{2}{41}+\frac{2}{3}\times\left.\frac{1}{41}\right|^{3}+\frac{2}{5}\times\left.\frac{1}{41}\right|^{5}\ \&c.$$

But the ſecond term of this ſeries, viz. $\left(\frac{2}{3}\times\left.\frac{1}{41}\right|^{3}=\right)\frac{2}{206763}$ is a quantity too ſmall to affect the calculation. When the rate of intereſt is leſs, ſuppoſe 4 per cent. the fraction whoſe powers conſtitute the ſeries will be ſtill ſmaller; for then $r =$

(1,04 or) $\frac{26}{25}$; $r + 1 = \frac{51}{25}$; and $r - 1 = \frac{1}{25}$; whence $\frac{1}{51} = \frac{r-1}{r+1}$; and $\Big(\frac{2}{3} \times \overline{\frac{1}{51}}\Big|^{3}$ will$\Big)$ be a quantity, ſtill ſmaller than the former; therefore the firſt term of the ſeries will, in all caſes, be ſufficiently exact; and the rule above found will agree with that of Mr. *De Moivre.*

Notwithſtanding the table, hereto annexed, of the values of annuities on lives ſecured by land; it is thought convenient to inſert the computation of the values of thoſe three, which have been uſed as examples in the queſtions of the ſecond volume.

EXAMPLE I.

What is the value of an annuity (ſecured by land) for the life of a perſon, aged 43, allowing compound intereſt at 4 per cent.?

By example 1. queſt. 56. Vol. 2. the value of an annuity for a life aged 43 years, is 12,683.

And, in this caſe, $P = 25$; $n = 43$; and $p = 0,1852$.

Whence $1 - p =$,8148

Multiply by 25

Then (2×43=)86)20,3700(,237: therefore (12,683 + 0,237 =) 12,920 will be the value required.

EXAMPLE II.

What is the value of an annuity (ſecured by land) for the life of a perſon, aged 54, at 4 per cent.?

By the 2d example to queſt. 56. Vol. 2. the life aged 54 is worth 10,478. And here $P = 25$; $n = 32$; and $p =$,2851.

Therefore $1 - p =$,7149

Multiply by 25

Then (2×32=)64)17,8725(,279; and (10,478 + 0,279 =) 10,757 will be the value required.

EXAMPLE III.

What is the value of an annuity (ſecured by land) for the life of a perſon aged 66, at 4 per cent.?

By the third example of the ſame queſtion, the life of 66 is worth 7,333; and here $P = 25$; $n = 20$; and $p = ,4564$

Whence $1 - p = ,5436$
Multiply by 25
Then $(2 \times 20 =) 40) 13,5900 (,340$; and $(7,333 + 0,340 =) 7,673$.

Corol. 1. From Mr. *De Moivre*'s ſolution of this queſtion, may be derived a commodious method of calculating the values of theſe annuities, *de novo*; that is, when the values of the former kind of annuities are not given.

For it appears, that $\frac{1}{a}$ or $\frac{\overline{r+1}}{\overline{r-1} \times 2}$ is a conſtant factor, in the negative term of the expreſſion $(P - \frac{1-p}{a\,n} P$ or$)$ $P - \frac{\overline{r+1}}{\overline{r-1} \times 2} \times \frac{1-p}{n} P$; If, therefore, the values of $\frac{1}{a}$ be calculated for the ſeveral rates of intereſt, as below, the value of the annuity will become

$$\left(P - \frac{1}{a} \times \frac{1-p}{n} P \text{ or}\right) P \times \overline{1 - \frac{1}{a} \times \frac{1-p}{n}}$$

$$\text{Now, when the rate of intereſt is} \begin{Bmatrix} 3 \\ 3\frac{1}{2} \\ 4 \\ 4\frac{1}{2} \\ 5 \\ 6 \end{Bmatrix} \text{per cent. Then } \frac{1}{a} \left(\text{or } \frac{\overline{r+1}}{\overline{r-1} \times 2}\right) = \begin{cases} 33,833 \\ 29,071 \\ 25,5 \\ 22,722 \\ 20,5 \\ 17,167 \end{cases}$$

Corol. 2. *If the value of an annuity (ſecured by land) for a life of a given age, be required to be computed, according to a given table of obſervations; then find (by the rule in pag.* 161. *Vol.* 2.*) the value of the common annuity, for*

that

that life; and (by queſt. 104. *Vol.* 2.) *find* n *the complement of that life; then, to the value of the annuity above found, add,* $\frac{1-P}{2n}P$, *the quotient above deſcribed, and the ſum will be the value required.*

QUESTION II.

A, whoſe complement is *m*, is poſſeſſed of an annuity for his life (ſecured by land) which he ſells to *B*, for *n* years certain, on condition that at the expiration thereof (if *A* be then alive) the annuity ſhall return to him: the ſeveral intereſts of *A* and *B* in that annuity are required?

SOLUTION.

By queſtion 57. Vol. 2, the value of an annuity (not ſecured by land) for *n* years certain, if a life whoſe complement is *m* ſhall live ſo long, is $\overline{1-\frac{1}{r^n}}\times P+\frac{n}{mr^n}P-\overline{1-\frac{1}{r^n}}\times\frac{Q}{m}$; To which, if $\left(\overline{1-\frac{1}{r^n}}\times\frac{P}{2m}\right)$ the ſum of *n* terms of the ſeries $\frac{1}{2mr}+\frac{1}{2mr^2}+\frac{1}{2mr^3}$, be added, the reſult will (by arguing as in queſt. 1.) appear to be the value of the intereſt of *B* in the annuity; viz.

$$\overline{1-\frac{1}{r^n}}\times P+\overline{1-\frac{1}{r^n}}\times\frac{P}{2m}+\frac{n}{mr^n}P-\left(\overline{1-\frac{1}{r^n}}+\frac{Q}{m}\right.,$$

$$\text{Or } \overline{1+\frac{1}{2m}}\times\overline{1-\frac{1}{r^n}}\times P+\frac{n}{mr^n}P-\left(\overline{1-\frac{1}{r^n}}\times\frac{Q}{m}\right.;$$

And, if the above value of *B*'s intereſt in the annuity, be taken from $\left(P + \overline{1 - \frac{1}{r^m}} \times \frac{P}{2m} - \overline{1 - \frac{1}{r^m}} \times \frac{Q}{m}\right)$ the value of the annuity for *A*'s life, the remainder will be the value of the intereſt of *A* in that annuity, viz.

$$= \frac{1}{r^n} P + \overline{\frac{1}{r^n} - \frac{1}{r^m}} \times \frac{P}{2m} - \frac{n}{mr^n} P - \overline{\left(\frac{1}{r^n} - \frac{1}{r^m}\right.} \times \frac{Q}{m};$$

Or $\overline{1 - \frac{n}{m}} \times \frac{P}{r^n} - \overline{\frac{Q}{m} - \frac{P}{2m}} \times \overline{\frac{1}{r^n} - \frac{1}{r^m}};$

That is $\frac{m-n}{m} \times \frac{1}{r^n} P - \frac{Q - \frac{1}{2}P}{m} \times \overline{\frac{1}{r^n} - \frac{1}{r^m}};$

Or $\frac{P}{m} \times \frac{m-n}{r^n} - \overline{rP - \tfrac{1}{2}} \times \overline{\frac{1}{r^n} - \frac{1}{r^m}}$

But $rP - \frac{1}{2} \left(= \frac{r}{r-1} - \frac{1}{2} = \frac{r+1}{\overline{r-1} \times 2} = \right) \frac{1}{a}$, by page 5 queſtion 1. Therefore, if the values of $\frac{1}{a}$ be taken from the table in page 7; the value of *A*'s remaining intereſt will become

$$\frac{P}{m} \times \frac{m-n}{r^n} - \frac{1}{a} \times \overline{\frac{1}{r^n} - \frac{1}{r^m}}.$$

EXAMPLE.

Suppoſe *A*, aged 54, would ſell to *B*. his intereſt in ſuch an annuity, on his own life, for 20 years certain; what ought *B* to pay for the ſame; and what is the value of the intereſt therein, which he has not diſpoſed of?

Here $P=25$; $Q=650$; $n=20$; $m=32$;
$\frac{1}{r^n}=,4564$; $1-\frac{1}{r^n}=,5436$; and $\frac{1}{r^m}=,2851$;

Then $25\times,5436=13,59$; $\frac{13,59}{2\times 32}=,212$; and $13,59+,212=13,802$;

Now $\frac{20\times 25\times ,4564}{32}=7,131$; and $\frac{650\times ,5436}{32}=$ (11,042;

Therefore $(13,802+7,131-11,042=)$ 9,891 is the value of *B*'s purchaſe.

Again $m-n=12$; $\frac{1}{a}=25,5$; and $,4564-,2851=,1713$;

Then $12\times ,4564=5,4768$; $25,5\times ,1713=4,3683$;
And $5,4768-4,3683=1,1085$; Therefore $\left(\frac{1,1085\times 25}{32}=\right)$ 0,866 will be the value of what *A* has not ſold.

Now if 9,891, the value of *B*'s purchaſe, be added to 0,866 the value not purchaſed; their ſum, 10,757, will be the value of the whole annuity.

Otherwiſe; if tables of the values of ſuch annuities on lives are at hand: then, ſince the numerators, $2m-2m+1$, and $2n-2n+1$, of the leaſt terms of the two ſeries (expreſſing the values of ſuch annuities for the lives, whoſe complements are *m* and *n*) are ſeverally unity; and ſince the common difference of the numerators of every ſuch ſeries is 2; therefore the numerators of any number of terms, whatſoever (reckoning from the leaſt) of both the ſeries, which expreſs the values of annuities, for two different lives, will be ſeverally equal: that is, if there be two lives, whoſe complements are *m*, the greater, and *d* the leſſer; then, the numerators of the laſt *d* terms of the ſeries, expreſſing the values of the life, whoſe complement is *m*, will be $2d-1$, $2d-3$, $2d-5$, &c, the ſame with thoſe of the ſeries, which exhibits the value of the life, whoſe complement is *d*.

For example, Let $m = 5$, and $n = 3$; then the numerators of the terms, expreſſing the value of the life, whoſe complement is $\left\{\begin{matrix}5\\3\end{matrix}\right\}$ will be $\left\{\begin{matrix}9,\ 7,\ 5,\ 3,\ 1;\\ 5,\ 3,\ 1.\end{matrix}\right.$

To apply this to the queſtion before us; let us put $\mathfrak{M}$ and $\mathfrak{D}$, for the values of annuities (ſecured by land) for two lives, whoſe complements are m, and $(m - n =)\ d$; then,

$$\mathfrak{M} = \frac{2m-1}{2mr} + \frac{2m-3}{2mr^2}\left(n\right) + \frac{2d-1}{2mr^{n+1}} + \left(\frac{2d-3}{2mr^{n+2}}\left(d\right)\right):$$

In which, $\frac{2m-1}{2mr} + \frac{2m-3}{2mr^2}\left(n\right)$ repreſents the part ſold, and $\frac{2d-1}{2mr^{n+1}} + \frac{2d-3}{2mr^{n+2}}\left(d\right)$ the part unſold: let the value of the firſt be denoted by ${}^{n}\mathfrak{M}$, and then that of the latter will be expreſſed by $\mathfrak{M} - {}^{n}\mathfrak{M}$:

$$\text{Then } \mathfrak{M} - {}^{n}\mathfrak{M} = \left(\frac{2d-1}{2mr^{n+1}} + \frac{2d-3}{2mr^{n+2}}\left(d\right) =\right)$$

$$\left(\frac{1}{m \times r^n} \times \overline{\frac{2d-1}{2r} + \frac{2d-3}{2r^2}\left(d\right)};\right.$$

$$\text{Therefore } mr^n \times \mathfrak{M} - {}^{n}\mathfrak{M} = \frac{2d-1}{2r} + \frac{2d-3}{2r^2}\left(d\right)$$

$$\text{But } \quad \mathfrak{D} = \frac{2d-1}{2dr} + \frac{2d-3}{2dr^2}\left(d\right)$$

$$\text{Or } \quad \mathfrak{D} = \frac{1}{d} \times \overline{\frac{2d-1}{2r} + \frac{2d-3}{2r^2}\left(d\right)};$$

$$\text{Therefore } d \times \mathfrak{D} = \frac{2d-1}{2r} + \frac{2d-3}{2r^2}\left(d\right);$$

$$\text{Whence } mr^n \times \overline{\mathfrak{M} - {}^{n}\mathfrak{M}} = d \times \mathfrak{D};$$

And the part unſold, viz.

$$\mathfrak{M} - {}^n\mathfrak{M} = \left(\frac{\mathfrak{D}d}{mr^n} =\right) \frac{1}{r^n} \times \mathfrak{D} \times \frac{m-n}{m};$$

And the part ſold, ${}^n\mathfrak{M} = \mathfrak{M} - \frac{1}{r^n} \times \mathfrak{D} \times \frac{m-n}{m};$

EXAMPLE.

If the age be 54, and the term for which the annuity is to be ſold, be 20 years.

$$\left.\begin{array}{l}\text{Then } \mathfrak{M} \text{ (the value of a life of 54)}\\ \quad\text{will be } 10{,}757\\ \text{And } \mathfrak{D} \text{ (the value of a life of } (54+\\ \quad 20=)74) = 5{,}057\end{array}\right\}\text{by table}$$

Alſo $\frac{1}{r^n}$ (the preſent worth of 1 £ due at the end of 20 years) $= 0{,}4564$, by table fol. 166. Vol. 2. and $\frac{m-n}{m} = \left(\frac{32-20}{32} = \frac{12}{32} =\right) \frac{3}{8}$:

Therefore $(5{,}057 \times 0{,}4564 \times \frac{3}{8} =)\ 0{,}866 = \mathfrak{M} - {}^n\mathfrak{M}$, the part unſold; and $(10{,}757 - 0{,}866 =)\ 9{,}891 = {}^n\mathfrak{M}$, the part ſold.

As (in the above ſolution) the ſymbol ${}^n\mathfrak{M}$ is put to denote the value of an annuity, certain, for n years, if a life (whoſe complement is m) ſhall continue ſo long; ſo, in the ſubſequent ſolutions, we ſhall uſe

$$\left.\begin{array}{l}{}^v\mathfrak{F}\\ {}^n\mathfrak{F}\\ {}^m\mathfrak{F}\\ {}^v\mathfrak{M}\\ {}^v\mathfrak{P}\end{array}\right\}\begin{array}{l}\text{For the va-}\\ \text{lue of an}\\ \text{annuity, for}\end{array}\left\{\begin{array}{c}v\\ n\\ m\\ v\\ v\end{array}\right\}\begin{array}{l}\text{Years, certain, if}\\ \text{a life, whoſe}\\ \text{complement is}\end{array}\left\{\begin{array}{c}t,\\ t,\\ t,\\ m,\\ n,\end{array}\right\}\begin{array}{l}\text{ſhall}\\ \text{live ſo}\\ \text{long.}\end{array}$$

If the numeral values of the above expreſſions be computed at 4 per cent; and t, m, n, v, are expounded by 43, 32, 20, 10; then it will appear that,

$${}^v\mathfrak{F} = 7{,}228;\quad {}^v\mathfrak{M} = 6{,}925;$$
$${}^n\mathfrak{F} = 10{,}838;\quad {}^n\mathfrak{M} = 9{,}891;$$
$${}^m\mathfrak{F} = 12{,}578;\quad {}^v\mathfrak{P} = 6{,}215.$$

Scholium. The latter method of ſolution might have been applied, to queſt. 57. Vol. 2. wherein is found the value of an annuity (not ſecured by land) for n years certain, if a life (whoſe complement is m) ſhall ſo long live: in which caſe let M and D repreſent the values of ſuch annuities, for the given life, and a life n years elder than the given one; then $M - \frac{1}{r^n} \times D \times \frac{m-n}{m}$, will be the anſwer; differing from the above, only, in writing M and D, inſtead of $\mathfrak{M}$ and $\mathfrak{D}$.

QUESTION III.

To find $\mathfrak{M}^{ii}$ the value of an annuity (ſecured by land) to continue during the joint lives of two perſons, of equal ages?

SOLUTION.

Let n be the common complement of thoſe ages; then

$$\Big(\frac{2n-1}{2n}\times\frac{2n-1}{2n}+\frac{2n-3}{2n}\times\frac{2n-3}{2n}\,(n)$$

or $\Big)\ \overline{\frac{n-1}{n}+\frac{1}{2n}}\times\overline{\frac{n-1}{n}+\frac{1}{2n}}+$

$\overline{\frac{n-2}{n}+\frac{1}{2n}}\times\overline{\frac{n-2}{n}+\frac{1}{2n}}\,(n)$ will repreſent the probabilities of receiving the firſt, ſecond, third, &c. payment.

Now $\overline{\frac{n-1}{n}+\frac{1}{2n}}\Big|^2=\overline{\frac{n-1}{n}}\Big|^2+\frac{n-1}{n}\times\frac{1}{n}+\frac{1}{4nn}$,

$\overline{\frac{n-2}{n}+\frac{1}{2n}}\Big|^2=\overline{\frac{n-2}{n}}\Big|^2+\frac{n-2}{n}\times\frac{1}{n}+\frac{1}{4nn}$,

$\overline{\frac{n-3}{n}+\frac{1}{2n}}\Big|^2=\overline{\frac{n-3}{n}}\Big|^2+\frac{n-3}{n}\times\frac{1}{n}+\frac{1}{4nn}$,

And theſe, being diſcounted according to the times, will give $\frac{n-1}{nr}\times\frac{n-1}{n}+\frac{n-1}{nnr}+\frac{1}{4nnr}$

$+\frac{n-2}{nr^2}\times\frac{n-2}{n}+\frac{n-2}{nnr^2}+\frac{1}{4nnr^2}$

$+\frac{n-3}{nr^3}\times\frac{n-3}{n}+\frac{n-3}{nnr^3}+\frac{1}{4nnr^3}$

&c. &c. &c.

for the value of the annuity required; which, putting N for the value of the ſingle life (as found by queſt. 56. vol. 2.) and N^{ii} for the value of the equal joint lives (found by queſt. 65 thereof) will become $N^{ii}+\frac{N}{n}+\frac{1-p}{4nn}P$.

EXAMPLE.

What is the value of an annuity (ſecured by land) for the joint lives of two perſons, each aged 48 years, allowing compound intereſt at 4 per cent.?

By queſt. 65. vol. 2. $N^{ii}=8,575$ and; $p=,2253$,

By table fol. 170. vol 2. $N=11,748$; $1-p=,7747$,

Here $n=38$; and $P=25$:

Then $38)11,748(0,309$; and $4\times\overline{38}^2=5776)\overset{25}{\overline{19,3675}}(,003$

Whence $(8,575+0,309+0,003=)8,887$ is the value required.

QUESTION IV.

To approximate to the value of an annuity (ſecured by land) to continue, during the joint lives of two perſons of given ages?

SOLUTION.

If m be the complement of the younger life, and n that of the elder; then $\frac{2n-1}{2nr}\times\frac{2m-1}{2m}+$

$2n-3$

$\frac{2n-3}{2nr^2} \times \frac{2m-3}{2m}$ (n) will repreſent the annuity required: and if we conceive it to conſiſt of the products of two arithmetical progreſſions, *viz.* $\frac{2n-1}{2nr} + \frac{2n-3}{2nr^2}$ (n) whoſe ſum is $\mathfrak{P}$, found per table; and $\frac{2m-1}{2m} + \frac{2m-3}{2m}$ (n) whoſe ſum is $\overline{1-\frac{n}{2m}} \times n$, by queſt. 106. vol. 2. their common differences being $\frac{1}{nr}$[*] and $\frac{1}{m}$; then the ſum of that ſeries will be (by queſt. 21. vol. 2.)
$\left(\frac{\mathfrak{P} \times \overline{1-\frac{n}{2m}} \times n}{n} + \frac{\overline{n+1}\,.\,n\,.\,\overline{n-1}}{2\,.\,2\,.\,3} \times \frac{1}{nr} \times \frac{1}{m}\right.$ or $\left.\right)$ $\mathfrak{P} \times \overline{1-\frac{n}{2m}} + \frac{\overline{n+1} \times \overline{n-1}}{2m \times 6r}$; that is (if we write $\frac{nn}{2m \times 6r}$ inſtead of $\frac{\overline{n+1} \times \overline{n-1}}{2m \times 6r}$ the difference of which expreſſions is, only, $\frac{1}{12mr}$) $\mathfrak{P} \times \overline{1-\frac{n}{2m}} + \frac{nn}{2m \times 6r}$, or $\left(\mathfrak{P} - \frac{n}{2m}\mathfrak{P} + \frac{nn}{2m \times 6r} =\right)$ $\mathfrak{P} - \overline{\mathfrak{P} - \frac{n}{6r}} \times \frac{n}{2m}$; which is therefore the value of the annuity required.

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[*] *In the ſcholium to queſt.* 64. *vol.* 2. *where the method of approximating to the values of joint lives firſt appears, the common difference of the ſuppoſed arithmetical progreſſion* $\frac{n-1}{nr}$, $\frac{n-2}{nr^2}$, $\frac{n-3}{nr^3}$, &c. (*which has the ſame differences with the above*) *was found to be* $\frac{1}{nr}$, *for which reaſon the ſame common difference is here aſſumed.*

EXAM-

EXAMPLE I.

What is the value of an annuity of 1£ (ſecured by land) for two joint lives of 43 and 54; allowing intereſt at 4 per cent?

Here $m = 43$; $n = 32$; $\mathfrak{N} = 10,757$ by queſt. 1. and $\frac{n}{6r} = 5,128$ by tab. laſt. vol. 2.

From 10,757 10,757;
Take 5,128

Rem. $5,629 \times \frac{32}{86} = 2,095$,

Remains 8,662, the anſwer.

EXAMPLE II.

What is the value of an annuity of 1£. (ſecured by land) for two joint lives of 43 and 66; allowing intereſt at 4 per cent?

Here $m = 43$; $n = 20$; $\mathfrak{N} = 7,673$; and $\frac{n}{6r} = 3,205$.

From 7,673 7,673;
Take 3,205

Rem. $4,468 \times \frac{20}{86} = 1,039$,

6,634, the anſwer.

EXAMPLE III.

What is the value of an annuity of 1£. (ſecured by land) for two joint lives of 54 and 66; allowing intereſt at 4 per cent?

Here $m = 32$; $n = 20$; $\mathfrak{N} = 7,673$; and $\frac{n}{6r} = 3,205$.

From 7,673 7,673;
Take 3,205

Rem. $4,468 \times \frac{20}{64} = 1,397$.

6,276, the anſwer.

COROL.

COROL.

If the two lives are of equal ages, then the approximation will become $(\mathfrak{P} - \overline{\mathfrak{P} - \frac{n}{6r}} \times \frac{n}{2n}$ or $\mathfrak{P} - \overline{\mathfrak{P} - \frac{n}{6r}} \times \frac{1}{2} =)$ $\overline{\mathfrak{P} + \frac{n}{6r}} \times \frac{1}{2}$.

Therefore the value of an annuity (ſecured by land) on two equal joint lives of 43, will be

$$\left(\frac{12{,}920 + 6{,}891}{2} =\right) 9{,}905;$$

That, of two equal joint lives of 48, will be

$$\left(\frac{12{,}002 + 6{,}089}{2} =\right) 9{,}046;$$

That, of two equal joint lives of 54, will be

$$\left(\frac{10{,}757 + 5{,}128}{2} =\right) 7{,}942;$$

And, that of two equal joint lives of 66, will be

$$\left(\frac{7{,}673 + 3{,}205}{2} =\right) 5{,}439.$$

SCHOLIUM.

By queſt. 106. vol. 1. it appears, that the expectation of two joint lives (whoſe complements are m and n) may be expreſſed by $\frac{n}{2} - \frac{nn}{6m}$; where $\frac{n}{2}$ ſignifies the expectation of the ſingle life, whoſe complement is n.

Now ſince theſe expreſſions, $\frac{n}{2}$ and $\frac{n}{2} - \frac{nn}{6m}$, are ſeverally the ſums of the two ſeries $\frac{2n-1}{2n} + \frac{2n-3}{2n}\,(n)$, and $\frac{2n-1}{2n} \times \frac{2m-1}{2m} + \frac{2n-3}{2n} \times \frac{2m-3}{2m}\,(n)$; it follows, that the expreſſion, $\frac{nn}{6m}$, is the ſum of that ſeries, which is compoſed of the differences of the terms of the two former; *viz.* the ſeries

$$\left(1-\frac{2m-1}{2m}\times\frac{2n-1}{2n}+1-\frac{2m-3}{2m}\times\frac{2n-3}{2n}\,(n)\right.$$

or $\left.\right)\ \frac{1}{2m}\times\frac{2n-1}{2n}+\frac{3}{2m}\times\frac{2n-3}{2n}\,(n)$.

Again ſince $\mathfrak{P}$, and $\mathfrak{P}-\mathfrak{P}-\frac{n}{6r}\times\frac{n}{2m}$ appear (by queſt. 1. and this queſtion) to be ſeverally the ſums of the two ſeries $\frac{2n-1}{2nr}+\frac{2n-3}{2nr^2}\,(n)$ and $\frac{2n-1}{2nr}\times\frac{2m-1}{2m}+\frac{2n-3}{2nr^2}\times\frac{2m-3}{2m}\,(n)$; it follows, that, $\mathfrak{P}-\frac{n}{6r}\times\frac{n}{2m}$, will be the ſum of that ſeries, which is compoſed of the differences of the terms of thoſe two ſeries, *viz.* the ſeries

$$\left(1-\frac{2m-1}{2m}\times\frac{2n-1}{2nr}+1-\frac{2m-3}{2m}\times\frac{2n-3}{2nr^2}\,(n)\right.$$

or $\left.\right)\ \frac{1}{2m}\times\frac{2n-1}{2nr}+\frac{3}{2m}\times\frac{2n-3}{2nr^2}\,(n)$: the truth of both which, the reader may render more evident, by applying the reſult of queſt. 21. vol. 2. to the ſummation of them.

Now the firſt two ſeries, thus compared together, viz. $\frac{2n-1}{2n}+\frac{2n-3}{2n}\,(n)$, and $\frac{1}{2m}\times\frac{2n-1}{2n}+\frac{3}{2m}\times\frac{2n-3}{2n}\,(n)$, exhibit ſeverally the expectations of the duration of the ſingle life, and the differences between thoſe and the expectations of the duration of the joint lives: and the latter pair of ſeries, viz. $\frac{2n-1}{2nr}+\frac{2n-3}{2nr^2}\,(n)$, and $\frac{1}{2m}\times\frac{2n-1}{2nr}+\frac{3}{2m}\times\frac{2n-3}{2nr^2}\,(n)$ exhibit the preſent worths of the annual payments, which are analogous to thoſe expectations of duration: therefore whenever, in inveſtigations the expectations or probabilities of the duration of lives, we meet with either of the ſeries, firſt named, or their ſums $\frac{n}{2}$ and $\frac{nn}{6m}$, we may ſafely conclude, that the values of the annuities, correſponding to thoſe probabilities will be expreſſed by the ſums of the latter two ſeries, viz. by $\mathfrak{P}$ and $\mathfrak{P} - \frac{n}{6r} \times \frac{n}{2m}$.

Thus, ſince the expectation of two joint lives is $\frac{n}{2} - \frac{nn}{6m}$; we may conclude, that an annuity (ſecured by land) for thoſe joint lives will be worth $\mathfrak{P} - \mathfrak{P} - \frac{n}{6r} \times \frac{n}{2m}$; as it appeared to be, by the inveſtigation of this queſtion.

COROL.

Hence (ſince the expectations both of two and three joint lives, and of the longeſt of two or three lives, are denoted by the additions or ſubtractions of $\frac{n}{2}$, $\frac{nn}{6m}$, $\frac{nnn}{12mt}$, or expreſſions ſimilar thereto) it follows, that if the value of the annuity, correſponding to the expectation, $\frac{nnn}{12mt}$, be found; then all the queſtions relating to annuities for two or three lives, may be anſwered by adding and ſubtracting the values of the correſponding annuities, in the ſame manner as the expreſſions are added or ſubtracted in the values of their expectations.

The finding the value of an annuity, which ſhall correſpond to the expectation, $\frac{n^3}{12mt}$, requires; firſt, that we diſcover that ſeries of probabilities, whoſe ſum is, $\frac{n^3}{12mt}$, the given expectation; and ſecondly, that the preſent worth of all the annual payments, expreſſed by thoſe probabilities be found.

QUESTION

QUESTION V.

To find the terms of that ſeries, of which the expreſſion, $\frac{n^3}{12mt}$, is the ſum?

SOLUTION.

By queſt. 107. vol. 2. the value of the expectation of three joint lives was found to be

$$\left(\frac{n}{2} - \frac{nn}{6} \times \frac{1}{m} + \frac{1}{t} + \frac{n^3}{12mt} \text{ or}\right) \frac{n}{2} - \frac{nn}{6m} - \frac{nn}{6t} + \frac{n^3}{12mt};$$

which whole expreſſion i therefore the ſum of the ſeries,

$$\frac{2n-1}{2n} \times \frac{2m-1}{2m} \times \frac{2t-1}{2t} + \frac{2n-3}{2n} \times \frac{2m-3}{2m} \times \frac{2t-3}{2t} (n).$$

Now it has been above remarked, that

$$\frac{n}{2} = \frac{2n-1}{2n} + \frac{2n-3}{2n} + \frac{2n-5}{2n} \left(n\right); \text{ and}$$

$$\frac{nn}{6m} = \frac{1}{2m} \times \frac{2n-1}{2n} + \frac{3}{2m} \times \frac{2n-3}{2n} \left(n\right);$$

And conſequently, that

$$\frac{nn}{6t} = \frac{1}{2t} \times \frac{2n-1}{2n} + \frac{3}{2t} \times \frac{2n-3}{2n} \left(n\right):$$

It follows, therefore, that (if x, y, and z, repreſen the firſt, ſecond, third, &c. term of the required ſeries.)

$$\text{Then } \frac{2n-1}{2n} - \frac{1}{2m} \times \frac{2n-1}{2n} - \frac{1}{2t} \times \frac{2n-1}{2n} +$$

$$\left(= \frac{2n-1}{2n} \times \frac{2m-1}{2m} \times \frac{2t-1}{2t};\right.$$

$$\text{and } \frac{2n-3}{2n} - \frac{3}{2m} \times \frac{2n-3}{2n} - \frac{3}{2t} \times \frac{2n-3}{2n} + y$$

$$\left(= \frac{2n-3}{2n} \times \frac{2m-3}{2m} \times \frac{2t-3}{2t};\right.$$

Alſ

Alſo $\frac{2n-5}{2n} - \frac{5}{2m} \times \frac{2n-5}{2n} - \frac{5}{2t} \times \frac{2n-5}{2n} + z$

$$\Big(= \frac{2n-5}{2n} \times \frac{2m-5}{2m} \times \frac{2t-5}{2t};$$

&c. &c.

Which equations being tranſpoſed, and ſeverally di-ided by $\frac{2n-1}{2n}$, $\frac{2n-3}{2n}$, $\frac{2n-5}{2n}$ &c. will become,

$$\frac{2n}{2n-1}x = \Big(\frac{2m-1}{2m} \times \frac{2t-1}{2t} - 1 + \frac{1}{2m} + \frac{1}{2t} = \Big)$$

$$\Big(\frac{1}{4mt};$$

$$\frac{2n}{2n-3}y = \Big(\frac{2m-3}{2m} \times \frac{2t-3}{2t} - 1 + \frac{3}{2m} + \frac{3}{2t} = \Big)$$

$$\Big(\frac{9}{4mt};$$

$$\frac{2n}{n-5}z = \Big(\frac{2m-5}{2t} \times \frac{2t-5}{2t} - 1 + \frac{5}{2m} + \frac{5}{2t} = \Big)$$

$$\Big(\frac{25}{4mt};$$

$$\text{For}\left\{\begin{array}{l} \frac{2m-1}{2m} \times \frac{2t-1}{2t} = \frac{4mt-2m-2t+1}{4mt}; \\ \frac{2m-3}{2m} \times \frac{2t-3}{2t} = \frac{4mt-6m-6t+9}{4mt}; \\ \frac{2m-5}{2m} \times \frac{2t-5}{2t} = \frac{4mt-10m-10t+25}{4mt}; \end{array}\right.$$

$$\text{And}\left\{\begin{array}{l} 1 - \frac{1}{2m} - \frac{1}{2t} = \frac{4mt-2m-2t}{4mt}; \\ 1 - \frac{3}{2m} - \frac{3}{2t} = \frac{4mt-6m-6t}{4mt}; \\ 1 - \frac{5}{2m} - \frac{5}{2t} = \frac{4mt-10m-10t}{4mt}; \end{array}\right.$$

Then, if from the three former fractions, theſe three latter be taken; the remainders will be $\frac{1}{4mt}$, $\frac{9}{4mt}$, and $\frac{25}{4mt}$, as was above aſſerted.

$$\text{Whence } x = \left(\frac{2n-1}{2n} \times \frac{1}{4mt} =\right) \frac{2n-1}{2n} \times \frac{1}{2m} \times \frac{1}{2t};$$

$$y = \left(\frac{2n-3}{2n} \times \frac{9}{4mt} =\right) \frac{2n-3}{2n} \times \frac{3}{2m} \times \frac{3}{2t};$$

$$z = \left(\frac{2n-5}{2n} \times \frac{25}{4mt} =\right) \frac{2n-5}{2n} \times \frac{5}{2m} \times \frac{5}{2t};$$

Therefore the required ſeries will be

$$\frac{2n-1}{2n} \times \frac{1}{2m} \times \frac{1}{2t} + \frac{2n-3}{2n} \times \frac{3}{2m} \times \frac{3}{2t} + \Bigg(\frac{2n-5}{2n} \times \frac{5}{2m} \times \frac{5}{2t}\ (n).$$

If the reader inveſtigates the ſum of this ſeries, by queſt. 22. vol. 1. (and in the reſult, writes nn for $\overline{n+1} \times \overline{n-1}$,) the ſum thereof will appear to be $\frac{n^3}{12mt}$, which operation will be a farther confirmation of the truth of this proceſs.

QUESTION VI.

To find the preſent value of the annual payments, expreſſed by the above ſeries; or, in other words, to find the ſum of the ſeries $\frac{2n-1}{2nr} \times \frac{1}{2m} \times \frac{1}{2t} + \frac{2n-3}{2nr^2} \times \frac{3}{2m} \times \frac{3}{2t}\ (n)$?

SOLU-

SOLUTION.

Here the ſums of the three ſeparate ſeries, whoſe products compoſe the above, are $\mathfrak{P}$, $\frac{nn}{2m}$, and $\frac{nn}{2t}$; their common differences, $\frac{1}{nr}$, $-\frac{1}{m}$,* and $-\frac{1}{t}$; and their firſt terms, $\frac{2n-1}{2nr}$, $\frac{1}{2m}$, and $\frac{1}{2t}$: therefore (by queſt. 22. vol. 2.) the ſum will be $\left(\frac{1}{nn}\times\overline{\mathfrak{P}\times\frac{nn}{2m}\times\frac{nn}{2t}}\right.$

$$+\frac{\overline{n+1}\,.\,n\,.\,\overline{n-1}}{2\,.\,2\,.\,3}\times\overline{\frac{2n-1}{2\,nmtr}-\frac{1}{2\,mntr}-\frac{1}{2\,tnmr}}$$

$$\left(-\frac{\overline{n+1}\,.\,n\,.\,\overline{n-1}^{2}}{2\,.\,2\,.\,2}\times\frac{1}{nmtr};\right.$$

or $\left(\frac{nn\mathfrak{P}}{4mt}+\frac{\overline{n+1}\,.\,\overline{n-1}\,.\,\overline{2n-3}}{4\,.\,6\,.\,mtr}-\frac{\overline{n+1}\,.\,\overline{n-1}^{2}}{4\times 2mtr}\right.$:
In which (writing nn for $\overline{n+1}\times\overline{n-1}$) we ſhall have
$\frac{nn\mathfrak{P}}{4mt}+\frac{nn}{4mt}\times\frac{2n-3}{6r}-\frac{nn}{4mt}\times\frac{n-1}{2r}$:

But $\frac{2n-3}{6r}-\frac{n-1}{2r}=\left(\frac{2n-3-3n+3}{6r}=\right)-\frac{n}{6r}$;

Therefore $\left(\frac{nn\mathfrak{P}}{4mt}-\frac{nn}{4mt}\times\frac{n}{6r}=\right)\frac{nn}{4mt}\times\overline{\mathfrak{P}-\frac{n}{6r}}$ will be the value required.

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* *In the ſolutions of queſtions 21 and 22. vol. 2. the arithmetical progreſſions were aſſumed to be decreaſing; and their common differences were, notwithſtanding, deemed affirmative; now, in this operation, there are two encreaſing progreſſions, and their common differences are conſidered as negative, becauſe otherwiſe, the reſult will be the ſame, as would have aroſe from decreaſing progreſſions.*

QUESTION VII.

A and *B*, (whoſe complements of life are t and m) have an annuity (ſecured by land) for their joint lives; which they will ſell to *C*, for n, a number of years, leſs than either of thoſe complements; on condition, that (if they both ſurvive that period) the annuity ſhall return to them again; the value of *C*'s intereſt in that annuity is required?

SOLUTION.

If t be greater than m; then (by queſtion 4,) $\frac{2m-1}{2mr}\times\frac{2t-1}{2t}+\frac{2m-3}{2mr^2}\times\frac{2t-3}{2t}\left(m\right)$ will repreſent the whole annuity; therefore $\frac{2m-1}{2mr}\times\frac{2t-1}{2t}+\frac{2m-3}{2mr^2}\times\frac{2t-3}{2t}\left(n\right)$ will repreſent that part thereof propoſed to be ſold to *C*.

Now, if this be conceived to conſiſt of the products of the terms of two arithmetical progreſſions, viz. $\frac{2m-1}{2mr}+\frac{2m-3}{2mr^2}\left(n\right)$ whoſe ſum is $\left(\frac{P}{m}\times\overline{\frac{m-n}{r^n}-\frac{1}{a}\times\overline{\frac{1}{r^n}-\frac{1}{r^m}}}\text{ or}\right){}^{n}\mathfrak{M}$, by queſt. 2. and $\frac{2t-1}{2t}+\frac{2t-3}{2t}\left(n\right)$ whoſe ſum is $\overline{1-\frac{n}{2t}}\times n$, by queſt. 106. vol. 2. their common differences being $\frac{1}{mr}$ and $\frac{1}{t}$; then the ſum of that ſeries will (by queſtion 21. vol. 2.) be

$$\left(\frac{{}^{n}\mathfrak{M}\times\overline{1-\frac{n}{2t}}\times n}{n}+\frac{\overline{n+1}\,.\,n\,.\,\overline{n-1}}{2\,.\,2\,.\,3}\times\frac{1}{mr^t};\right.$$

Or) ${}^{n}\mathfrak{M} \times \overline{1 - \frac{n}{2t}} + \frac{\overline{n+1} \,.\, n \,.\, \overline{n-1}}{2t \,.\, 6\,rm}$; that is (writing nn for $\overline{n+1} \,.\, \overline{n-1}$)

$$\left({}^{n}\mathfrak{M} - \frac{n}{2t}\,{}^{n}\mathfrak{M} + \frac{n}{2t} \times \frac{nn}{6rm} \text{ or} \right)$$

$$ {}^{n}\mathfrak{M} - \overline{{}^{n}\mathfrak{M} - \frac{nn}{6rm}} \times \frac{n}{2t}.$$

EXAMPLE.

What is the value of an annuity (ſecured by land) for 20 years certain, if two perſons of the ages 43 and 54, ſhall both live ſo long; allowing compound intereſt at 4 per cent?

Here $n = 20$; $m = 32$; $t = 43$; $\frac{n}{6r} = 3{,}205$ and ${}^{n}\mathfrak{M}$ (by exa. 1. queſt. 2.) $= 9{,}891$.

Whence $\frac{nn}{6rm} = \frac{3{,}205 \times 20}{32} = 2{,}003$; and $9{,}891 - 2{,}003 = 7{,}888$:

Alſo $\frac{7{,}888 \times 20}{2 \times 43} = 1{,}835$; Th. ($9{,}891 - 1{,}835 =$) $8{,}056$ will be the value required.

COROL. I.

If the two perſons, on whoſe joint lives the annuity is held, are of equal ages: then $t = m$; and the value required will become ${}^{n}\mathfrak{M} - \overline{{}^{n}\mathfrak{M} - \frac{nn}{6rm}} \times \frac{n}{2m}$.

COROL. II.

If the complement of one of the two perſons, on whoſe joint lives the annuity is held, be equal to the number of years for which the annuity is propoſed to be ſold; then ${}^{a}\mathfrak{M} = \mathfrak{P}$; and $m = n$; whence the value required will become ($\mathfrak{P} - \overline{\mathfrak{P} - \frac{nn}{6rn}} \times \frac{n}{2t}$ or) $\mathfrak{P} - \overline{\mathfrak{P} -} \frac{n}{6r} \times \frac{n}{2t}$; the ſame with the value of an annuity, on the two joint lives, whoſe complements are n and t.

QUESTION VIII.

To find $\mathfrak{P}^{iii}$ the value of an annuity (ſecured on land) to continne during the joint lives of three perſons of equal ages?

SOLUTION.

If n be the common complement of thoſe lives; then

$$\left.\overline{\frac{n-1}{n} + \frac{1}{2n}}\right|^3, \left.\overline{\frac{n-2}{n} + \frac{1}{2n}}\right|^3, \left.\overline{\frac{n-3}{n} + \frac{1}{2n}}\right|^3,$$

&c. will be the probabilities of receiving the firſt, ſecond, third, &c. payment.

Now $\left.\overline{\frac{n-1}{n} + \frac{1}{2n}}\right|^3 = \left.\overline{\frac{n-1}{n}}\right|^3 + 3\times\left.\overline{\frac{n-1}{n}}\right|^2 \times \frac{1}{2n}$

$\left(+ 3 \times \frac{n-1}{n} \times \frac{1}{4nn} + \frac{1}{8n^3};\right.$

$\left.\overline{\frac{n-2}{n} + \frac{1}{2n}}\right|^3 = \left.\overline{\frac{n-2}{n}}\right|^3 + 3\times\left.\overline{\frac{n-2}{n}}\right|^2 \times \frac{1}{2n}$

$\left(+ 3 \times \frac{n-2}{n} \times \frac{1}{4nn} + \frac{1}{8n^3};\right.$

$\left.\overline{\frac{n-3}{n} + \frac{1}{2n}}\right|^3 = \left.\overline{\frac{n-3}{n}}\right|^3 + 3\times\left.\overline{\frac{n-3}{n}}\right|^2 \times \frac{1}{2n}$

$\left(+ 3 \times \frac{n-3}{n} \times \frac{1}{4nn} + \frac{1}{8n^3};\right.$

&c. &c.

Therefore $\left.\overline{\frac{n-1}{n}}\right|^{3} \times \frac{1}{r} + \left.\overline{\frac{n-1}{n}}\right|^{2} \times \frac{3}{2nr} +$

$$\left(\frac{n-1}{n} \times \frac{3}{4nnr} + \frac{1}{8n^{3}r}\right.$$

$$+ \left.\overline{\frac{n-2}{n}}\right|^{3} \times \frac{1}{r^{2}} + \left.\overline{\frac{n-2}{n}}\right|^{2} \times \frac{3}{2nr^{2}} +$$

$$\left(\frac{n-2}{n} \times \frac{3}{4nnr^{2}} + \frac{1}{8n^{3}r^{2}}\right.$$

$$+ \left.\overline{\frac{n-3}{n}}\right|^{3} \times \frac{1}{r^{3}} + \left.\overline{\frac{n-3}{n}}\right|^{2} \times \frac{3}{2nr^{3}} +$$

$$\left(\frac{n-3}{n} \times \frac{3}{4nnr^{3}} + \frac{1}{8n^{3}r^{3}}\right.$$

&c. &c &c.

Will repreſent the value of the annuity required;

Or (putting N, N^{ii}, and N^{iii} for the values of 1, 2, and 3 equal joint lives, as found in queſtions, 56, 65, and 72, vol. 2.)

$$\mathfrak{N}^{iii} = N^{iii} + \frac{3N^{ii}}{2n} + \frac{3N}{4nn} + \frac{1-p}{8n^{3}}P.$$

COROLLARY.

Since $\mathfrak{N} = N + \frac{1-p}{2n}P$

$$\mathfrak{N}^{ii} = N^{ii} + \frac{2}{2n}N + \frac{1-p}{4n^{2}}P$$

$$\mathfrak{N}^{iii} = N^{iii} + \frac{3}{2n}N^{ii} + \frac{3}{4n^{2}}N + \frac{1-p}{8n^{3}}P$$

Therefore $\mathfrak{N}^{iv} = N^{iv} + \frac{4}{2n}N^{iii} + \frac{6}{4n^{2}}N^{ii} +$

$$\left(\frac{4}{8n^{3}}N + \frac{1-p}{16n^{4}}P\right.$$

And the law of continuation is manifeſt.

QUESTION IX.

To approximate to the value of an annuity (ſecured by land) for the joint lives, whoſe complements are n, m and t; n being the leaſt, and t the greateſt?

SOLUTION.

By queſt. 107. vol. 2. the expectation of the three joint lives is $\frac{n}{2} - \frac{nn}{6m} - \frac{nn}{6t} + \frac{n^3}{12mt}$; which expreſſion conſiſts of four terms, which are the ſeveral ſums of the four ſeries, $\frac{2n-1}{2n} + \frac{2n-3}{2n} \left(n \right)$;

$$\frac{2n-1}{2n} \times \frac{1}{2m} + \frac{2n-3}{2n} \times \frac{3}{2m} (n);$$

$$\frac{2n-1}{2n} \times \frac{1}{2t} + \frac{2n-3}{2n} \times \frac{3}{2t} (n);$$

And $\frac{2n-1}{2n} \times \frac{1}{2m} \times \frac{1}{2t} + \frac{2n-3}{2n} \times \frac{3}{2m} \times \frac{3}{2t} (n)$.

And, if the terms of each of thoſe ſeries be ſeverally multiplied by $\frac{1}{r}$, $\frac{1}{r^2}$, $\frac{1}{r^3}$, &c. then the ſums of the reſulting ſeries will (by queſtions 1, 4, and 6.) be $\mathfrak{P}$, $\overline{\mathfrak{P} - \frac{n}{6r}} \times \frac{n}{2m}$, $\overline{\mathfrak{P} - \frac{n}{6r}} \times \frac{n}{2t}$, and $\overline{\mathfrak{P} - \frac{n}{6r}} \times \frac{nn}{4mt}$; which (being combined by the ſame ſigns as $\frac{n}{2} - \frac{nn}{6m} - \frac{nn}{6t} + \frac{n^3}{12mt}$) will give

$$\mathfrak{P} - \overline{\mathfrak{P} - \frac{n}{6r}} \times \frac{n}{2m} - \overline{\mathfrak{P} - \frac{n}{6r}} \times \frac{n}{2t}$$

$+ \overline{\mathfrak{P} - \frac{n}{6r}} \times \frac{nn}{4mt}$ for the value required; which may be reduced to

$$(\mathfrak{P} - \mathfrak{P} - \frac{n}{6r} \times \overline{\frac{n}{2m} + \frac{n}{2t} - \frac{nn}{4mt}} \text{ or})$$

$$\mathfrak{P} - \overline{\mathfrak{P} - \frac{n}{6r}} \times \frac{\overline{\overline{m+t} \times 2 - n} \times n}{4mt}.$$

But, in order (by another proof) to render the truth of this method of proceeding, quite evident; let us inveſtigate (by queſtion 22. vol. 2.) the ſum of the ſeries $\frac{2n-1}{2nr} \times \frac{2m-1}{2m} \times \frac{2t-1}{2t} + \frac{2n-3}{2nr^2} \times \frac{2m-3}{2m} \times \frac{2t-3}{2t}$ (n), which exhibits the value of the joint lives required.

Here the ſums of the three conſtituting ſeries are, $\mathfrak{P}$, $\overline{1 - \frac{n}{2m}} \times n$, and $\overline{1 - \frac{n}{2t}} \times n$; their common differences, $\frac{1}{nr}$, $\frac{1}{m}$, and $\frac{1}{t}$; and firſt terms, $\frac{2n-1}{2nr}$, $\frac{2m-1}{2m}$, and $\frac{2t-1}{2t}$; whence their ſum will be

$$\left(\frac{1}{nn} \times \mathfrak{P} \times \overline{1 - \frac{n}{2m}} \times n \times \overline{1 - \frac{n}{2t}} \times n\right.$$

$$+ \frac{\overline{n+1} . n . \overline{n-1}}{2 . 2 . 3} \times \overline{\frac{2n-1}{2nmtr} + \frac{2m-1}{2mntr} + \frac{2t-1}{2tnmr}}$$

$$- \frac{\overline{n+1} . n . \overline{n-1}^2}{2 . 2 . 2} \times \frac{1}{nr} \times \frac{1}{m} \times \frac{1}{t}$$

$$\text{Or}) \; \mathfrak{P} \times \overline{1 - \frac{n}{2m}} \times \overline{1 - \frac{n}{2t}} +$$

$$\frac{\overline{n+1} . \overline{n-1} . \overline{2n + 2m + 2t - 3}}{6r . 4mt} - \frac{\overline{n+1} . \overline{n-1}^2}{2r . 4mt},$$

which, if we write nn for $\overline{n+1} . \overline{n-1}$, and expand $\left(\overline{1 - \frac{n}{2m}} \times \overline{1 - \frac{n}{2t}} \text{ or}\right) \frac{2m-n}{2m} \times \frac{2t-n}{2t}$ by multiplication,

plication, will become ($\mathfrak{P} \times \frac{\overline{4mt - 2mn - 2tn} \times m}{4mt}$

$$+ \frac{nn \times \overline{2n + 2m + 2t - 3}}{6r \,.\, 4mt} - \frac{nn \times \overline{n - 1}}{2r \,.\, 4mt}$$

Or) $\mathfrak{P} \times \frac{4mt - 2mn - 2tn + nn}{4mt} + \frac{n}{6r} \times \frac{2mn + 2n.n + 2tn - 3n}{4mt}$

$$\left(- \frac{n}{6r} \times \frac{3nn - 3n}{4mt} ;\right.$$

That is ($\mathfrak{P} \times \frac{4mt - 2mn - 2tn + nn}{4mt} + \frac{n}{6r} \times \frac{2mn + 2tn - nn}{4mt}$

Or) $\mathfrak{P} \times \frac{4mt}{4mt} - \frac{2mn + 2tn - nn}{4mt} \mathfrak{P}$

$$\left(+ \frac{2mn + 2nt - nn}{4mt} \times \frac{n}{6r} ;\right.$$

Which may be reduced to

$$(\mathfrak{P} - \overline{\mathfrak{P} - \frac{n}{6r}} \times \frac{2mn + 2tn - nn}{4mt}$$

Or) $\mathfrak{P} - \overline{\mathfrak{P} - \frac{n}{6r}} \times \frac{\overline{\overline{m + t} \times 2 - n} \times n}{4mt}$, the ſame reſult as was obtained by the other proceſs.

Having thus eſtabliſhed the principle mentioned, in ſchol. queſt. 4. *viz.* that whenever the expectations, or the ſums of the probabilities of the receiving any annuity, are denoted by $\frac{n}{2}$, $\frac{nn}{6m}$, $\frac{n^3}{12mt}$, or ſuch like expreſſions, any how compounded; that, then the value of the annuity itſelf may be denoted by $\mathfrak{P}$, $\mathfrak{P} - \frac{n}{6r} \times \frac{n}{2m}$, $\mathfrak{P} - \frac{n}{6r} \times \frac{nn}{4mt}$ or ſuch like expreſſions, compounded as the former; we ſhall in all future ſolutions take the latter, for the values of ſuch annuities, whoſe expectations, or the ſums of whoſe probabilities, are expreſſed by the former; without repeating either the Argument whereon the ſame is founded, or the proof thereof; becauſe thereby we ſhall greatly ſhorten our Operations, as ſufficiently appears by the two proceſſes, uſed in the ſolution of this queſtion.

EXAMPLE.

What is the value of an annuity (ſecured by land) for the joint lives of three perſons, aged 43, 54, and 66, allowing compound intereſt at 4 per cent?

Here $\mathfrak{P} = 7{,}674$; $n = 20$; $m = 32$; $t = 43$ and $\frac{n}{6r} = 3{,}205$;

$$\begin{array}{llllll}
m = & 32 & 32 & & \mathfrak{P} = & 7{,}674 \\
t = & \underline{43} & \underline{43} & - & \frac{n}{6r} = & \underline{3{,}205} \\
 & 75 & 96 & & \text{Rem}^r & 4{,}469 \\
 & \underline{2} & \underline{128} & & & \\
 & 150 & 1376 & & & \\
-n = & \underline{20} & \underline{4} & & \text{multiply by} & \underline{2600} \\
 & 130 & 5504 = 4mt & & & 2681400 \\
 & \underline{20} & & & & \underline{8938} \quad 7{,}674 = \mathfrak{P} \\
 & 2000 & & & & 5504)11619{,}400(\underline{2{,}111} \\
 & & & & & 5{,}563 \text{ the anſ.}
\end{array}$$

COROL. I.

If the two youngeſt lives are equal, then $t = m$, and the value of the annuity will become

$$\mathfrak{P} - \overline{\mathfrak{P} - \frac{n}{6r}} \times \frac{\overline{4m - n} \times n}{4mm}.$$

Hence the value of an annuity (ſecured by land) on the joint lives of three perſons, two of which are aged 54, and the other 66, allowing intereſt at 4 per cent. will be 5,318.

COROL. II.

If the two elder lives are equal, then $m = n$; and the value will be $\mathfrak{P} - \mathfrak{M} - \frac{n}{6r} \times \frac{2t + n}{4t}$:

And the value of an annuity (ſecured by land) on the joint lives of three perſons, two of which are aged 66, and the third 43, allowing intereſt at 4 per cent, will be 4,920.

COROL. III.

If the three lives are of equal ages, then $t = m = n$; and the joint lives will become $\mathfrak{M} - \mathfrak{M} - \frac{n}{6r} \times \frac{3}{4}$:

And the value of an annuity (ſecured by land) for the joint lives of three perſons, each aged 66, will be 4,321.

QUESTION X.

A, *B*, and *C*, whoſe Complements of life are t, m, and n (n being the leaſt) have an annuity (ſecured by land) for their joint lives; which they would ſell to *D*, for, v, a number of years leſs than either of their complements; on condition, that (if they all ſurvive that period) the annuity ſhall return to them again: what is *D*'s purchaſe worth?

SOLUTION.

By arguing as before, the propoſed annuity will be repreſented by the ſeries $\frac{2n-1}{2nr} \times \frac{2m-1}{2m} \times \frac{2t-1}{2t}$ $+ \frac{2n-3}{2nr^2} \times \frac{2m-3}{2m} \times \frac{2t-3}{2t}$ (v); which may be conceived as conſtituted of the products of the terms of 3 arithmetical progreſſions: viz.

$$\left.\begin{array}{l|c|c|c|c|c|c|}
\frac{2n-1}{2nr}+\frac{2n-3}{2nr^2}(v) & & \frac{2n-1}{2nr} & & {}^{v}\mathfrak{R} & & \frac{1}{nr} \\
\frac{2m-1}{2m}+\frac{2m-3}{2m}(v) & \text{whole firſt term is} & \frac{2m-1}{2m} & \text{ſum} & \overline{1-\frac{v}{2m}}\times v & \text{and common difference} & \frac{1}{m} \\
\frac{2t-1}{2t}+\frac{2t-3}{2t}(v) & & \frac{2t-1}{2t} & & \overline{1-\frac{v}{2t}}\times v & & \frac{1}{t}
\end{array}\right. ;$$

Therefore the ſum thereof will (by queſt. 22. vol. 2.) be

$$\left(\frac{{}^{v}\mathfrak{R}\times\overline{1-\frac{v}{2m}}\times v\times\overline{1-\frac{v}{2t}}\times v}{v\,v}\right.$$

$$+\frac{\overline{v+1}\;v\,.\,\overline{v-1}}{2\,.\,2\,.\,3}\times\overline{\frac{2n-1}{2nrmt}+\frac{2m-1}{2mnrt}+\frac{2t-1}{2turm}}$$

$$-\frac{\overline{v+1}\,.\,v\,.\,\overline{v-1}^{2}}{2\,.\,2\,.\,2\,.\,nrmt}\text{ or) }{}^{v}\mathfrak{R}\times\overline{1-\frac{v}{2m}}\times\overline{1-\frac{v}{2t}}$$

$$+\frac{\overline{v+1}\,.\,v\,.\,\overline{v-1}}{2\,.\,2\,.\,3}\times\frac{2t+2m+2n-3}{2nmtr}$$

$$-\frac{\overline{v+1}\,.\,v\,.\,\overline{v-1}^{2}}{2\,.\,2\,.\,2\,.\,nmtr};$$

which, by writing vv for $\overline{v+1}\times\overline{v-1}$, and expanding $\left(\overline{1-\frac{v}{2m}}\times\overline{1-\frac{v}{2t}}\right.$ or) $\frac{2m-v}{2m}\times\frac{2t-v}{2t}$ by multiplication, will become

$$\left({}^{v}\mathfrak{R}\times\frac{4mt-2mv-2tv+vv}{4mt}+\frac{v^3\times\overline{2t+2m+2n-3}}{6r\times 4nmt}\right.$$

$$-\frac{v^3\times\overline{v-1}}{2r\times 4nmt}\text{ or) }{}^{v}\mathfrak{R}\times\frac{4mt-2mv-2tv+vv}{4mt}$$

$$+\frac{vv}{6rn}\times\frac{2tv\times 2mv+2nv-3v-3v\times\overline{v-1}}{4mt};$$

That is ($^{v}\mathfrak{P} \times \frac{4mt}{4mt} - {}^{v}\mathfrak{P} \times \frac{2mv + 2tv - vv}{4mt}$

$+ \frac{vv}{6rn} \times \frac{2tv + 2mv - vv}{4mt} + \frac{vv}{6rn} \times \frac{2nv - 2v}{4mt}$

or) $^{v}\mathfrak{P} - {}^{v}\mathfrak{P} - \frac{vv}{6rn} \times \frac{2mv + 2tv - vv}{4mt}$

$+ \frac{vv}{6rn} \times \frac{2nv - 2vv}{4mt}$; which may be reduced to $^{v}\mathfrak{P}$

$- \overline{\overline{v\mathfrak{P} - \frac{vv}{6rn}} \times \overline{\overline{m+t} \times 2 - v} - \frac{vv}{6rn} \times \overline{n-v} \times 2} \times \frac{v}{4mt}$.

EXAMPLE.

What is the value of an annuity, for 10 years certain, if three joint lives, whoſe complements are 43, 32, an 20, ſhould live ſo long?

Here (by queſt. 2.) $v\mathfrak{P} = 6{,}215$; $v = 10$; $\frac{v}{6r}$ $= 1{,}603$; $n = 20$; $m = 32$; $t = 43$; $\frac{vv}{6rn} = \frac{1{,}603 \times 1}{20}$ $= 0{,}8015$; $\overline{32+43} \times 2 - 10 = 140$; $\overline{20-10} \times 2 = 20$ Now $6{,}215 - 0{,}8015 = 5{,}4135$; $5{,}4135 \times 140 = 757{,}89$ $0{,}8015 \times 20 = 16{,}03$; and $757{,}890 - 16{,}03 = 741{,}86$;

Then $\frac{741{,}86 \times 10}{4 \cdot 32 \cdot 43} = 1{,}348$; and $6{,}215 - 1{,}34$ $= 4{,}870$ will be the value required.

COROL. I.

If the two youngeſt lives are equal, then $t = m$ and th value required will be

$$^{v}\mathfrak{P} - \overline{\overline{v\mathfrak{P} - \frac{vv}{6rn}} \times \overline{4m - v} - \frac{vv}{6rn} \times \overline{n-v} \times 2} \times \frac{v}{4m}$$

COROL

COROL. II.

If the two elder lives be equal, then $m=n$; but no other alteration will happen in the firſt given expreſſion.

COROL. III.

If the three lives are of equal ages, then n may be wrote for m, in the expreſſion given in *Corol.* 1; but no other alteration will happen therein.

QUESTION XI.

To find $L.\mathfrak{P}^{ii}$ the value of an annuity (ſecured by land) to continue during the longeſt of two equal lives?

SOLUTION.

By arguing as in queſt. 76. vol. 2. if, from the ſum of the values of the ſingle lives, we take the value of the joint lives; the remainder will be the anſwer.

Here $2\mathfrak{P} = 2N + \frac{1-p}{2n}2P$;

And $\mathfrak{P}^{ii} = N^{ii} + \frac{2}{2n}N + \frac{1-p}{4n^2}P$;

Therefore $L.\mathfrak{P}^{ii} = 2N - N^{ii} - \frac{2}{2n}N + \frac{1-p}{2n}2P$

$$\left(-\frac{1-p}{4n^2}P\right.:$$

Now $2N - N^{ii} = L.N^{ii}$,

And let $\frac{1-p}{2n}P = A$; and $\frac{1-p}{4nn}P = \left(\frac{A}{2n}=\right)B$;

Then $L.\mathfrak{P}^{ii} = L.N^{ii} - \frac{2}{2n}N + 2A - B$.

The ſolution of this queſtion, and thoſe of queſtions the 3d and 8th, are ſufficient to ſhew, that the exact values of annuities (ſecured by land) for combined lives, cannot be obtained without a calculation, much more tedious and difficult, than that of the values of ſuch annuities, not ſo ſecured: and ſince the approximations to the values of the former are rather eaſier, than thoſe to the latter, and equally near the truth, it hath been thought expedient, to give only the approximations to the ſolutions of the following queſtions.

QUESTION XII.

To approximate to the value of an annuity (ſecured by land) for the longeſt of two lives, whoſe complements are n, the leſſer, and m the greater?

SOLUTION.

By queſt. 108. vol. 2. the expectation of the longeſt of thoſe lives is $\frac{m}{2} + \frac{nn}{6m}$; and the ſum of the values of the annuities, correſponding to thoſe expectations, is $\mathfrak{M} + \mathfrak{N} - \frac{n}{6r} \times \frac{n}{2m}$; the value of the annuity required.

EXAMPLE.

What is the value of an annuity (ſecured by land) for the longeſt of two lives, of the ages 54 and 66, allowing compound intereſt at 4 per cent?

By exam. 2. queſt. 1. $\mathfrak{M} = 10{,}757$;

By exam. 3. queſt. 4. $\mathfrak{N} - \frac{n}{6r} \times \frac{n}{2m} = 1{,}397$;

Therefore, 12,154,
will be the value required.

Or (according to the rule in vol. 2. page 234)
To the value of the life of 54, *viz.* $\mathfrak{M}$ = 10,757;
Add the value of the life of 66, *viz.* $\mathfrak{N}$ = 7.674;
And, from the ſum 18,431,
Take the value of the joint lives (ex. 3. queſt. 4.) 6.277;
Remains the value required 12,154.

COROL.

When the two lives are equal, then $\mathfrak{M} = \mathfrak{N}$, and $m = n$; whence the value of the annuity required will become $(\mathfrak{N} + \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{1}{2}$ or$)$ $3\,\overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{1}{2}$.

QUESTION XIII.

A and *B* who are poſſeſſed of an annuity (ſecured by land) for the longeſt of their two lives, propoſe to ſell the ſame to *C*, for a number of years, leſs than the complement of either of their lives; on condition, that if both, or either of them, ſurvive that period, the annuity ſhall revert to them again; required the value of *C*'s purchaſe?

SOLUTION.

Let the complements of the lives of the two annuitants be repreſented by t and m; and the time for which the annuity is ſold by n; then

From the ſum of the values of annuities for n years certain, on the ſingle lives, viz. $\quad {}^{n}\mathfrak{T} + {}^{n}\mathfrak{M}$

Take the value of an annuity, for n years certain, on their joint lives (queſt. 7.) $\quad {}^{n}\mathfrak{M} - {}^{n}\overline{\mathfrak{M} - \frac{nn}{6rm}} \times \frac{n}{2t}$

The remainder, viz. $\quad {}^{n}\mathfrak{T} + \overline{{}^{n}\mathfrak{M} - \frac{nn}{6rm}} \times \frac{n}{2t}$,

will be the value of *C*'s intereſt in the annuity.

EX.

EXAMPLE.

What is the value of an annuity (ſecured by land) for twenty years certain, if either of two perſons, of the age 43 and 54, ſhall live ſo long?

Here ${}^{n}\mathfrak{F}$ is by queſtion 2 10,838;

And $\overline{{}^{n}\mathfrak{M} - \frac{nn}{6rm}} \times \frac{n}{2t}$ (is by queſt. 7,) 1,835;

Therefore, 12,673;

will be the value required.

QUESTION XIV.

To approximate to the value of an annuity (ſecured by land) for the longeſt of three lives, whoſe complements are n, m, and t.

SOLUTION.

By queſt. 109. vol. 2. the expectation of thoſe lives i $\frac{t}{2} + \frac{n:m}{6t} + \frac{n^3}{12mt}$; and conſequently the annuity required will be worth $(\mathfrak{F} + \overline{\mathfrak{M} - \frac{m}{6r}} \times \frac{m}{2t} +$

$\overline{(\mathfrak{N} - \frac{n}{6r}} \times \frac{nn}{4m}$

or) $\overline{\mathfrak{F} + \overline{\mathfrak{M} - \frac{m}{6r}} \times m + \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{nn}{2m}} \times \frac{1}{2t}$.

EXAMPLE.

What is the value of an annuity (ſecured by land) f the longeſt of three lives, whoſe ages are 43, 54, a 66, allowing compound intereſt at 4 per cent?

Here (by queſt. 11) $\left.\begin{matrix}\\ \mathfrak{F} = 12{,}920;\end{matrix}\right\}$ $\mathfrak{M} = 10{,}757;$ and $\mathfrak{N} = 7{,}673;$

$\left.\begin{matrix} t = 43;\ m = 32; \\ n = 20;\end{matrix}\right\}$ $\frac{m}{6r} = 5{,}128;$ and $\frac{n}{6r} = 3{,}205:$

Remr. $5{,}629;$ $4{,}468;$

Then $5{,}629 \times 32 = 180{,}128;$ and $4{,}468 \times \frac{400}{64}$ $(= 27{,}925;$

Alſo $\frac{180{,}128 + 27{,}925}{86} = \left(\frac{208{,}053}{86} =\right) 2{,}419:$

Laſtly $(12{,}920 + 2{,}419 =)$ 15,339 is the value required.

Or (by proceeding in the manner of the rule in queſt. 80. vol. 2. page 244.)

To the value of the life of	43	queſt. 1.	12,920,	
Add that of	54		10,757.	
And that of	66		7,673,	
Alſo the value of the three joint lives (queſt. 9.)			5,562.	
And from their ſum,				36,912;
Take the ſum of the values of the joint lives of	43 and 54	queſt. 4.	8,662	21,572;
	43 and 66		6,634	
	54 and 66		6,276	
Remains the anſwer				15,340

COROL. I.

If the two younger lives are equal, then $\mathfrak{F} = \mathfrak{M}$, and $t = m$; whence the above value will become

$$\left(\mathfrak{M} + \overline{\mathfrak{M} - \frac{m}{6r}} \times \frac{m}{2m} + \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{nn}{4mm} \text{ or}\right)$$

$$\mathfrak{M} + \overline{\mathfrak{M} - \frac{m}{6r}} \times \tfrac{1}{2} + \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{nn}{4mm}.$$

And the value of an annuity (ſecured by land) to continue as long as either of three perſons (two of whom are aged 54, and the other 66) ſhall be alive, allowing compound intereſt at 4 per cent. will be 14,007.

COROL. II.

If the two elder lives are equal, then $\mathfrak{M}=\mathfrak{N}$, and $m=n$; whence the value will be

$$(\overline{\mathfrak{T}+\mathfrak{N}-\frac{n}{6r}}\times\frac{n}{2t}+\overline{\mathfrak{N}-\frac{n}{6r}}\times\frac{nn}{4nt}\text{ or})$$

$$\overline{\mathfrak{T}+\mathfrak{N}-\frac{n}{6r}}\times\overline{\frac{n}{2t}+\frac{n}{4t}};\text{ that is }\overline{\mathfrak{T}+\mathfrak{N}-\frac{n}{6r}}\times\frac{3n}{4t};$$

whence the value of an annuity (ſecured by land) to continue as long as either of three perſons (two of whom are aged 66 and the other 43) ſhall be alive, allowing compound intereſt at 4 per cent. will be 14,478.

COROL. III.

If the three lives are of equal ages, then $\mathfrak{T}=\mathfrak{M}=\mathfrak{N}$, and $t=m=n$; whence the value of an annuity on the longeſt of them will be

$$\overline{\mathfrak{N}+\mathfrak{N}-\frac{n}{6r}}\times\frac{n}{2n}+\overline{\mathfrak{N}-\frac{n}{6r}}\times\frac{nn}{4nn},\text{ or}$$

$$(\overline{\mathfrak{N}+\mathfrak{N}-\frac{n}{6r}}\times\overline{\frac{1}{2}+\frac{1}{4}}=)\ \overline{\mathfrak{N}+\mathfrak{N}-\frac{n}{6r}}\times\frac{3}{4}:$$

Therefore the value of an annuity (ſecured by land) for the longeſt of three lives, each aged 66, allowing compound intereſt at 4 per cent. will be 11,024.

SCHOLIUM.

The value of an annuity (ſecured by land) to continue as long as any two, of three perſons of given ages, are alive, may (by ſchol. 2. queſt. 85. vol. 2.) be found as follows.

Let n, m, t, repreſent the complements, and $\mathfrak{N}$, $\mathfrak{M}$, $\mathfrak{T}$, the values of the lives of the eldeſt, ſecond, and youngeſt perſon; $\overline{\mathfrak{N}\mathfrak{M}}$, $\overline{\mathfrak{N}\mathfrak{T}}$. $\overline{\mathfrak{M}\mathfrak{T}}$. the values of their joint lives, taken two and two; and $\overline{\mathfrak{N}\mathfrak{M}\mathfrak{T}}$ the value of their three joint lives. Then

$$\text{Then } +\overline{\mathfrak{M}\mathfrak{F}} = \mathfrak{M} - \overline{\mathfrak{M} - \frac{m}{6r}} \times \frac{m}{2t},$$

$$+\overline{\mathfrak{N}\mathfrak{F}} = \mathfrak{N} - \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{n}{2t},$$

$$+\overline{\mathfrak{N}\mathfrak{M}} = \mathfrak{N} - \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{n}{2m},$$

$$-2\times\overline{\mathfrak{N}\mathfrak{M}\mathfrak{F}} = -2\mathfrak{N} + \overline{\mathfrak{N} - \frac{n}{6r}} \times \overline{\frac{n}{m} + \frac{n}{t} - \frac{nn}{2mt}};$$

And the reſult will be

$$(\mathfrak{M} - \overline{\mathfrak{M} - \frac{m}{6r}} \times \frac{m}{2t} + \overline{N - \frac{n}{6r}} \times \overline{\frac{n}{2m} + \frac{n}{2t} - \frac{nn}{2mt}},$$

$$)\,\mathfrak{M} - \frac{1}{2t} \times \overline{\mathfrak{M} - \frac{m}{6r}} \times m - \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{\overline{t+m-n}\times n}{m};$$

ill be the value required.

EXAMPLE III.

What is the value of an annuity (ſecured by land) to ntinue as long as any two, of three perſons, of the ages , 54, and 43, ſhall be alive?

Here $n = 20$; $m = 32$; $t = 43$; $\mathfrak{M} = 10,757$; $\mathfrak{M} - \frac{m}{6r} = 5,629$; $5,629 \times 32 = 180,128$; $\mathfrak{N} - \frac{n}{6r} = 4,468$; $\frac{\overline{t+m-n}\times n}{m} = \left(\frac{55.20}{32}\right) \frac{275}{8}$; $4,468 \times \frac{275}{8} = 153,875$; $180,128 - 153,587 = 26,541$; And $\frac{26,541}{2 \cdot 43} = 0,309$: therefore $(10,757 - 0,309 =)$ 10,448, will be the value required.

COROL. I.

f the two younger perſons are of equal ages; then $t = m$; and,

$$\mathfrak{M} - \frac{1}{2m} \times \overline{\overline{\mathfrak{M} - \frac{m}{6r}} \times m - \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{\overline{2m - n} \times n}{m}},$$

will be the value required.

COROL. II.

If the two elder perſons are of equal ages; then $\mathfrak{M} = \mathfrak{N}$, and $m = n$; whence,

$\mathfrak{N} + \frac{1}{2t} \times \overline{\mathfrak{N} - \frac{n}{6r}} \times \overline{t - n}$, will be the valu required.

COROL. III.

If the three perſons are of equal ages; then the valu required will be $\mathfrak{N}$.

QUESTION XV.

A, *B*, and *C*, whoſe complements of life are n, m, an t; (whereof n is the leaſt and t the greateſt) being poſſeſſed of an annuity, for the longeſt of their three live agree with *D*, to ſell him the ſame for v, a number years leſs than either of their complements; upon condition, that if all, or any of them, be alive at the expir tion of that time, the annuity ſhall revert to them; wh is the value of *D*'s purchaſe?

SOLUTION.

To $v\mathfrak{T} + v\mathfrak{M} + v\mathfrak{N}$, the values of annuities for years certain, on the three ſingle lives (by queſt. 2.)

$$v\mathfrak{N} - v\mathfrak{N} - \frac{vv}{6rn} \times \frac{2mv + 2tv - vv}{4mt}$$

$$+ \frac{vv}{6rn} \times \frac{2nv - 2vv}{4mt},$$ the value of an annuity for years certain on the three joint lives (by queſt. 10); from the ſum ($v\mathfrak{T} + v\mathfrak{M} + 2 \times v\mathfrak{N} -$

$$\overline{v\mathfrak{N} - \frac{vv}{6rn}} \times \frac{2mv + 2tv - vv}{4mt} + \frac{vv}{6rn} \times \frac{2nv - 2vv}{4mt}\Big)$$

ake $v\mathfrak{M} - \overline{v\mathfrak{M} - \frac{vv}{6rm}} \times \frac{v}{2t}$, $v\mathfrak{N} - \overline{v\mathfrak{N} - \frac{vv}{6rn}} \times \frac{v}{2t}$,

d $v\mathfrak{N} - \overline{v\mathfrak{N} - \frac{vv}{6rn}} \times \frac{v}{2m}$, the values of annuies for v years certain, for the two joint lives, whoſe mplements are ſeverally *m*, *t*; *n*, *t*; and *n*, *m*; the mainder, viz.

$$v\mathfrak{F} + \overline{v\mathfrak{M} - \frac{vv}{6rm}} \times \frac{v}{2t} - \overline{v\mathfrak{N} - \frac{vv}{6rn}} \times \frac{2mv + 2tv - vv}{4mt}$$

$$\overline{\frac{vv}{6rn} \times \frac{2nv - 2vv}{4mt}} + \overline{v\mathfrak{N} - \frac{vv}{6rn}} \times \frac{v}{2t}$$

$\overline{v\mathfrak{N} - \frac{vv}{6rn}} \times \frac{v}{2m}$, will be the value required.

But $\frac{v}{2t} + \frac{v}{2m} = \frac{2mv + 2tv}{4mt}$, whence the above ill be reduced to $v\mathfrak{F} + \overline{v\mathfrak{M} - \frac{vv}{6rm}} \times \frac{v}{2t}$

$$\overline{v\mathfrak{N} - \frac{vv}{6rn}} \times \frac{vv}{4mt} + \frac{vv}{6rn} \times \frac{2nv - 2vv}{4mt} \text{ or) } v\mathfrak{F}$$

$$\overline{v\mathfrak{M} + \frac{v}{2m} \times v\mathfrak{N} -} \frac{vv}{6rm} \times \frac{v}{2t} + \frac{vv}{6rn} \times \frac{2nv - 3vv}{4mt};$$

hat is $v\mathfrak{F}$

$$\overline{v\mathfrak{M} + \frac{v}{2m} \times v\mathfrak{N} - \frac{vv}{6rm} + \frac{vv}{6rn} \times \frac{2n - 3v}{2m}} \times \frac{v}{2t}.$$

QUESTION XVI.

It is required to find the ſum of *n* terms of the ſeries $\frac{2m-1}{2m} \times \frac{2t-1}{2t} + \frac{2m-3}{2m} \times \frac{2t-3}{2t} + \frac{2m-5}{2m} \times \frac{2t-5}{2t}$ c.?

SOLU-

SOLUTION.

By queſt. 106. vol. 2. the ſum of the ſeries $\frac{2m-1}{2m}$ + $\frac{2m-3}{2m}+\frac{2m-5}{2m}$ (n) is $\overline{1-\frac{n}{2m}}\times n$, and the ſum $\frac{2t-1}{2t}+\frac{2t-3}{2t}+\frac{2t-5}{2t}$ (n) is $\overline{1-\frac{n}{2t}}\times n$; the common differences being ſeverally $\frac{1}{m}$, and $\frac{1}{t}$; therefo (by queſt. 21. vol. 2.) the ſum of the required ſeries will

$$\left(\frac{\overline{1-\frac{n}{2m}}\times n\times\overline{1-\frac{n}{2t}}\times n}{n}+\frac{\overline{n+1}.n.\overline{n-1}}{2.2.3}\times\frac{1}{m}\times\right.$$

or) $\overline{1-\frac{n}{2m}}\times\overline{1-\frac{n}{2t}}\times n+\frac{\overline{n+1}.n.\overline{n-1}}{3.4mt}$

That is $\overline{\overline{1-\frac{n}{2m}}\times\overline{1-\frac{n}{2t}}+\frac{\overline{n+1}.\overline{n-1}}{3.4mt}}\times$

But $\overline{1-\frac{n}{2m}}\times\overline{1-\frac{n}{2t}}=1-\frac{n}{2m}-\frac{n}{2t}+\frac{nn}{4mt}$

And (if we write $\frac{nn}{3.4mt}$ for $\frac{\overline{n+1}.\overline{n-1}}{3.4mt}$) then th above expreſſion will become

$$\overline{1-\frac{n}{2m}-\frac{n}{2t}+\frac{nn}{4mt}+\frac{nn}{3.4mt}}\times n:$$

Now $\frac{nn}{4mt}+\frac{nn}{3.4mt}=\left(\overline{\frac{1}{4}+\frac{1}{12}}\times\frac{nn}{mt}=\right)\frac{1}{3}\times\frac{nn}{mt}$

And, therefore, the ſum required will be

$$\overline{1-\frac{n}{2m}-\frac{n}{2t}+\frac{nn}{3mt}}\times n.$$

QUESTIO

QUESTION XVII.

It is required to find the ſum of v terms of the ſeries $\frac{2n-1}{2n}\times\frac{2m-1}{2m}\times\frac{2t-1}{2t}+\frac{2n-3}{2n}\times\frac{2m-3}{2m}\times\frac{2t-3}{2t}$ &c.?

SOLUTION.

By queſt. 106. vol. 2. the ſum of v terms of the ſeveral component arithmetical progreſſions are as follows;

$$\left.\begin{array}{l}\frac{2t-1}{2t}+\frac{2t-3}{2t}\ (v)=\overline{1-\frac{v}{2t}}\times v;\\ \frac{2m-1}{2}+\frac{2m-3}{2m}\ (v)=\overline{1-\frac{v}{2m}}\times v;\\ \frac{2n-1}{2n}+\frac{2n-3}{2n}\ (v)=\overline{1-\frac{v}{2n}}\times v;\end{array}\right\}\begin{array}{c}\text{their common dif-}\\ \text{ferences being}\end{array}\left\{\begin{array}{l}\frac{1}{t};\\ \frac{1}{m};\\ \frac{1}{n}:\end{array}\right.$$

Therefore by queſt. 22. vol. 2. their ſum will be

$$\left(\frac{\overline{1-\frac{v}{2t}}\times v\times\overline{1-\frac{v}{2m}}\times v\times\overline{1-\frac{v}{2n}}\times v}{vv}\right. +$$

$v+1$

$$\frac{\overline{v+1}\,.\,v\,.\,\overline{v-1}}{2\,.\,2\,.\,3}\times\overline{\frac{2t-1}{2t}\times\frac{1}{m}\times\frac{1}{n}+\frac{2m-1}{2m}\times\frac{1}{t}\times\frac{1}{n}+\frac{2n-1}{2n}\times\frac{1}{t}\times\frac{1}{m}}$$

$$-\frac{\overline{v+1}\,.\,v\,.\,\overline{v-1}^{2}}{2\,.\,2\,.\,2}\times\frac{1}{t}\times\frac{1}{m}\times\frac{1}{n}.\ \text{or)}$$

$$\overline{1-\frac{v}{2t}}\times\overline{1-\frac{v}{2m}}\times\overline{1-\frac{v}{2n}}\times v+\frac{\overline{v+1}\,.v.\,\overline{v-1}}{2\,.\,2\,.\,3}\times\frac{2\times\overline{t+m+n}-3}{2tmn}-\frac{\overline{v+1}\,.v.\,\overline{v-1}^{2}}{2.\,2\,.\,2tmn};$$

That is (by writing vv for $\overline{v+1}\times\overline{v-1}$)

$$\overline{1-\frac{v}{2t}}\times\overline{1-\frac{v}{2m}}\times\overline{1-\frac{v}{2n}}\times v+\frac{v^3}{3}\times\frac{2\times\overline{t+m+n}-3}{8tmn}-\frac{v^3\times\overline{v-1}}{8tmn}:$$

But $\overline{1-\frac{v}{2t}}\times\overline{1-\frac{v}{2m}}\times\overline{1-\frac{v}{2n}}\times v$ will, if multiplied, become

$$v-\frac{v^2}{2t}-\frac{v^2}{2m}-\frac{v^2}{2n}+\frac{vvv}{4tm}+\frac{vvv}{4tn}+\frac{vvv}{4mn}-\frac{v^4}{8tmn};$$

And $\frac{v^3}{3}\times\frac{2t+2m+2n-3}{8tmn}$ will become $\Big(\frac{2tv^3}{3\,.\,8tmn}+\frac{2mv^3}{3\,.\,8tmn}+\frac{2nv^3}{3\,.\,8tmn}$

$-\frac{3v^3}{3\,.\,8tmn}$ or) $\frac{v^3}{3\,.\,4mn}+\frac{v^3}{3\,.\,4tn}+\frac{v^3}{3\,.\,4tm}-\frac{v^3}{8\,.\,tmn};$

Alſ

Alſo $-\frac{v^3 \times \overline{v-1}}{8tmn} = -\frac{v^4}{8tmn} + \frac{v^3}{8tmn}$

Let now theſe be properly ranged under each other and added as below

$$v - \frac{v^2}{2t} - \frac{v^2}{2m} - \frac{v^2}{2n} + \frac{v^3}{4tm} + \frac{v^3}{4tn} + \frac{v^3}{4mn} \qquad - \frac{v^4}{8tmn};$$

$$+ \frac{v^3}{3 \cdot 4tm} + \frac{v^3}{3 \cdot 4tn} + \frac{v^3}{3 \cdot 4mn} - \frac{v^3}{8tmn} \qquad ;$$

$$+ \frac{v^3}{8tmn} \qquad - \frac{v^4}{8tmn}.$$

The ſum $\left(v - \frac{v^2}{2t} - \frac{v^2}{2m} - \frac{v^2}{2n} + \frac{v^3}{3tm} + \frac{v^3}{3tn} + \frac{v^3}{3mn} - \frac{v^4}{4tmn}\right.$

Or) $\overline{1 - \frac{v}{2t} - \frac{v}{2m} - \frac{v}{2n} + \frac{vv}{3tm} + \frac{vv}{3tn} + \frac{vv}{3mn} - \frac{v^3}{4tmn}} \times v$ will be the value required.

QUESTION

QUESTION XVIII.

The complement, *n*, of a life being known; to find th probability of that life's failing, in any like portion o time, during the poſſibility of its exiſtence.

SOLUTION.

Since the probability of the given life's continuing th firſt year is $\frac{n-1}{n}$; therefore the probability of its failin in that time will be $\left(1-\frac{n-1}{n}=\frac{n-n+1}{n}=\right)\frac{1}{n}$

When the firſt year is expired, then the complement o the life becomes $n-1$; the probability of its continuanc another year will be $\left(\frac{n-1-1}{n-1}=\right)\frac{n-2}{n-1}$; and that o its failing in that year $\left(1-\frac{n-2}{n-1}=\frac{n-1-n+2}{n-1}=\right)\frac{1}{n-1}$: But if th probability is to be found at the beginning of the firſt yea then $\frac{n-1}{n}$ the probability of the life's continuing the fi year muſt be multiplied into $\frac{1}{n-1}$ the above found probability; becauſe the life's failing, in the ſecond year, depends on its having ſurvived the firſt; and therefore $\left(\frac{n-1}{n}\times\frac{1}{n-1}=\right)\frac{1}{n}$ will be the probability of the lif failing in the ſecond year.

In like manner $\left(\frac{n-2}{n}\times\frac{1}{n-2}=\right)\frac{1}{n}$ will be th probability of its failing in the third year, &c.

By reaſoning in the ſame manner, the probability o the given life's failing, in any half year, will be $\frac{1}{2n}$; in any month, $\frac{1}{12n}$; or in any day $\frac{1}{365n}$.

QUESTIO

QUESTION XIX.

The probability p, that the lives of two perſons, of different ages, will be both extinct, in a given ſpace of time, being known; to find the probability that either of them will, within that ſpace of time, die before the other?

SOLUTION.

Let the complement of the younger life be m; and hat of the elder, n.

Since the probability of the younger life's failing, in ny year, month, or day, is $\frac{1}{m}$, $\frac{1}{12m}$, or $\frac{1}{365m}$; and hat of the elder life's failing, in the like times, is $\frac{1}{n}$, $\frac{1}{12n}$, or $\frac{1}{365n}$, by queſt. 18. Therefore, in any uch interval, the probability of the failing of the younger life, is to the probability of the failing of the elder fe; as $\frac{1}{m}$, to $\frac{1}{n}$; or, as n to m.

Let now the probability of the younger life's failing firſt e denoted by y; and that of the elder, by x.

Then, as $n : m :: y : x$; Therefore $x = \frac{my}{n}$, and $y = \frac{nx}{m}$.

But $x + y = p$, by the queſtion.

Therefore $x + \frac{nx}{m} = p$; and $y + \frac{my}{n} = p$;

And $mx + nx = mp$; and $ny + my = np$;

Whence $x = \frac{mp}{m+n}$: and $y = \frac{np}{n+m}$.

COROL.

If the two lives are of equal ages, then $m = n$: and x $= \left(\frac{np}{n+n} = \frac{np}{2n} = \right)\frac{p}{2}$; alſo $y = \left(\frac{np}{n+n} = \right)\frac{p}{2}$,

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QUESTION XX.

There is an eſtate of 1 £. per annum, uſually let on a leaſe for two lives; of which one is dropped: what fine ought to be paid to the leſſor, for filling up the leaſe?

OR,

A is poſſeſſed of an annuity (ſecured by land) which, upon his deceaſe, is to deſcend to *B*, if he be then living, for his life only; what is the value of *B*'s intereſt in that annuity?

OR,

What is the value of the reverſion of an annuity (ſe-cured by land) for one life, after one?

CASE I.

When the expectant is elder than the poſſeſſor.

Then (putting n and m for the complements, and $\mathfrak{N}$ and $\mathfrak{M}$ for the values of the lives of the expectant and poſſeſ-ſor) by arguing as in queſt. 94. vol. 2.

If from the value of the expectant's life	$\mathfrak{N}$,
We take the value of the joint lives of both poſſeſſor and expectant	$\mathfrak{N} - \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{n}{2m}$;
The remainder will be the value of the reverſion	$+\overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{n}{2m}$.

E X

EXAMPLE I.

If the expectant be 66, and the poſſeſſor 54, then (by exam. 3, queſt. 4.) $\mathfrak{N} - \frac{n}{6r} \times \frac{n}{2m} = 1,397$ is the value required.

EXAMPLE II.

If the expectant be 66, and the poſſeſſor 43: then (by exam. 2. queſt. 4.) 1,039 will be the value required.

EXAMPLE III.

If the expectant be 54 and the poſſeſſor 43: then (by exam. 1. queſt. 4.) 2,095 will be the value required.

CASE II.

When the poſſeſſor is elder than the expectant.

Then put m, n, for the complements, and $\mathfrak{M}$, $\mathfrak{N}$, for the values of the lives of the expectant and poſſeſſor; and

From the value of the expectant's life } $\mathfrak{M}$,

Take the value of the joint lives } $\mathfrak{N} - \overline{\mathfrak{N} - \frac{n}{6r} \times \frac{n}{2m}}$

And the remainder, } $\mathfrak{M} - \mathfrak{N} + \overline{\mathfrak{N} - \frac{n}{6r} \times \frac{n}{2m}}$, will be the value of the reverſion required.

EXAMPLE I.

If the expectant be 54, and the poſſeſſor 66.

Then $\mathfrak{M} = 10,757$; $\mathfrak{N} = 7,673$; and $\mathfrak{N} - \frac{n}{6r} \times \frac{n}{2m} = 1,397$;

Therefore (10,757—7,673+1,397=) 4,481 will be the value required.

EXAMPLE II.

If the expectant be 43, and the poſſeſſor 66.

Then $\mathfrak{M}=12{,}920$; $\mathfrak{P}=7{,}673$; and $\mathfrak{P}-\frac{n}{6r}\times\frac{n}{2m}=1{,}039$;

Therefore (12,920—7,673+1,039=) 6,286 will be the value required.

EXAMPLE III.

If the expectant be 43, and the poſſeſſor 54. then $\mathfrak{M}=12{,}920$; $\mathfrak{P}=10{,}757$; and $\mathfrak{P}-\frac{n}{6r}\times\frac{n}{2m}=2{,}095$;

Therefore (12,920—10,757+2,095=)4,258 will be the value required.

CASE III.

If the expectant and poſſeſſor be of the ſame age.

Then (by writing n for m in the firſt caſe) we ſhall have $\left(\mathfrak{P}-\frac{n}{6r}\times\frac{n}{2n}\text{ or}\right)\mathfrak{P}-\frac{n}{6r}\times\frac{1}{2}$ for the value required.

EXAMPLE.

If both expectant and poſſeſſor be aged 66;

Then $\mathfrak{P}-\frac{n}{6r}=4{,}468$ (by exam. 2. queſt. 4.) an $\left(\frac{4.468}{2}=\right)$ 2,234 will be the value required.

QUESTION XXI.

The reſpective ages of two perſons, *B* the elder, and *A* the younger, being given; it is required to find the probability, that *B* has of ſurviving *A*.

SOLUTION.

Let the complements of the lives of *B* and *A*, be (ſeverally) denoted by n and m: then the terms of the ſeries $\frac{2n-1}{2n}$, $\frac{2n-3}{2n}$, $\frac{2n-5}{2n}$, &c. will be the ſeveral expectations of the life of *B*, for the firſt, ſecond, third, &c. years; or the probabilities of the continuance of that life, to the expiration of the whole, or at leaſt of the half of each of thoſe years (by queſt. 105. vol. 2.) Alſo $\frac{1}{m}$ will be the probability of the failing of the life of *A*, in the firſt year; that is, of the deceaſe of *A*, either in the firſt, or in the ſecond, half of that year.

Now, if the expectation of *B*'s life, for the firſt year (*viz.* $\frac{2n-1}{2n}$) be multiplied by $\frac{1}{m}$, the probability of *A*'s dying in that year; then the product, $\frac{2n-1}{2n} \times \frac{1}{m}$, will be the probability of the ſurvivorſhip's taking place in that year: for if *A* be ſuppoſed to die in the firſt half year, then the expectation, $\frac{2n-1}{2n}$, includes the probability of *B*'s living, at leaſt, to the end of that half year, and conſequently of his ſurviving *A* for that time; and if *A* be ſuppoſed not to die until the ſecond half year, then the ſame expectation includes the probability of *B*'s living to the end of the year, and (by that means) of his ſurviving *A*, for that time alſo.

If it were certain that the ſurvivorſhip would take place in the firſt year, then this calculation would proceed no farther; but, ſince both the perſons may outlive the firſt year, we muſt proceed to find the probability of its taking place in the ſecond: now the expectation of *B*'s life, for the ſecond year, is $\frac{2n-3}{2n}$ and the probability that *A*, having ſurvived the firſt year, ſhall die in the ſecond is (by queſt. 18.) $\frac{1}{m}$; conſequently their product, $\frac{2n-3}{2n} \times \frac{1}{m}$, will (for the ſame reaſons as before) be the probability of the ſurvivorſhip's taking place in the ſecond year.

And, if we continue to argue in the ſame manner, it will appear that $\frac{2n-5}{2n} \times \frac{1}{m}$, $\frac{2n-7}{2n} \times \frac{1}{m}$, &c. will be the probabilities of the ſurvivorſhip's taking place, in the third, fourth, &c. years; and conſequently, that

$$\left(\frac{2n-1}{2n} \times \frac{1}{m} + \frac{2n-3}{2n} \times \frac{1}{m} + \frac{2n-5}{2n} \times \frac{1}{m}\,(n) \text{ or}\right) \frac{1}{m} \times \overline{\frac{2n-1}{2n} + \frac{2n-3}{2n} + \frac{2n-5}{2n}}\,(n)$$

will be the whole probability of *B*'s ſurviving *A*.

But ſince (by queſt. 105. vol. 2.) the ſum of the ſeries $\frac{2n-1}{2n} + \frac{2n-3}{2n} + \frac{2n-5}{2n}\,(n)$ is $\frac{n}{2}$; and, ſince every term of that ſeries is above multiplied by the conſtant factor, $\frac{1}{m}$; it follows, that $\left(\frac{n}{2} \times \frac{1}{m} =\right) \frac{n}{2m}$ will be the whole probability of *B*'s ſurviving *A*.

QUESTION XXII.

The ſame lives being propoſed, as in queſt. 21. it is required to find the probability, that *A*, the younger, has of ſurviving *B*, the elder?

If we aſſume *m* and *n* for their complements of life, then (arguing as before) the terms of the ſeries $\frac{2m-1}{2m}$, $\frac{2m-3}{2m}$, $\frac{2m-5}{2m}$, &c. which expreſs the yearly expectations of *A*'s life, are to be ſeverally multiplied by $\frac{1}{n}$, the yearly probability of *B*'s dying: but ſince that probability does (by the hypotheſis) become a certainty, at the expiration of *n* years; it follows, that *n* terms only of the ſeries reſulting (viz. $\frac{2m-1}{2mn} + \frac{2m-3}{2mn} + \frac{2m-5}{2mn}$) will exhibit the value required.

Now, (by queſt. 106. vol. 2.) the ſum of *n* terms of the ſeries, $\frac{2m-1}{2m} + \frac{2m-3}{2m} + \frac{2m-5}{2m}$ &c. is $\overline{1-\frac{n}{2m}} \times n$; which ſum, being multiplied by the conſtant factor. $\frac{1}{n}$, produces $1-\frac{n}{2m}$ for the probability required.

SCHOLIUM.

Here it may be remarked, that the ſum of the two probabilities of ſurvivorſhip above found, *viz.* $\frac{n}{2m}$, and $1-\frac{n}{2m}$, is unity; for it may be eſteemed a certainty, that one of the two perſons will ſurvive the other.

COROL.

Hence, if the two lives propoſed be of equal ages, then their probabilities of ſurvivorſhip will be equal, *viz.* $\left(\frac{n}{2n}\text{ or}\right)\ \frac{1}{2}$,

QUESTION XXIII.

A has left a legacy of (or an eſtate which will ſell for) $\mathfrak{P}$ pounds, to *B*, if he be alive at the time of *A*'s deceaſe; but if not, then the heirs or aſſigns of *B* are not to receive the benefit thereof; what is the preſent worth of *B*'s intereſt, in that legacy or eſtate?

CASE I.

If the expectant be elder than the poſſeſſor;

Let n be the complement of the expectant's life, and m that of the poſſeſſor; then (by queſt. 21.) the probability of his receiving the legacy in the firſt, ſecond, third, &c. years, will be, $\frac{2n-1}{2nm}$, $\frac{2n-3}{2nm}$, $\frac{2n-5}{2nm}$, &c. and conſequently, the preſent worths of the ſum $\mathfrak{P}$, ſo to be received, will be $\frac{2n-1}{2nmr}\mathfrak{P}$, $\frac{2n-3}{2nmr^2}\mathfrak{P}$, $\frac{2n-5}{2nmr^3}\mathfrak{P}$, &c. in each of which, $\frac{\mathfrak{P}}{m}$, is a conſtant factor; and therefore

$$\left(\frac{\mathfrak{P}}{m} \times \overline{\frac{2n-1}{2nr} + \frac{2n-3}{2nr^2} + \frac{2n-5}{2nr^3}}(n) = \right) \frac{\mathfrak{P}\mathfrak{N}}{m},$$

will be the preſent worth of *B*'s intereſt in the eſtate.

EXAMPLE I.

Suppoſe *B*, aged 66, is to receive a legacy of twenty five pounds, or an eſtate in fee ſimple of 1£ per ann. if he ſurvives *A*, who is 54 years of age: what is that expectation worth, allowing compound intereſt at 4 per cent?

Here $\mathfrak{P} = 25$; $\mathfrak{N} = 7{,}673$; and $m = 32$;

Th. $\left(\frac{7{,}673 \times 25}{32} = \right)$ 5·995£. will be the value required.

Note, The ſymbol $\mathfrak{P}$, in the above ſolution mentioned, denotes the legacy, or the preſent worth of the eſtate in fee ſimple (of what annual rent ſoever) ſuppoſed to devolve to the expectant on the death of the poſſeſſor: whereas *P*, the ſymbol (in this work) uſed before this queſtion, denotes the preſent worth of an eſtate of only 1£ per ann. Therefore theſe two values will coincide only in ſuch a caſe, as is propoſed in the above example.

EXAMPLE II.

If *B*, aged 66, is to receive the aforeſaid legacy, or eſtate, if he ſurvives *A*, aged 43.

Then $\mathfrak{P}=25$; $\mathfrak{N}=7{,}673$; and $m=(86-43=)\ 43$.

Whence $\left(\frac{7{,}673\times25}{43}=\right)$ 4,461£. the value required.

EXAMPLE III.

If *B*, aged 54, is to receive the ſame, if he ſurvives *A*, aged 43.

Then $\mathfrak{P}=25$; $\mathfrak{N}=10{,}757$; and $m=43$:

Whence $\left(\frac{10{,}757\times25}{43}=\right)$ 6,254£. is the value required.

CASE II.

If the expectant be younger than the poſſeſſor; let *n* e the complement of the poſſeſſor's life, and *m* that of he expectant.

Then, by comparing together the arguments in queſt. 2. and in the firſt caſe of this, it will appear, that

$$\frac{\mathfrak{P}}{n}\times\overline{\frac{2m-1}{2mr}+\frac{2m-3}{2mr^2}+\frac{2m-5}{2mr^3}(n)},$$ will be the preent value requierd.

But $\frac{2m-1}{2mr}+\frac{2m-3}{2mr^2}+\frac{2m-5}{2mr^3}(n)$ is (by queſt. 2.)

equal to $(\mathfrak{M} - \frac{P}{m} \times \overline{\frac{m-n}{r^n} - \frac{1}{a} \times \frac{1}{r^n} - \frac{1}{r^n}}$

Or) ${}^n\mathfrak{M}$: Therefore $\frac{\mathfrak{P}}{n} \times {}^n\mathfrak{M}$ will be the value required.

EXAMPLE I.

Suppoſe *B*, aged 54, is to receive a legacy of 25£. or an eſtate is fee ſimple of 1£. per ann. if he ſurvives *A*, who is 66 years of age; what is that expectation worth, allowing compound intereſt at 4 per cent?

Here $\mathfrak{P} = 25$; ${}^n\mathfrak{M} = 9{,}891$ (by queſt. 2.) and $n = 205$.

Therefore $\left(\frac{25}{20} \times 9{,}891 = \right)$ 12,364 will be the value required.

EXAMPLE II.

Let *B*, aged 43, be to receive the ſame legacy or eſtate if he ſurvives *A*, aged 66.

Here $\mathfrak{P} = 25$; ${}^n\mathfrak{M} = 10{,}838$; and $n = 20$;

Th. $\left(\frac{10{,}838 \times 25}{20} = \right)$ 13,547£. will be the value required.

EXAMPLE III.

If *B*, aged 43, is to receive the ſame, if he ſurvives *A*, aged 54.

Here $\mathfrak{P} = 25$; ${}^n\mathfrak{M} = 12{,}578$; and $n = 32$

Th. $\left(\frac{12{,}578 \times 25}{32} = \right)$ 9,827£. will be the value required.

SCHOLIUM.

By queſtion 22. the ſum of the probabilities of the younger life's ſurviving the elder; *viz.* the ſum of the ſeries $\frac{2m-1}{2mn}+\frac{2m-3}{2mn}+\frac{2m-5}{2mn}\,(n)$ appeared to be $1-\frac{n}{2m}$, and (by caſe 2. of this) the ſum of the ſeries $\frac{2m-1}{2mr}+\frac{2m-3}{2mr^2}+\frac{2m-5}{2mr^3}\,(n)$ is ${}^{n}\mathfrak{M}$: therefore, in all caſes, wherein the ſum of the probabilities of ſurvivorſhip is denoted by the expreſſion, $1-\frac{n}{2m}$, or one ſimilar thereto, we may write, $\frac{{}^{n}\mathfrak{M}}{n}$, for the preſent value of 1£. dependent on that probability.

CASE III.

If the poſſeſſor and expectant are of equal ages.

Then (putting n for their common complement) the value required will be $\overline{\frac{2n-1}{2nr}+\frac{2n-3}{2nr^2}+\frac{2n-5}{2nr^3}\,(n)}\times\frac{\mathfrak{P}}{n}$

Or $\frac{\mathfrak{M}\mathfrak{P}}{n}$.

EXAMPLE.

Suppoſe both poſſeſſor and expectant, to be aged 66, the eſtate and rate of intereſt being as before.

Then $\mathfrak{M}=7{,}673$; $\mathfrak{P}=25$; and $n=20$;

Whence $\frac{7{,}673\times 25}{20}=\big)$ 9,591 will be the value required.

COROL.

If the expectant ſhould, during the life of the poſſeſſor, want to borrow a ſum of money, on his intereſt in the eſtate or legacy, dependent on the above ſurvivorſhip; then the utmoſt that can be advanced to him thereon, will be the above found value of his intereſt therein, in which caſe he ſells his whole expectation; but if he borrows a leſſer ſum, then the 25th queſtion naturally ariſes.

SCHOLIUM.

The reader will obſerve, that there is a very conſiderable difference between the two reverſions, or ſurvivorſhips, propoſed in queſtions 20 and 23; in the former the expectant is (for the time which he ſurvives the poſſeſſor) to receive divers annual ſums; whereas, in the latter, he is to receive on the death of the poſſeſſor only one groſs ſum or legacy: or thus, in the former caſe, the expectant is (at the death of the poſſeſſor) to enjoy an annuity for the remainder of his life, which annuity will be of different values, according to the age at which he ſhall happen to come into poſſeſſion; whereas in the latter, (if he ſurvives the poſſeſſor) he and his heirs are to enjoy an annuity for ever; which perpetuity will, therefore, be of the ſame value, at any time, when it may come into his poſſeſſion.

For this reaſon, therefore, they are calculated in manners ſo different; viz. the former caſe depends on the ſolution of (queſt. 94. vol. 2.) wherein the probabilities of the poſſeſſor's dying before the expiration of one, two, 3, &c. years, are ſeverally multiplied into the probabilities of the expectant's living, to the end of thoſe years; and therefore the probabilities of receiving a payment of the annuity in the ſecond year, doth not depend on the not having received it in the firſt: but in the ſecond caſe the probability of the poſſeſſor's dying in any one year, is conſtantly multiplied into the other's expectation of life, for the firſt, ſecond, third, &c. years; which conſtant probability will (by queſt. 18.) appear, in the ſecond year, to ſuppoſe the poſſeſſor's having ſurvived the firſt; and conſequently, the probability of receiving the legacy, in the ſecond year, does (in this calculation) entirely depend on the not having received it in the firſt.

This may ſerve to account for a ſimilar difference in the manner of calculating queſtions of thoſe kinds, in the enſuing parts of this work.

QUESTION XXIV.

Things being as in the laſt queſtion, ſuppoſe that *A* has (in caſe *B* dies before him) left the legacy, or eſtate, (worth $\mathfrak{P}$ pounds) to *C* and his heirs; what is the preſent worth of their intereſt in that reverſion?

SOLUTION.

If *B* had no intereſt in the legacy, or eſtate, then *C*, or his heirs, would be entituled to the ſame, immediately on the death of *A*: if, therefore, from the value of the reverſion of the legacy or eſtate, after the life of *A*, there be taken the intereſt of *B* therein; the remainder will be the value of the expectation of *C*, or his heirs.

CASE I.

If *B*, the expectant, be elder than *A*, the poſſeſſor. Let n, m, repreſent the complements of their lives; and $\mathfrak{N}$, $\mathfrak{M}$ the values of annuities for them. Then (by queſtion 8. vol. 2.) the preſent value of 1£. to be received on the death of *A* will be $1 - \overline{r - 1} \times \mathfrak{M}$; whence the value of the legacy, or eſtate in queſtion will be $\overline{1 - \overline{r - 1} \times \mathfrak{M}} \times \mathfrak{P}$; and (by caſe 1. queſtion 23.) the preſent value of *B*'s intereſt therein is $\frac{\mathfrak{P}\mathfrak{N}}{m}$; therefore the preſent value of the intereſt of *C*, and his heirs, therein will be

$$\overline{1-\overline{r-1}\times\mathfrak{M}}\times\mathfrak{P}-\frac{\mathfrak{N}\mathfrak{P}}{m}\text{ or }\overline{1-\overline{r-1}\times\mathfrak{M}-\frac{\mathfrak{N}}{m}}\times\mathfrak{P}.$$

EXAMPLE.

Suppoſe *A* and *B* to be aged 54 and 66; intereſt at per cent. and the legacy 25£.

Then $\mathfrak{M} = 10,757$; $\mathfrak{N} = 7,673$; $m = 32$; $r -$ $= 0,04$; and $\mathfrak{P} = 25$: now $10,757 \times 0,04 = ,4302$ $\frac{7,673}{32} = ,240$;

And $1 - 0,430 - 0,240 = 0,330$; theref. (25×,0,330= 8,25 will be the value required.

CASE II.

If *B*, the expectant, be younger than *A*, the poſſeſſo Let *m* repreſent the complement of the expectant, an *n* that of the poſſeſſor.

Alſo let $\mathfrak{N}$ repreſent thevalue of an annuity for the po ſeſſor's life, and ${}^{n}\mathfrak{M}$ that of an annuity for *n* years, the expectant lives ſo long.

Then (by queſt. 89. vol. 2.) the value of $\mathfrak{P}$ pound be received at the death of the poſſeſſor, will be $\overline{1-\overline{r-1}\times\mathfrak{N}}\times\mathfrak{P}$; and (by caſe 2. queſt. 23.) value of *B*'s intereſt in the legacy or eſtate, will $\frac{\mathfrak{P}\;{}^{n}\mathfrak{M}}{n}$; therefore ($\overline{1-\overline{r-1}\times\mathfrak{N}}\times\mathfrak{P}-\frac{\mathfrak{P}\;{}^{n}\mathfrak{M}}{n}$ or)

$$\overline{1-\overline{r-1}\times\mathfrak{N}-\frac{{}^{n}\mathfrak{M}}{n}}\times\mathfrak{P}$$ will be the value quired.

EXAMPLE.

Suppoſe *A* and *B*, to be aged 66 and 54; intereſt at per cent, and the legacy 25£.

Then $\mathfrak{N} = 7,673$; ${}^{n}\mathfrak{M} = 9,891$; $n = 20$; $r -$ $= 0,04$; and $\mathfrak{P} = 25$.

Now $7{,}673 \times 0{,}04 = {,}30692$; $\frac{9{,}891}{20} = 0{,}4995$;

And $1 - 0{,}307 - 0{,}500 = 0{,}193$; Therefore ($25 \times 0{,}193 =$) 4,825 will be the value required.

CASE III.

If the expectant and poſſeſſor be of the ſame age.

Then (putting *n* for the complement and $\mathfrak{P}$ for the value of a life of that age) $\overline{1 - \overline{r - 1} \times \mathfrak{P} - \frac{\mathfrak{P}}{n}} \times \mathfrak{p}$ will be the value required.

EXAMPLE.

Suppoſe *A* and *B* to be each 66 years of age; intereſt at 4 per cent. and the legacy 25 pounds.

Then $\mathfrak{P} = 7{,}673$; $n = 20$; $r - 1 = 0{,}04$; and $\mathfrak{p} = 25$£.

Then $7{,}673 \times 0{,}04 = {,}30692$; $\frac{7{,}673}{20} = 0{,}38365$;

And $1 - 0{,}307 - 0{,}384 = 0{,}309$; Th. ($0{,}309 \times 25 =$) 7,725 will be the value required.

As the values of the reverſions of an eſtate or legacy, dependent on the failure of any of the ſurvivorſhips hereafter calculated, may be obtained, by arguing in the ſame manner as in this queſtion, no more inſtances thereof are inſerted in this work.

QUESTION XXV.

B (who is entituled to a legacy of or an eſtate worth $\mathfrak{p}$ pounds, if he ſurvives the death of *A*) would borrow a ſum of money of *C* upon the credit thereof; on condition that if he, *B*, dies before *A*, then *C* ſhall entirely loſe his money: what ſum ought *C* to receive on the death of *A*, if *B* be then living, for every pound now lent?

SOLU-

SOLUTION.

Since C is to loſe the ſum lent, unleſs B ſurvives A; it is plain, that he has purchaſed the expectation of a ſum of money, dependent on that ſurvivorſhip; the preſent values of which, in the three poſſible caſes, were computed in queſtion XXIII; if, therefore, equations be ſeverally made between unity, or one pound the preſent worth given in the queſtion, and the reſults of thoſe caſes; then the values of $\mathfrak{P}$, (the ſum receivable when the ſurvivorſhip takes place) being found, by reducing thoſe equations, will be the anſwers.

CASE I.

If the expectant be elder than the poſſeſſor.

Let m be the complement of the poſſeſſor, and $\mathfrak{R}$ the value of an annuity (ſecured by land) for the expectant's life.

Then $1 = \frac{\mathfrak{P}\mathfrak{R}}{m}$; whence $\frac{m}{\mathfrak{R}} = \mathfrak{P}$.

EXAMPLE I.

What ſum ought C to receive at the death of A, aged 54, for every pound lent, on the contingency that B, (who is 66) ſhall ſurvive A?

Here $m = 32$; and $\mathfrak{R} = 7,673$; therefore $\mathfrak{P} = \left(\frac{32}{7,673} =\right)$ 4,171 will be the ſum required.

EXAMPLE II.

If the expectant be 66, and the poſſeſſor 43 years old.

Then $m = 43$; and $\mathfrak{R} = 7,673$; therefore $\mathfrak{P} = \left(\frac{43}{7,673} =\right)$ 5,604 will be the ſum required.

EXAMPLE III.

If the expectant be 54, and the poſſeſſor 43 years old. Then $m = (86 - 43 =)$ 43; and $\mathfrak{P} = 10{,}757$; therefore $\mathfrak{P} = \left(\frac{43}{10{,}757} = \right)$ 3,998 will be the ſum required.

CASE II.

If the expectant be younger than the poſſeſſor.

Let n be the complement of the poſſeſſor's life, and ${}^{n}\mathfrak{M}$ be the value of an annuity (ſecured by land) for n years certain, if the expectant live ſo long, found by queſt. 2.

Then $1 = \frac{\mathfrak{P}}{n} \times {}^{n}\mathfrak{M}$; whence $\frac{n}{{}^{n}\mathfrak{M}} = \mathfrak{P}$.

EXAMPLE I.

If the expectant be 54, and the poſſeſſor 66 years of age. then $n = 20$; and ${}^{n}\mathfrak{M} = 9{,}891$; therefore $\left(\frac{20}{9{,}891} = \right)$ 2,022 will be the ſum required.

EXAMPLE II.

If the expectant be 43 and the poſſeſſor 66 years of age.

Here $n = 20$; and ${}^{n}\mathfrak{M} = 10{,}838$; therefore $\left(\frac{20}{10{,}838} = \right)$ 1,846 will be the ſum required.

EXAMPLE III.

If the expectant be 43 and the poſſeſſor 54 years of age.

Here $n = 32$; and ${}^{n}\mathfrak{M} = 12{,}578$; therefore $\left(\frac{32}{12{,}578} = \right)$ 2,544 will be the ſum required.

CASE

CASE III.

If the expectant and poſſeſſor are of the ſame age.

Let n be the complement and $\mathfrak{N}$ the value of an ann ity (ſecured by land) for a life of that age.

Then $1 = \frac{\mathfrak{N}\mathfrak{P}}{n}$; whence $\frac{n}{\mathfrak{N}} = \mathfrak{P}$.

EXAMPLE.

If the perſons are each aged 66.

Then $n = 20$; and $\mathfrak{N} = 7,673$; Therefore $\frac{20}{7,67}$ $= 2,607$ will be the ſum required.

The ſame method of proceeding will give the ſum money which ought to be received for 1£. lent; wh the repayment depends on any other kind of ſurvivorſhi

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QUESTION XXVI.

Things being as in the laſt queſtion, if (inſtead of ceiving a ſum of money on the deceaſe of *A*) *C* ſhou chuſe an annuity for the remainder of *B*'s life, to co mence at the deceaſe of *A*; it is required to determi how much per annum he ſhould receive for every po now lent.

SOLUTION.

Here it is evident that *C* purchaſes the reverſion of annuity for *B*'s life after the deceaſe of *A*; the values which in all the poſſible caſes will (if x be put to rep ſent the annual payment) be found by queſtion 20: therefore equations be ſeverally made between unity, 1 pound, the given preſent worth of ſuch annuity; the reſults of thoſe caſes; then the values of x bei found, by reducing thoſe equations, will be the anſwe

CAS

CASE I.

When the expectant is elder than the poſſeſſor.

Then $\mathfrak{P} - \frac{n}{6r} \times \frac{n}{2m} x = 1$; therefore $x = \frac{2m}{\mathfrak{P} - \frac{n}{6r} \times n}$.

EXAMPLE.

If the expectant be 66, and the poſſeſſor 54.

Then the reverſion of an annuity of 1£. is (per ex. 1. caſe 1. queſt. 20) 1,397; and $\left(\frac{1}{1,397} =\right)$ 0,716 will be the annual payment required.

CASE II.

When the poſſeſſor is elder than the expectant.

Then $\mathfrak{M} - \mathfrak{P} + \overline{\mathfrak{P} - \frac{n}{6r}} \times \frac{n}{2m} \times x = 1$; th. $x = \frac{1}{\mathfrak{M} - \mathfrak{P} + \overline{\mathfrak{P} - \frac{n}{6r}} \times \frac{n}{2m}}$

EXAMPLE.

If the expectant be 54, and the poſſeſſor 66.

Then $\left(\frac{1}{4,481} =\right)$ 0,223 will be the annual payment required.

CASE

CASE III.

When the poſſeſſor aud expectant are of the ſame ag

Then $\mathfrak{P} - \frac{n}{6r} \times \frac{1}{2}x = 1$; Th. $x = \frac{2}{\mathfrak{P} - \frac{n}{6r}}$

EXAMPLE.

If both expectant and poſſeſſor be aged 56.

Then $\left(\frac{1}{2,234} =\right)$ 0,448 will be the annual payme required.

The ſame method of proceeding will give the ann payment, which ought to be made in conſequence of t lending of 1£. when the commencement of ſuch pa ments depends on any other kind of reverſion.

QUESTION XXVII.

B (who will become poſſeſſed for life of a conſidera eſtate, if he ſurvives *A*, the preſent poſſeſſor) would bo row of *C* a ſum of money upon the credit thereof: an it is agreed between them, that *C* ſhall be repaid t ſum (ſo borrowed) by an annuity (ſecured on the lan for his (*C*'s) life, to commence on the deceaſe of *A* (if be then alive) and to continue as long as *B* ſhall poſſe the eſtate; that is, as long as *B* ſhall live: what ſu ought *C* to lend, in conſideration of the reverſion of annuity of 1£. ſo circumſtanced?

OR,

What is the value of the reverſion of an annuity (ſecu ed by land) for the joint lives of *B* and *C*, after the d ceaſe of *A*?

SOL

SOLUTION.

By the ſolution of queſt. 95. vol. 2. fol. 293; if from he value of the joint lives of the two expectants, be tak- n the value of the three joint lives; the remainder will e the value of the reverſion.

CASE I.

When the poſſeſſor is younger than the expectants. Let *t* be the complement of the poſſeſſor's life; *m* and *n*, thoſe of the expectants; and $\mathfrak{P}$ the value of the ſingle ife, whoſe complement, *n*, is the leaſt.

Then (by queſt. 4.) $\mathfrak{P} - \overline{\mathfrak{P} - \frac{n}{6r}} \times \frac{n}{2m}$ will be he value of the joint lives of the two expectants;

And (by queſt. 9.) $\mathfrak{P} - \overline{\mathfrak{P} - \frac{n}{6r}} \times \overline{\frac{n}{2m} + \frac{n}{2t} - \frac{nn}{4mt}}$ ill be the value of the three joint lives.

Therefore, $\left(\overline{\mathfrak{P} - \frac{n}{6r}} \times \overline{\frac{n}{2t} - \frac{nn}{4mt}} \text{ or}\right)$ $\overline{\mathfrak{P} - \frac{n}{6r}} \times \overline{1 - \frac{n}{2m}} \times \frac{n}{2t}$ will be the value of the eversion required.

EXAMPLE.

If the poſſeſſor be 43, and the expectants 54 and 66. en $t = 43$; $m = 32$; $n = 20$; $\mathfrak{P} = 7{,}673$; and $\frac{n}{6r} = 3{,}205$: alſo $\frac{n}{2m} = \left(\frac{20}{2 \times 32} = \right) \frac{5}{16}$

Now from $\mathfrak{P} = 7{,}673$; and from $1 = \frac{16}{16}$;

Take $\frac{n}{6r} = 3{,}205$; take $\frac{n}{2m} = \frac{5}{16}$;

Remains 4,468: Remains $\frac{11}{16}$.

Then $\left(4{,}468 \times \frac{11}{16} \times \frac{20}{2.43} = \right)$ 0,714 will be the value required.

COROL. I.

If the expectants are of equal ages; then $m = n$; whence $1 - \frac{n}{2m} = \left(1 - \frac{1}{2} = \right) \frac{1}{2}$; and the value of the reverſion will become $\overline{\mathfrak{R} - \frac{n}{6r}} \times \frac{n}{4t}$.

COROL. II.

If the three perſons are of equal ages; then $t = m = n$; and the value of the reverſion will become $\overline{\mathfrak{R} - \frac{n}{6r}} \times \frac{1}{4}$.

CASE II.

When the expectants are one elder, and the other younger than the poſſeſſor.

Let m be the complement of the poſſeſſor's life; t and n thoſe of the expectants; and $\mathfrak{R}$ the value of the ſingle life, whoſe complement n is the leaſt.

Then from, $\mathfrak{R} - \overline{\mathfrak{R} - \frac{n}{6r}} \times \frac{n}{2t}$, the joint lives of the expectants;

Take, $\mathfrak{R} - \overline{\mathfrak{R} - \frac{n}{6r}} \times \overline{\frac{n}{2m} + \frac{n}{2t} - \frac{nn}{4mt}}$, the three joint lives;

And the remainder $\left(\overline{\mathfrak{R} - \frac{n}{6r}} \times \overline{\frac{n}{2m} - \frac{nn}{4mt}}\right.$ or $\left.\overline{\mathfrak{R} - \frac{n}{6r}} \times \overline{1 - \frac{n}{2t}} \times \frac{n}{2m}\right)$ will be the value of the reverſion required.

EXAMPLE.

EXAMPLE.

If the poſſeſſor be 54; and the expectants 43 and 66, then $t = 43$; $m = 32$; $n = 20$; and $\mathfrak{P} - \frac{n}{6r} = 4,468$ (as in the example to caſe 1.); $\frac{n}{2t} = \left(\frac{20}{2.43} = \right) \frac{10}{43}$ and $\left(1 - \frac{10}{43} = \right) \frac{33}{43}$.

Whence $\left(4,468 \times \frac{33}{43} \times \frac{20}{2.32} = \right) 1,072$, will be the value of the reverſion required.

COROL. I.

If the younger expectant be of the ſame age with the poſſeſſor; then $t = m$; and the value of the reverſion will become $\overline{\mathfrak{P} - \frac{n}{6r}} \times \overline{1 - \frac{n}{2m}} \times \frac{n}{2m}$.

COROL. II.

If the elder expectant be of the ſame age with the poſſeſſor; then $m = n$; whence the value of the reverſion will become $\overline{\mathfrak{P} - \frac{n}{6r}} \times \overline{1 - \frac{n}{2t}} \times \frac{1}{2}$.

CASE III

When the poſſeſſor is elder than the expectants.

Let n be the complement of the poſſeſſor's life, t and m thoſe of the poſſeſſors; $\mathfrak{M}$ and $\mathfrak{P}$ the values of the ſingle lives, whoſe complements are m and n.

Then from, $\mathfrak{M} - \overline{\mathfrak{M} - \frac{m}{6r}} \times \frac{m}{2t}$, the joint lives of the expectants;

Take, $\mathfrak{N} - \overline{\mathfrak{N} - \frac{n}{6r}} \times \overline{\frac{n}{2m} + \frac{n}{2t} - \frac{nn}{4mt}}$; the three joint lives; and the Remainder will be the anſwer.

But here, ſince there is no common factor in both expreſſions; the joint lives of the two expectants, and the three joint lives, muſt be ſeparately computed; and their difference found by ſubtraction.

EXAMPLE.

If the poſſeſſor be 66, and the expectants 43 and 54.
Then (by exam. 1. queſt. 4.) the joint lives of the expectants $\}$ = 8,662,
And (by queſt. 9.) the three joint lives = 5,562;
Th. the value of the reverſion will be 3,100.

COROL.

If the expectants are of equal ages; then $t = m$; the joint lives of the expectants will become $\mathfrak{M} - \overline{\mathfrak{M} - \frac{m}{6r}} \times \frac{1}{2}$; and the three joint lives

$$\left(\mathfrak{N} - \overline{\mathfrak{N} - \frac{n}{6r}} \times \overline{\frac{n}{m} - \frac{nn}{4mm}} \text{ or}\right) \mathfrak{N} - \overline{\mathfrak{N} - \frac{n}{6r}} \times \overline{1 - \frac{n}{4m}} \times \frac{n}{m};$$

but theſe values muſt (as before) be ſeparately computed; and their difference will be the anſwer.

QUESTION

QUESTION XXVIII.

There is an eſtate of 1£. per ann. uſually let on a leaſe for three lives; of which two are already fallen: what fine ought to be paid to the leſſor, for filling up the leaſe?

OR,

What is the value of the reverſion of an annuity (ſecured by land) for the longeſt of two lives, after the deceaſe of one?

SOLUTION.

By the ſolution of queſt. 97. vol. 2. fol. 308; if, from the value of the longeſt of three lives the value of the poſſeſſor's life be taken, the remainder will be the value of the reverſion.

CASE I.

When the poſſeſſor is younger than the expectants.

Let the complement of the poſſeſſor's life be denoted by *t*; and thoſe of the expectants by *m* and *n*; and let $\mathfrak{F}$, $\mathfrak{M}$ and $\mathfrak{N}$ be ſeverally the values of annuities on thoſe lives, whoſe complements are *t*, *m* and *n*.

Then, from $\mathfrak{F} + \overline{\mathfrak{M} - \frac{m}{6r}} \times \frac{m}{2t} + \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{nn}{4mt}$, the value of the longeſt of the three lives (by queſt. 14.); take $\mathfrak{F}$ the value of the poſſeſſor's life; and the remainder $\left(\overline{\mathfrak{M} - \frac{m}{6r}} \times \frac{m}{2t} + \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{nn}{4mt} \text{ or}\right)$ $\overline{\overline{\mathfrak{M} - \frac{m}{6r}} \times m + \overline{\mathfrak{N} - \frac{n}{6r} \times \frac{nn}{2m}}} \times \frac{1}{2t}$ will be the anſwer.

COROL. I.

If the expectants are of equal ages: then $m = n$, and $\mathfrak{M} = \mathfrak{N}$; whence $\left(\overline{\mathfrak{N} - \frac{n}{6r}} \times \overline{n + \frac{n}{2}} \times \frac{1}{2t} \text{ or}\right)$ $\overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{3n}{4t}$ will be the anſwer.

COROL. II.

If the three lives are of equal ages; then $\overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{3}{4}$ will be the value of the reverſion required.

CASE II.

When the expectants are one elder, and the other younger, than the poſſeſſor.

Then (changing the ſymbols, as in caſe 2. queſt. 27. and proceeding as before.)

$$\mathfrak{F} - \mathfrak{M} + \overline{\mathfrak{M} - \frac{m}{6r}} \times \frac{m}{2t} + \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{nn}{4mt}$$

will be the value of the reverſion required.

COROL. I.

If the younger expectant be of the ſame age with the poſſeſſor; then $\mathfrak{F} = \mathfrak{M}$, and $t = m$; whence, $\overline{\mathfrak{M} - \frac{m}{6r}} \times \frac{1}{2} + \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{nn}{4mm}$, will be the value required.

COROL. II.

If the elder expectant be of the ſame age with the poſſeſſor; then $\mathfrak{M} = \mathfrak{N}$, and $m = n$, whence

$\mathfrak{F} - \mathfrak{N} + \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{3n}{4t}$ will be the value required.

CASE III.

When the poſſeſſor is elder than the expectants.

Then (uſing the ſymbols as in caſe 3. queſt. 27. and proceeding as before) $\mathfrak{F} - \mathfrak{N} + \overline{\mathfrak{M} - \frac{m}{6r}} \times \frac{m}{2t} + \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{nn}{4mt}$, will be the value of the reverſion required.

COROL.

When the expectants are of equal ages; then $\mathfrak{F} = \mathfrak{M}$, and $t = m$; whence $\mathfrak{M} - \mathfrak{N} + \overline{\mathfrak{M} - \frac{m}{6r}} \times \frac{1}{2} + \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{nn}{4mm}$ will be the value required.

QUESTION XXIX.

There is an eſtate of 1£. per annum, uſually let on a leaſe for three lives; of which one is dropt; what fine ought to be paid to the leſſor, for filling up the leaſe?

OR,

What is the value of the reverſion of an annuity (ſecured by land) for one life, after the longeſt of two lives?

SOLUTION.

By the ſolution of queſt. 101. vol. 2. fol. 318. if, from the value of the longeſt of the three lives, the value of the longeſt of the two poſſeſſors lives be taken; the remainder will be the value of the reverſion.

CASE I.

When the expectant is elder than the poſſeſſors.

Let n repreſent the complement of the expectant, m, t, thoſe of the poſſeſſors; and $\mathfrak{N}$, $\mathfrak{M}$, $\mathfrak{T}$, the values of the ſingle lives, whoſe complements are ſeverally n, m, t.

Then, if from $\overline{\mathfrak{T}+\mathfrak{M}-\frac{m}{6r}}\times\frac{m}{2t}+\overline{\mathfrak{N}-\frac{n}{6r}}\times\frac{nn}{4mt}$, the value of the longeſt of the 3 lives; we take $\overline{\mathfrak{T}+\mathfrak{M}-\frac{m}{6r}}\times\frac{m}{2t}$ the value of the longeſt of the poſſeſſor's lives; the remainder, $\overline{\mathfrak{N}-\frac{n}{6r}}\times\frac{nn}{4mt}$, will be the value of the reverſion required.

EXAMPLE.

If the two poſſeſſors are of the ages 43 and 54, and the expectant 66.

Then $\mathfrak{N}-\frac{n}{6r}=4{,}468$; and $\left(4{,}468\times\frac{20\cdot 20}{4\cdot 3^2\cdot 43}=\right)$ 0,325, will be the value of the reverſion required.

COROL. I.

If the poſſeſſors are of equal ages,

Then $t=m$; and $\left(\overline{\mathfrak{N}-\frac{n}{6r}}\times\frac{nn}{4mm}\text{ or}\right)\overline{\mathfrak{N}-\frac{n}{6r}}\times\frac{n}{2m}$ will be the value of the reverſion required.

COROL. II.

If the 3 perſons are of equal ages; then $t = m = n$; and the value of the reverſion will become $\overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{1}{4}$.

CASE II.

When the poſſeſſors are, the one elder, and the other younger than the expectant.

Here m will repreſent the complement of the expectant, n and t thoſe of the poſſeſſors.

Then, if from $\mathfrak{F} + \overline{\mathfrak{M} - \frac{m}{6r}} \times \frac{m}{2t} + \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{nn}{4mt}$;

We take $\mathfrak{F} + \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{n}{2t}$;

The remainder $(\overline{\mathfrak{M} - \frac{m}{6r}} \times \frac{m}{2t} - \overline{\mathfrak{N} - \frac{n}{6r}} \times \overline{\frac{n}{2t} - \frac{nn}{4mt}}$

Or) $\overline{\mathfrak{M} - \frac{m}{6r}} \times \frac{m}{2t} - \overline{\mathfrak{N} - \frac{n}{6r}} \times \overline{1 - \frac{n}{2m}} \times \frac{n}{2t}$

will be the value of the reverſion required.

EXAMPLE.

If the expectant is 54 years old; the poſſeſſors being 43 and 66.

Then $\overline{\mathfrak{M} - \frac{m}{6r}} = 5{,}629$; $\overline{\mathfrak{N} - \frac{n}{6r}} = 4{,}468$; $\frac{n}{2m} =$ $(\frac{20}{64} =)\ \frac{5}{16}$; And $\overline{1 - \frac{n}{2m}} = \frac{11}{16}$:

Then $(5{,}629 \times \frac{32}{86} - 4{,}468 \times \frac{11}{10} \times \frac{20}{86} =)$ 1,381 will be the value required.

COROL. I.

If the elder poſſeſſor and expectant are of the ſame age.

Then $m = n$, and $\mathfrak{M} = \mathfrak{N}$; whence the value of the reverſion will become

$$\overline{\mathfrak{N}-\frac{n}{6r}}\times\frac{n}{2t}-\overline{\mathfrak{N}-\frac{n}{6r}}\times\overline{1-\frac{n}{2n}}\times\frac{n}{2t};$$

That is

$$\left(\overline{\mathfrak{N}-\frac{n}{6r}}\times\overline{1-\overline{1-\frac{1}{2}}}\times\frac{n}{2t}=\right)\overline{\mathfrak{N}-\frac{n}{6r}}\times\frac{n}{4t}$$

COROL. II.

If the younger poſſeſſor and expectant are of the ſame age.

Then $t = m$; and the value of the reverſion will become

$$\left(\overline{\mathfrak{M}-\frac{m}{6r}}\times\frac{m}{2m}-\overline{\mathfrak{N}-\frac{n}{6r}}\times\overline{1-\frac{n}{2m}}\times\frac{n}{2m}\right.$$

$$\text{Or})\ \overline{\mathfrak{M}-\frac{m}{6r}}\times\frac{1}{2}-\overline{\mathfrak{N}-\frac{n}{6r}}\times\overline{1-\frac{n}{2m}}\times\frac{n}{2m};$$

CASE III

When the expectant is younger than the poſſeſſors.

Let t be the complement of the expectant; m, n, thoſe of the Poſſeſſors; and $\mathfrak{F}$, $\mathfrak{M}$, $\mathfrak{N}$, the values of the ſingle lives, whoſe complements are t, m, n.

Then from the value of the longeſt of the three lives, viz.

$$\mathfrak{F}+\overline{\mathfrak{M}-\frac{m}{6r'}}\times\frac{m}{2t}+\overline{\mathfrak{N}-\frac{n}{6r}}\times\frac{nn}{4mt};$$

take the value of the longeſt of the two poſſeſſor's lives, viz.

$$\mathfrak{M}+\overline{\mathfrak{N}-\frac{n}{6r}}\times\frac{n}{2m};$$

and the remainder

$$\left(\mathfrak{F}-\mathfrak{M}+\overline{\mathfrak{M}-\frac{m}{6r}}\times\frac{m}{2t}-\overline{\mathfrak{N}-\frac{n}{6r}}\times\overline{\frac{n}{2m}-\frac{nn}{4mt}}\right.$$

Or) $\mathfrak{F} - \mathfrak{M} + \overline{\mathfrak{M} - \frac{m}{6r}} \times \frac{m}{2t} - \overline{\mathfrak{N} - \frac{n}{6r}} \times \overline{1 - \frac{n}{2t}} \times \frac{n}{2m}$ will be the value of the reverſion required.

EXAMPLE.

If the two poſſeſſors are of the ages 54 and 66, and the expectant 43.
Then $\mathfrak{F} = 12{,}920$; $\mathfrak{M} = 10{,}757$; $\mathfrak{N} = 7.673$; $n = 20$; $m = 32$; $t = 43$; $\overline{\mathfrak{M} - \frac{m}{6r}} = 5{,}629$; $\overline{\mathfrak{N} - \frac{n}{6r}} = 4{,}468$; $\frac{n}{2t} = \left(\frac{20}{86} =\right) \frac{10}{43}$; and $1 - \frac{n}{2t} = \frac{33}{43}$:
Th. $(12{,}920 - 10{,}757 + 5{,}629 \times \frac{32}{86} - 4{,}468 \times \frac{33}{43} \times \frac{20}{64} =)$ 3,187 will be the value of the reverſion required.

COROL.

If the poſſeſſors are of equal ages.
Then $\mathfrak{M} = \mathfrak{N}$, and $m = n$; whence the value of the reverſion will become,

$$\mathfrak{F} - \mathfrak{N} + \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{n}{2t} - \overline{\mathfrak{N} - \frac{n}{6r}} \times \overline{\frac{n}{2n} - \frac{nn}{4nt}};$$

That is $\left(\mathfrak{F} - \mathfrak{N} + \overline{\mathfrak{N} - \frac{n}{6r}} \times \overline{\frac{n}{2t} - \frac{1}{2} + \frac{n}{4t}} =\right)$

$$\mathfrak{F} - \mathfrak{N} + \overline{\mathfrak{N} - \frac{n}{6r}} \times \overline{\frac{n}{4t} - \frac{1}{2}}.$$

QUESTION XXX.

A will be entituled to an annuity (ſecured by land) for the remainder of his life, after the deceaſe of *B*; if *B* ſurvives *C*, the preſent poſſeſſor thereof; but if *B* dies before ſore *C*, then the annuity will (upon *C*'s deceaſe) deſcend to *D*, a perſon of the ſame age with *A*; the reſpective intereſts of *A* and *D*, in that annuity, are required.

SOLUTION.

Since *A* and *D* are of equal ages; they are, between them, entituled to the reverſion of an annuity, for one life of that age, after the longeſt of the two lives of *B* and *C*: and conſequently, the value thereof being found, it is farther required to divide that reverſion, properly between them.

Now the probabilities, that either one, or the other, of them will receive a payment in the firſt, ſecond, third, &c. year, are (by queſt. 101. vol. 2.) the continual products of the two probabilities of the dying of *B* and *C*, and the probability of the ſurviving of a perſon of the age of *A*, taken ſucceſſively for thoſe times.

Now if m and n repreſent the reſpective complements of the lives of *B* and *C*; and p repreſents the probability of their being both extinct, at the end of one, two, or three years; then (by queſt. 19.) $\frac{mp}{m+n}$ will exhibit the probability that the elder will within thoſe times die before the younger; and $\frac{np}{n+m}$ the probability that the younger will ſo die before the elder; and theſe factors, $\frac{m}{m+n}$ and $\frac{n}{m+n}$, being to be applied to every probability, they will be conſtant factors, in every term of the ſeries, which expreſſes the required probabilities.

If therefore the value of the reverſion of an annuity, for the life of *A*, after the longeſt of the lives of *B* and *C*, be multiplied by $\frac{m}{n.+n}$ the product will (when *C* is elder than *B*) be the value of *A*'s intereſt, in the annuity; and if the value of the ſame reverſion, be multiplied by

n

$\frac{n}{m+n}$ the product will (upon the ſame ſuppoſition) be the value of *D*'s intereſt therein.

But, for as much as *A* and *D* are ſuppoſed to be of equal ages, the queſtion will not be altered by ſuppoſing them to be but one perſon; and that the diverſity conſiſts, in that perſon's Intereſt, depending on the different ſurvivorſhips of the elder, or the younger of the poſſeſſor's.

If *A*, *B*, and *C*, repreſent the names of the youngeſt, ſecond, and eldeſt perſon, then the queſtion will admit of the following varieties, applicable to the three caſes given in the laſt queſtion.

CASE I.

If *C*, the eldeſt, is the expectant; *B* and *A*, the two younger, being poſſeſſors; then,

Firſt, if *C*'s expectation depends upon *A*'s ſurviving *B*; then $\left(\frac{t}{t+m} \times \mathfrak{R} - \frac{n}{6r} \times \frac{nn}{4mt} \text{ or}\right)$ $\mathfrak{R} - \frac{n}{6r} \times \frac{nn}{\overline{t+m} \times 4m}$ will be the value of his intereſt in the reverſion.

Secondly, if it depends upon *B*'s ſurviving *A*; then $\left.\frac{m}{t+m} \times \mathfrak{R} - \frac{n}{6r} \times \frac{nn}{4mt} \text{ or}\right)$ $\mathfrak{R} - \frac{n}{6r} \times \frac{nn}{\overline{t+m} \times 4t}$ will be that value.

COROL. I.

If the poſſeſſors are of equal ages; then $t=m$, and both $\frac{t}{t+m}$, and $\frac{m}{t+m}$, will be equal to $\frac{1}{2}$; whence the value of *C*'s intereſt in the reverſion will be

$(\mathfrak{R} - \frac{n}{6r} \times \frac{nn}{4mm} \times \frac{1}{2} \text{ or}) \ \mathfrak{R} - \frac{n}{6r} \times \frac{nn}{8mm}$.

COROL. II.

If the three perſons are of equal ages; then the value required will be $\overline{\mathfrak{N}-\frac{n}{6r}}\times\frac{1}{8}$.

CASE II.

When the poſſeſſors are *C*, who is elder, and *A*, who is younger, than *B* the expectant; then

Firſt, if *B*'s expectation depends upon *A*'s ſurviving *C*; then

$$\left(\frac{t}{t+n}\times\overline{\overline{\mathfrak{M}-\frac{m}{6r}}\times\frac{m}{2t}-\overline{\mathfrak{N}-\frac{n}{6r}}\times\overline{1-\frac{n}{2m}}\times\frac{n}{2t}}\text{ or}\right)$$

$$\overline{\mathfrak{M}-\frac{m}{6r}}\times\frac{m}{\overline{t+n}\times 2}-\overline{\mathfrak{N}-\frac{n}{6r}}\times\overline{1-\frac{n}{2m}}\times\frac{n}{\overline{t+n}\times 2};$$

That is,

$$\overline{\overline{\mathfrak{M}-\frac{m}{6r}}\times m-\overline{\mathfrak{N}-\frac{n}{6r}}\times\overline{1-\frac{n}{2m}}\times n}\times\frac{1}{\overline{t+n}\times 2},$$

will be the value of his intereſt in the reverſion.

EXAMPLE.

There is a copyhold eſtate held on the lives of two perſons; *C* the father in poſſeſſion, and *A* the ſon in expectation, of the ages 66 and 43; *A* has a wife, *B*, aged 54, who by the cuſtom of the manor, will have her life in the premiſes, if her huſband comes into poſſeſſion, and ſhe ſurvives him: what is the preſent value of her intereſt in the eſtate?

By the example to caſe 2. queſt. 29, the value of the reverſion of an annuity for a life of 54, after the longeſt liver of two perſons, of the ages 43 and 66, is 1,381; alſo $\frac{t}{t+n}=\frac{43}{63}$; therefore $(1{,}381\times\frac{43}{63}=)\,0{,}942$ will be the value required.

Secondly, If *B*'s expectation depends upon *C*'s ſurviving *A*; then

$$\left(\frac{n}{t+n}\times\left\{\begin{array}{l}\overline{\mathfrak{M}-\frac{m}{6r}}\times\frac{m}{2t}\\ -\overline{\mathfrak{N}-\frac{n}{6r}}\times\overline{1-\frac{n}{2m}}\times\frac{n}{2t}\end{array}\right.\text{ or }\right)$$

$$\left.\begin{array}{l}\overline{\mathfrak{M}-\frac{m}{6r}}\times m-\overline{\mathfrak{N}-\frac{n}{6r}}\\ \qquad\times\overline{1-\frac{n}{2m}}\times n\end{array}\right\}\times\frac{n}{\overline{t+n}\times 2t}$$

will be the value thereof.

EXAMPLE.

The ſame caſe being propoſed, as in the laſt example, if (inſtead of *B*'s being the wife of the ſon, *A*) ſhe be the ſecond wife of the father *C*; then, the value of her intereſt in the eſtate will be found, by multiplying (1,381) the value of the reverſion of the annuity for a life of 54, after the longeſt liver of two perſons, aged 43 and 66, by $\left(\frac{n}{t+n} =\right) \frac{20}{63}$; which product (viz. $1{,}381 \times \frac{20}{63}$) is equal to 0,439;

And, if (0,942) the reſult in the former example be added to (0,439) the reſult in this; the ſum (1,381) will be the value of the whole reverſion.

COROL. I.

If the elder poſſeſſor *C*, and the expectant *B*, are of the ſame age; then $m = n$, and $\mathfrak{M} = \mathfrak{N}$, whence,

Firſt, if the expectation depends upon *A*'s ſurviving *C*;

then $\left(\overline{\mathfrak{N} - \frac{n}{6r}} \times \overline{n - 1 - \tfrac{1}{2}} \times n \times \frac{1}{\overline{t+n} \times 2} \text{ or}\right)$

$\overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{n}{\overline{t+n} \times 4}$ will be the value thereof.

Secondly, if it depends upon *C*'s ſurviving *A*; then

$\left(\overline{\mathfrak{N} - \frac{n}{6r}} \times \overline{n - 1 - \tfrac{1}{2}} \times n \times \frac{n}{\overline{t+n} \times 2t} \text{ or}\right)$

$\overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{nn}{\overline{t+n} \times 4t}$ will be the value thereof.

COROL. II.

If the younger poſſeſſor *A*, and the expectant *B*, are of the ſame age; then $t = m$; whence

Firſt, if the expectation depends upon *A*'s ſurviving *C*; then

$$\left.\begin{array}{l}\overline{\mathscr{M} - \frac{m}{6r}} \times m - \overline{\mathfrak{N} - \frac{n}{6r}} \\ \qquad \times \overline{1 - \frac{n}{2m}} \times n\end{array}\right\} \times \frac{1}{\overline{m+n} \times 2}$$

will be the value thereof.

Secondly, if it depends upon *C*'s ſurviving *A*; then

$$\left.\begin{array}{l}\overline{\mathfrak{M} - \frac{m}{6r}} \times m - \overline{\mathfrak{N} - \frac{n}{6r}} \\ \qquad \times \overline{1 - \frac{n}{2m}} \times n\end{array}\right\} \times \frac{n}{\overline{m+n} \times 2m}$$

will be the value thereof.

CASE III.

When the expectant *A* is younger than the poſſeſſors *B*, and *C*; then

Firſt, if the expectation depends upon *B*'s ſurviving *C*; then

$$\frac{m}{m+n} \times \left\{\begin{array}{l}\mathfrak{F} - \mathfrak{M} + \overline{\mathscr{M} - \frac{m}{6r}} \times \frac{m}{2t} \\ \qquad - \overline{\mathfrak{N} - \frac{n}{6r}} \times \overline{1 - \frac{n}{2t}} \times \frac{n}{2m}\end{array}\right.$$

will be the value required.

Secondly, if it depends upon *C*'s ſurviving *B*; then

$$\frac{n}{m+n} \times \left\{\begin{array}{l}\mathfrak{F} - \mathscr{M} + \overline{\mathfrak{M} - \frac{m}{6r}} \times \frac{m}{2t} \\ \qquad - \overline{\mathfrak{N} - \frac{n}{6r}} \times \overline{1 - \frac{n}{2t}} \times \frac{n}{2m}\end{array}\right.$$

will be the value thereof.

COROL.

COROL.

If the poſſeſſors *B* and *C* are of equal ages; then $m = n$, and both $\frac{m}{m+n}$, and $\frac{n}{m+n}$, will be equal to $\frac{1}{2}$; whence the value of *A*'s intereſt in the reverſion will be

$$\frac{1}{2} \times \overline{\mathfrak{F} - \mathfrak{N} + \overline{\mathfrak{N} - \frac{n}{6r}} \times \overline{\frac{3n}{4t} - \frac{1}{2}}}.$$

QUESTION XXXI.

The reſpective ages of three perſons, *A*, *B*, and *C*, being given; it is required to find the probability, that any two of them ſhall ſurvive the third.

CASE I.

If one, or both, of the ſurvivors are elder than the perſon to be ſurvived; that is, if the complements of the ſurvivors be *n* and *t*, that of the ſurvived being *m*; or if the complements of the ſurvivors be *n* and *m*, that of the ſurvived being *t*.

Then, the expectation of the joint lives of the ſurvivors will be expreſſed, by the ſeries $\frac{2n-1}{2n} \times \frac{2t-1}{2t} + \frac{2n-3}{2n} \times \frac{2t-3}{2t}\,(n)$; or, by the ſeries $\frac{2n-1}{2n} \times \frac{2m-1}{2m} + \frac{2n-3}{2n} \times \frac{2m-3}{2m}\left(n\right)$:

And (by arguing as in queſt. 21.) the probabilities of their, ſeverally, ſurviving the perſons, whoſe complements are *m* and *t*, will be, $\frac{2n-1}{2n} \times \frac{2t-1}{2t} \times \frac{1}{m}$

+ 2n—

$+\frac{2n-3}{2n}\times\frac{2t-3}{2t}\times\frac{1}{m}\,(n)$ and $\frac{2n-1}{2n}\times\frac{2m-1}{2m}\times\frac{1}{t}$ $+\frac{2n-3}{2n}\times\frac{2m-3}{2m}\times\frac{1}{}\,(n)$; the ſums of which will (by queſtion 106. vol. 2.) be $\frac{n}{2m}-\frac{nn}{6tm}$, and $\frac{n}{2t}-\frac{nn}{6mt}$, which were required.

CASE II.

If both the ſurvivors be younger, than the ſurvived.

Let the complements of the ſurvivors be denoted by m nd t; and that of the ſurvived by n.

Then by arguing (as in queſt. 22.) the probability required will be expreſſed by the ſeries,

$$\frac{2m-1}{2m}\times\frac{2t-1}{2t}\times\frac{1}{n}+\frac{2m-3}{2m}\times\frac{2t-3}{2t}\times\frac{1}{n}\,(n);$$

The ſum of which, will, (by queſt. 16.) be

$$\left(1-\frac{n}{2m}-\frac{n}{2t}+\frac{nn}{3mt}\times n\times\frac{1}{n}\text{ or}\right)1-\frac{n}{2m}-\frac{n}{2t}+\frac{nn}{3mt};$$

becauſe $\frac{1}{n}$ is a common factor, in every term of the ſeries.

QUESTION XXXII.

A has (by his will) left a legacy of $\mathfrak{P}$ pounds to *B*, if both he (*B*) and his wife (*C*) ſhall be alive at the time of his (*A*'s) deceaſe; what is the preſent value of *B*'s expectation?

CASE I.

If either, or both, of the expectants be elder than the poſſeſſor.

Then (by queſt. 31. caſe, 1.) the ſum of the ſeries of annual probabilities will be, either $\frac{n}{2t} - \frac{nn}{6mt}$, or $\frac{n}{2m} - \frac{nn}{6mt}$;

That is, either $\frac{1}{t} \times \overline{\frac{n}{2} - \frac{nn}{6m}}$, or $\frac{1}{m} \times \overline{\frac{n}{2} - \frac{nn}{6t}}$: consequently, the ſum of the correſponding ſeries of preſent worths will be, either $\frac{1}{t} \times \overline{\mathfrak{N} - \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{n}{2m}}$, or $\frac{1}{m} \times \overline{\mathfrak{N} - \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{n}{2t}}$:

Whence, either $\frac{\mathfrak{P}}{t} \times \overline{\mathfrak{N} - \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{n}{2m}}$, or $\frac{\mathfrak{P}}{m} \times \overline{\mathfrak{N} - \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{n}{2t}}$, will be the value required.

EXAMPLE I.

If *A* be aged 43 years; *B* and *C* being 66, and 54; and the legacy 25£.

Then $t = 43$; $\mathfrak{P} = 25$;

And $\mathfrak{N} - \mathfrak{N} - \frac{n}{6r} \times \frac{n}{2m} = 6{,}276$, by example 3. queſt. 4.

Therefore $\frac{6{,}276 \times 25}{43} =$) 3,649 will be the value required.

EXAMPLE II.

If *A* be aged 54 years; *B* and *C* being 66 and 43; and the legacy 25£.

Then $\mathfrak{N} - \mathfrak{N} - \frac{n}{6r} \times \frac{n}{2t} = 6{,}634$, by ex. 2. queſt. 4.

And $\left(\frac{6{,}634 \times 25}{32} = \right)$ 5,183 will be the value required.

CASE II.

If both the expectants be younger than the poſſeſſor, then (by queſt. 31. caſe 2.) the ſeries of annual probabilities will be $\frac{2m-1}{2m} \times \frac{2t-1}{2t} \times \frac{1}{n} + \frac{2m-3}{2m} \times \frac{2t-3}{2t} \times \frac{1}{n}$ (n.)

Now (by queſt. 7.) the ſum of n terms of the ſeries of preſent worths, $\frac{2m-1}{2mr} \times \frac{2t-1}{2t} + \frac{2m-3}{2mr^2} \times \frac{2t-3}{2t}$ &c. is ${}^{n}\mathfrak{M} - \overline{{}^{n}\mathfrak{M} - \frac{nn}{6rm}} \times \frac{n}{2t}$, which being multipl. by $\frac{1}{n}$, the common factor of the above ſeries of annual probabilities, will become $\frac{{}^{n}\mathfrak{M}}{n} - \overline{{}^{n}\mathfrak{M} - \frac{nn}{6rm}} \times \frac{1}{2t}$;

Therefore $\mathfrak{P} \times \overline{\frac{{}^{n}\mathfrak{M}}{n} - \overline{{}^{n}\mathfrak{M} - \frac{nn}{6rm}} \times \frac{1}{2t}}$ will be the value required.

EXAMPLE.

If *A* be aged 66; *B* and *C*, being 54 and 43; and the legacy 25£;

Then ${}^{n}\mathfrak{M} - \overline{{}^{n}\mathfrak{P} - \frac{nn}{6rm}} \times \frac{n}{2t} = 8{,}056$, by queſt. 7; and $\left(8{,}056 \times \frac{25}{20} = \right)$ 10,070 will be the value required.

SCHO-

SCHOLIUM.

Since the ſum of the ſeries of annual probabilities,

$$\frac{2m-1}{2m} \times \frac{2t-1}{2t} \times \frac{1}{n} + \frac{2m-3}{2m} \times \frac{2t-3}{2t} \times \frac{1}{n}\,(n),$$

is $1 - \frac{n}{2m} - \frac{n}{2t} + \frac{nn}{3mt}$; and the ſum of the corr ponding ſeries of preſent worths is

$$\frac{{}^{n}\mathfrak{M}}{n} - \overline{{}^{n}\mathfrak{M} - \frac{nn}{6rm}} \times \frac{1}{2t};$$ Therefore, wher the former appears in any operation, the value of correſponding annuity, or ſurvivorſhip, will be deno by the latter.

QUESTION XXXIII.

The reſpective ages of *A*, *B*, *C*, and *D*, being give it is required to find the probability, that any three them ſhall ſurvive the fourth.

CASE I.

If one or more of the ſurvivors are elder than the p ſon to be ſurvived; that is (putting *t*, *m*, *n* and *v*, ſeveral for the complements of the youngeſt, ſecond, third, a eldeſt) if the complements of the ſurvivors are either m, and t; or v, n and t; or v, n and m; the comp ments of the perſon to be ſurvived, being, ſeverally, n, m, or t.

Then (by arguing as in queſtion 21 and 31.) the prob bilities of the three ſurvivorſhips will be expreſſed by t three ſeries $\frac{2v-1}{2v} \times \frac{2m-1}{2m} \times \frac{2t-1}{2t} \times \frac{1}{n} +$

$$\left(\frac{2v-3}{2v} \times \frac{2m-3}{2m} \times \frac{2t-3}{2t} \times \frac{1}{n}\,(v)\right.$$

$2v-$

$$\frac{2v-1}{2v} \times \frac{2n-1}{2n} \times \frac{2t-1}{2t} \times \frac{1}{m} +$$

$$\Big(\frac{2v-3}{2v} \times \frac{2n-3}{2n} \times \frac{2t-3}{2t} \times \frac{1}{m}(v);$$

$$\frac{2v-1}{2v} \times \frac{2n-1}{2n} \times \frac{2m-1}{2m} \times \frac{1}{t} +$$

$$\Big(\frac{2v-3}{2v} \times \frac{2n-3}{2n} \times \frac{2m-3}{2m} \times \frac{1}{t}(v):$$

Which ſerieſes have each a factor common to all their terms, and therefore their ſums will (by queſt, 107. vol. 2.) be $\frac{v}{2n} - \frac{vv}{6mn} - \frac{vv}{6tn} + \frac{v^3}{12mtn}$,

$$\frac{v}{2m} - \frac{vv}{6nm} - \frac{vv}{6tm} + \frac{v^3}{12ntm},$$

And $\frac{v}{2t} - \frac{vv}{6nt} - \frac{vv}{6mt} + \frac{v^3}{12nmt}$, which were required.

CASE II.

If all the ſurvivors be younger than the perſon to be ſurvived; then, putting n, m, and t, for the complements of the ſurvivors, and v for that of the ſurvived, the probability required will (by arguing as in queſtion 22.) be expreſſed by the ſeries, $\frac{2n-1}{2n} \times \frac{2m-1}{2m} \times \frac{2t-1}{2t} \times \frac{1}{v}$

$$\Big(+\frac{2n-3}{2n} + \frac{2m-3}{2m} + \frac{2t-3}{2t} \times \frac{1}{v}(v):$$

The ſum of which, will (if we multiply the reſult of queſt. 17, by $\frac{1}{v}$, the conſtant factor in this ſeries) appear to be $1 - \frac{v}{2n} - \frac{v}{2m} - \frac{v}{2t} + \frac{vv}{3tm} + \frac{vv}{3tn} + \frac{vv}{3mn} - \frac{v^3}{4tmn}$; which is, therefore, the probability required.

QUES-

QUESTION XXXIV.

What is the preſent worth of a legacy of, or an eſtate worth, $\mathfrak{P}$ pounds, depending on the contingency of thre perſons ſurviving a fourth; the ages of thoſe perſons being given?

CASE I.

If one or more of the ſurvivors are elder, than the perſon to be ſurvived.

Then (by queſt. 33. caſe 1. retaining the ſame ſymbols as are therein uſed) the ſum of the ſeries of annual probabilities will be, either $\frac{v}{2n} - \frac{vv}{6mn} - \frac{vv}{6tn} + \frac{v^3}{12mtn}$, or $\frac{v}{2m} - \frac{vv}{6nm} - \frac{vv}{6tm} + \frac{v^3}{12ntm}$, or $\frac{v}{2t} - \frac{vv}{6nt} - \frac{vv}{6mt} + \frac{v^3}{12nmt}$; in the firſt of which $\frac{1}{n}$; in the ſecond, $\frac{1}{m}$; and in the third, $\frac{1}{t}$, are conſtant factors; therefore, if the preſent worths, correſponding to th probabilities, into which thoſe common factors are multiplied, be ſeverally multiplied by $\frac{\mathfrak{P}}{n}$, $\frac{\mathfrak{P}}{m}$, and $\frac{\mathfrak{P}}{t}$; th reſults will (by queſt. 9.) be

$$\overline{\mathfrak{P} - \overline{\mathfrak{P} - \frac{v}{6r}} \times \frac{\overline{\overline{m+t} \times 2 - v} \times v}{4mt}} \times \frac{\mathfrak{P}}{n},$$

$$\overline{\mathfrak{P} - \overline{\mathfrak{P} - \frac{v}{6r}} \times \frac{\overline{\overline{n+t} \times 2 - v} \times v}{4nt}} \times \frac{\mathfrak{P}}{m},$$

and $\overline{\mathfrak{P} - \overline{\mathfrak{P} - \frac{v}{6r}} \times \frac{\overline{\overline{n+m} \times 2 - v} \times v}{4nm}} \times \frac{\mathfrak{P}}{t}$

the values required.

CASE II.

If all the ſurvivors be younger, than the perſon to be ſurvived; then (by caſe 2. queſt. 33.) the ſeries of annual probabilities will be

$$\frac{2n-1}{2n} \times \frac{2m-1}{2m} \times \frac{2t-1}{2t} \times \frac{1}{v} + \frac{2n-3}{2n} \times \frac{2m-3}{2m} \times \frac{2t-3}{2t} \times \frac{1}{v}\ (v)$$;

and (by queſt. 10.) the ſum of v terms of the ſeries of preſent worths, $\frac{2n-1}{2nr} \times \frac{2m-1}{2m} \times \frac{2t-1}{2t} + \frac{2n-3}{2nr^2} \times \frac{2m-3}{2m} \times \frac{2t-3}{2t}$ &c. is ${}^{v}\mathfrak{P} -$

$$\left.\begin{matrix} \overline{{}^{v}\mathfrak{P} - \frac{vv}{6rn}} \times \overline{\overline{m+t} \times 2 - v} \\ - \frac{vv}{6rn} \times \overline{n-v} \times 2 \end{matrix}\right\} \times \frac{v}{4mt};$$

if, therefore, this expreſſion be multiplied by $\frac{\mathfrak{P}}{v}$ the reſult, *viz.*

$$\left.\left.\begin{matrix} {}^{v}\mathfrak{P} - \\ \overline{{}^{v}\mathfrak{P} - \frac{vv}{6rn}} \times \overline{\overline{m+t} \times 2 - v} \\ - \frac{vv}{6rn} \times \overline{n-v} \times 2 \end{matrix}\right\} \frac{v}{4mt} \right\} \times \frac{\mathfrak{P}}{v}$$

will be the value required.

EXAMPLE.

D, aged 74, hath, by his will, left a legacy of 25*£*. to *C*, his brother, aged 66; provided he, *C*; his wife, *B*, aged 54; and his daughter, *A*, aged 43; ſhall be, all of them, alive at the time of his, *D*'s, deceaſe: what is the value of *C*'s intereſt in the legacy?

By the example to queſt. 10. the expreſſion $\cdot\mathfrak{P}$ –

$$\left.\begin{array}{l} \overline{\cdot\mathfrak{P} - \frac{vv}{6rn}} \times \overline{\overline{m+t} \times 2 - v} \\ \quad - \frac{vv}{6rn} \times \overline{n - v} \times 2 \end{array}\right\} \times \frac{v}{4mt} = 4{,}870;$$

And $\left(4{,}870 \times \frac{25}{10} = \right)$ 12,175 will be the value of *C*'s intereſt in the legacy.

SCHOLIUM.

Since the ſum of the ſeries of annual probabilities,

$$\frac{2n-1}{2n} \times \frac{2m-1}{2m} \times \frac{2t-1}{2t} \times \frac{1}{v} + \frac{2n-3}{2n} \times \frac{2m-3}{2m} \times \frac{2t-3}{2t} \times \frac{1}{v}\ (\&c.),$$

is

$$1 - \frac{v}{2t} - \frac{v}{2m} - \frac{v}{2n} + \frac{vv}{3tm} + \frac{vv}{3tn} + \frac{vv}{3mn} - \frac{v^3}{4tmn};$$

and ſince the ſum of the correſponding ſeries of preſent worths is $\frac{\cdot\mathfrak{P}}{v}$ –

$$\left.\begin{array}{l} \overline{\cdot\mathfrak{P} - \frac{vv}{6rn}} \times \overline{\overline{m+t} \times 2 - v} \\ \quad - \frac{vv}{6rn} \times \overline{n - v} \times 2 \end{array}\right\} \times \frac{1}{4mt};$$

therefore, whenever the former occurs in any operation, the value of the correſponding annuity, or ſurvivorſhip, will be denoted by the latter.

QUESTION XXXV.

The reſpective ages of three perſons *A*, *B*, and *C* (of whom *A* is the youngeſt, and *C* the eldeſt) being given; it is required to find the probability, that *A* ſhall die firſt, *B* next, and *C* laſt.

SOLU-

SOLUTION.

Let the complements of *A*, *B*, and *C*, be ſeverally denoted by t, m, and n: then, ſince $\frac{2t-1}{2t}$ is the expectation of *A*'s life for the firſt year, $1-\frac{2t-1}{2t}$ will be the expectation of his death, at the beginning, or early in that year. If, therefore, this expectation of *A*'s death be multiplied by, $\frac{2n-1}{2n}\times\frac{1}{m}$, the probability of *C*'s ſurviving *B*, for that year (ſee queſt. 21.) then the product, $1-\frac{2t-1}{2t}\times\frac{2n-1}{2n}\times\frac{1}{m}$, will exhibit the probability of *A*'s dying firſt, and *B* next, in the firſt year, and that *C* ſhall ſurvive them both for that year: and (by arguing in the ſame manner) the probabilities of the like event's taking place, in the ſecond, third, &c. year, will be $1-\frac{2t-3}{2t}\times\frac{2n-3}{2n}\times\frac{1}{m}$, $1-\frac{2t-5}{2t}\times\frac{2n-5}{2n}\times\frac{1}{m}$, &c. and conſequently the probability required will be the ſum of this ſeries of products.

Now by actual multiplication theſe products will become

$$\frac{2n-1}{2n}\times\frac{1}{m}-\frac{2t-1}{2t}\times\frac{2n-1}{2n}\times\frac{1}{m},$$

$$\frac{2n-3}{2n}\times\frac{1}{m}-\frac{2t-3}{2t}\times\frac{2n-3}{2n}\times\frac{1}{m},$$

$$\frac{2n-5}{2n}\times\frac{1}{m}-\frac{2t-5}{2t}\times\frac{2n-5}{2n}\times\frac{1}{m};$$

&c. &c.

Where the affirmative ſeries, $\frac{2n-1}{2n} \times \frac{1}{m} + \frac{2n-3}{2n} \times$ &c. is the probability of *C*'s ſurviving *B* (by queſt. 21.) and the negative ſeries, $\frac{2t-1}{2t} \times \frac{2n-1}{2n} \times \frac{1}{m} + \frac{2t-3}{2t} \times \frac{2n-3}{2n} \times \frac{1}{m}$ &c. is the probability that both *A* and *C* will ſurvive *B*, by queſt. 31, caſe 1.

Hence, if we call the perſon propoſed to die firſt, the *poſſeſſor*; the perſon who is to die next, the *expectant*; and the third perſon the *ſurvivor*.

Then, from the probability of the ſurvivor's out-living the expectant; take the probability, that both ſurvivor and poſſeſſor ſhall out-live the expectant; and the remainder will be the probability of the propoſed order of ſurvivorſhip among the three perſons.

Now the probability of *C*'s ſurviving *B* is $\frac{n}{2m}$ (by queſt. 21.) and the probability, that both *A* and *C* ſhall ſurvive *B*, is $\frac{n}{2m} - \frac{nn}{6tm}$ (by caſe 1. queſt. 31.) therefore, their difference, $\frac{nn}{6tm}$, will be the probability required.

COROL. I.

If the ages of the poſſeſſor *A*, and expectant *B*, are equal; then $t = m$; and the probability required will become $\frac{nn}{6mm}$.

COROL. II.

If the ages of the expectant *B*, and ſurvivor *C*, are equal; then $m = n$; and ($\frac{nn}{6tn}$ or) $\frac{n}{6t}$ will be the probability required.

COROL.

COROL. III.

If the three perſons are of equal ages; then $t = m = n$; nd the probability will become $\left(\frac{nn}{6nn} = \right) \frac{1}{6}$.]

QUESTION XXXVI.

The ſame lives being conſidered, as in the laſt queſtin; it is required to find the probability, that *B*, ſhall die rſt; *A*, the youngeſt, next; and *C*, the eldeſt, laſt?

SOLUTION.

Here the complement of the poſſeſſor, is m; of the exectant, t; and of the ſurvivor, n.

From $\frac{n}{2t}$ the probability of *C*'s ſurviving *A*,

Take $\frac{n}{2t} - \frac{nn}{6mt}$; the probability that *B* and *C* hall ſurvive *A*;

Remains, $\frac{nn}{6mt}$, for the probability of the given order f ſurvivorſhip; which probability is the ſame, with e reſult of the laſt queſtion.

COROL. I.

If the ages of the poſſeſſor, *B*, and expectant *A*, are qual; then $t = m$; and the anſwer will be $\frac{nn}{6mm}$, as in orol. 1. queſt. 35.

COROL. II.

If the ages of the poſſeſſor, *B*, and ſurvivor, *C*, are equal; then $m=n$; and the reſult will be $\left(\frac{nn}{6nt}=\right)$ $\frac{n}{6t}$, as in corol. 2. queſt. 35.

QUESTION XXXVII.

In the ſame lives as before; it is required to determine the probability, that *A*, the youngeſt, ſhall die firſt; *C*, the eldeſt, next; and *B*, laſt.

Here the complement of the poſſeſſor, is t; of the expectant, n; and of the ſurvivor, m.

The probability of the ſurvivor's outliving the expectant will (by queſt. 22.) be $1-\frac{n}{2m}$;

And the probability, that both poſſeſſor and ſurvivor ſhall outlive the expectant, is (by caſe 2. queſtion 31.)

$1-\frac{n}{2m}-\frac{n}{2t}+\frac{nn}{3mt}$;

Which, being ſubtracted from the former, leaves $\frac{n}{2t}-\frac{nn}{3mt}$, for the probability required.

COROL. I.

If the poſſeſſor, *A*, and the ſurvivor, *B*, be of equal ages; then $t=m$; and, $\frac{n}{2m}-\frac{nn}{3mm}$, will be the probability required.

COROL.

COROL. II.

If the expectant, *C*, and ſurvivor, *B*, be of equal ages; hen $m = n$; and the anſwer will become $\frac{n}{2t} - \frac{nn}{3nt}$;

hat is $\left(\frac{n}{2t} - \frac{n}{3t} = \right) \frac{n}{6t}$, as in corol. 2. queſt. 35 nd 36.

QUESTION XXXVIII.

In the ſame lives as before; required the probability, at *C*, the eldeſt, ſhall die firſt; *A*, the youngeſt, next; nd *B*, laſt.

Here, the complement of the poſſeſſor, is n; of the expectant, t; and of the ſurvivor, m.

Therefore from $\frac{m}{2t}$;

Take $\frac{n}{2t} - \frac{nn}{6mt}$;

And the remainder $\left(\frac{m}{2t} - \frac{n}{2t} + \frac{nn}{6mt}\right.$ or)

$\overline{m - n + \frac{nn}{3m}} \times \frac{1}{2t}$ will be the probaility required.

COROL. I.

If the poſſeſſor C, and ſurvivor *B*, are of equal ages; en $m = n$; and $\frac{n}{2t} - \frac{n}{2t} + \frac{nn}{6nt}$ or) $\frac{n}{6t}$ will be the nſwer, as in corol. 2. queſt. 35, 36, and 37.

COROL. II.

If the expectant, *A*, and ſurvivor, *B*, are of equ ages; then $t = m$; and $\left(\frac{m}{2m} - \frac{n}{2m} + \frac{nn}{6mm} \text{ or } \right)$ $\frac{1}{2} - \frac{n}{2m} + \frac{nn}{6mm}$ will be the anſwer.

QUESTION XXXIX.

In the ſame lives as before; it is required to determin the probability, that *B*, ſhall die firſt; *C* the eldeſt, next; and *A* the youngeſt, laſt.

Here the complement of the poſſeſſor is m; of the expectant, n; and of the ſurvivor, t.

The probability of the ſurvivor's outliving the expectant is (by queſt. 22) $1 - \frac{n}{2t}$; and the probability, th both poſſeſſor and ſurvivor will outlive the expectant, i (by caſe 2. queſt. 31) $1 - \frac{n}{2m} - \frac{n}{2t} + \frac{nn}{3mt}$; whic being ſubtracted from the former, leaves, $\frac{n}{2m} - \frac{n}{3}$ for the probability required.

COROL. I.

If the poſſeſſor *B*, and the expectant *C*, are of equal age then $m = n$; and $\left(\frac{n}{2n} - \frac{nn}{3nt} \text{ or} \right)$ $\frac{1}{2} - \frac{n}{3t}$ will b the anſwer.

COROL. II.

If the poſſeſſor *B*, and ſurvivor *A*, are of equal ages; then $t = m$; and $\frac{n}{2m} - \frac{nn}{3mm}$ will be the probability required, as in corol. 1. queſt. 37.

QUESTION XL.

In the ſame lives as before; it is required to determine the probability, that *C*, the eldeſt, ſhall die firſt; *B*, next; and *A*, the youngeſt, laſt.

Here the complement of the poſſeſſor is n; of the expectant, m; and of the ſurvivor, t.

Then from $1 - \frac{m}{2t}$;

Take $\frac{n}{2m} - \frac{nn}{6tm}$;

The remainder, $1 - \frac{m}{2t} - \frac{n}{2m} + \frac{nn}{6tm}$,

will be the probability required.

COROL. I.

If the ages of the poſſeſſor *C*, and expectant *B*, are equal; then $m = n$; and the anſwer will be $1 - \frac{n}{2t} - \frac{n}{2n} + \frac{nn}{6nt}$;

That is $\left(1 - \frac{1}{2} - \frac{n}{2t} - \frac{n}{6t} =\right) \frac{1}{2} - \frac{n}{3t}$; as in corol 1. queſt. 39.

COROL. II.

If the ages of the expectant, *B*, and ſurvivor, *A*, b equal; then $t = m$; and the probability required wi become $1 - \frac{m}{2m} - \frac{n}{2m} + \frac{nn}{6mm}$;

That is $(1 - \frac{1}{2} - \frac{n}{2m} + \frac{nn}{6mm} =) \frac{1}{2} - \frac{n}{2m} +$ $\frac{nn}{6mm}$; as in corol. 2. queſt. 38.

The reſults of the laſt 6 queſtions and their corollari are inſerted in the following table; together with the nu merical anſwers, when the complements, *t*, *m*, and *n*, d note ſeverally, 43, 32 and 20.

A TABLE of the probabilities of all the ſeveral orders of ſurvivorſhip, that can poſſibly happen, among three perſons; *C*, the eldeſt; *B*, the ſecond; and *A*, the youngeſt.

Caſe	Poſſeſſor	Expectant	Survivor	If the ages are	SOLUTION. Put n, for the complement of the eldeſt life; m, for that of the ſecond; and t, for that of the youngeſt.	Numerical anſ. for the ages 66, 54, and 43.
1	*A*	*B*	*C*	all unequal	$\frac{nn}{6tm}$	0,0484
2	*B*	*A*	*C*	all unequal	$\frac{nn}{6tm}$	0,0484
3	*A*	*C*	*B*	all unequal	$\frac{n}{2t} - \frac{nn}{3tm}$	0,1358
4	*C*	*A*	*B*	all unequal	$\frac{m}{2t} - \frac{n}{2t} + \frac{nn}{6tm}$	0,1879
5	*B*	*C*	*A*	all unequal	$\frac{n}{2m} - \frac{nn}{3tm}$	0,2157
6	*C*	*B*	*A*	all unequal	$1 - \frac{m}{2t} - \frac{n}{2m} + \frac{nn}{6tm}$	0,3638

The

Caſe	Poſſeſſor	Expectant	Survivor	If the ages are	The ſum of the above 6 probabilities, taken either literally, or figurally, is unity; and they are the ſame, with thoſe given by Mr. *De Moivre*, in his treatiſe of annuities	Numerical anſwer for the ages 66, 54, and 42.
7	*A* *B*	*B* *A*	*C* *C*	$A = B$	$\frac{nn}{6mm}$	0,0651
8	*A* *B* *A* *C*	*B* *A* *C* *A*	*C* *C* *B* *B*	$B = C$	$\frac{n}{6t}$	0,0775
9	*A* *B*	*C* *C*	*B* *A*	$A = B$	$\frac{n}{2m} - \frac{nn}{3mm}$	0,1823
10	*C* *C*	*A* *B*	*B* *A*	$A = B$	$\frac{1}{2} - \frac{n}{2m} + \frac{nn}{6mm}$	0,2526
11	*B* *C*	*C* *B*	*A* *A*	$B = C$	$\frac{1}{2} - \frac{n}{3t}$	0,345
12	*A* *B* *A* *C* *B*	*B* *A* *C* *A* *C*	*C* *C* *B* *B* *A*	all equal	$\frac{1}{6}$	0,1667

QUESTION XLI.

The reſpective ages of three perſons, *A*, *B*, and *C*, of whom *A* is the youngeſt, and *C* the eldeſt, being given; it is required to find the probability, that any one of them ſhall ſurvive the other two.

CASE I.

When the ſurvivor is elder, than the two perſons to be ſurvived; that is, if *C* be to ſurvive *A* and *B*; this ſurvivorſhip will take place, when the three perſons die, either in the order *A*, *B*, *C*; or in the order *B*, *A*, *C*.

Let *n* be the ſurvivor's complement; and let *m*, and *t*, be the complements of the other two perſons *A*, and *B*. Then (by queſt. 35.) the probability of their dying in the order *A*, *B*, *C*, is $\frac{nn}{6tm}$;

And (by queſt. 36.) the probability of their dying in the order *B*, *A*, *C*, is $\frac{nn}{6tm}$;

Therefore, their ſum, $\frac{nn}{3tm}$, will be the probability of *C*'s ſurviving *A*, and *B*.

COROL. I.

If the two perſons, that are to be ſurvived, are of equal ages; that is, if $A = B$; then $t = m$; and the probability required will be $\frac{nn}{3mm}$.

COROL. II.

If the ſurvivor, and the elder of the perſons to be ſurvived, are of equal ages; that is, if $B = C$; then $m = n$; and the probability required will be $\frac{nn}{3tn} = \frac{n}{3t}$.

COROL.

COROL. III.

If the three perſons are of equal ages; then $t=m=n$; and the reſult will become $\frac{nn}{3nn} = \Big)\ \frac{1}{3}$.

CASE. II.

When the ſurvivor is, elder than one, and younger than the other, perſon to be ſurvived.

This ſurvivorſhip will take place, when the three perſons die, in the order *A*, *C*, *B*; or in the order *C*, *A*, *B*.

Let, therefore, the complement of the ſurvivor's life be denoted by m; and thoſe of the two perſons to be ſurvived, by n, and t.

Then (by queſt. 37.) the probability of their dying in the order *A*, *C*, *B*, will be $\frac{n}{2t} - \frac{nn}{3tm}$;

And (by queſt. 38.) the probability of their dying in the order *C*, *A*, *B*, will be $\frac{m}{2t} - \frac{n}{2t} + \frac{nn}{6tm}$;

Therefore, their ſum, $\frac{m}{2t} - \frac{nn}{6tm}$; will be the probability required.

COROL. I.

If the ſurvivor, and the younger perſon to be ſurvived, are of equal ages; that is, if $B = A$; then $t = m$; and the anſwer will be $\left(\frac{m}{2m} - \frac{nn}{6mm} \text{ or}\right) \frac{1}{2} - \frac{nn}{6mm}$.

COROL. II.

If the ſurvivor, and the elder perſon to be ſurvived, are of equal ages; that is, if $B = C$; then $m = n$; and the anſwer will become $\left(\frac{n}{2t} - \frac{nn}{6tn} = \frac{n}{2t} - \frac{n}{6t} =\right) \frac{n}{3t}$; as in corol. 2. caſe 1.

CASE III.

When the ſurvivor is younger, than the two perſons to be ſurvived.

This ſurvivorſhip will take place, when the three perſons die in the order *B*, *C*, *A*; or in the order *C*, *B*, *A*.

Let, therefore, the complement of the ſurvivor's life be denoted by t; and thoſe of the two perſons, to be ſurvived, by n, and m.

Then (by queſt 39.) the probability of their dying in the order *B*, *C*, *A*, will be $\frac{n}{2m} - \frac{nn}{3tm}$;

And (by queſt. 40.) the probability of their dying in the order *C*, *B*, *A*, will be $1 - \frac{m}{2t} - \frac{n}{2m} + \frac{nn}{6tm}$;

Therefore, their ſum, $1 - \frac{m}{2t} - \frac{nn}{6tm}$,

will be the probability required.

COROL. I.

If the two perſons, that are to be ſurvived, are of equal ages; that is, if $B = C$; then the probability required will be $\left(1 - \frac{n}{2t} - \frac{nn}{6tn} = 1 - \frac{n}{2t} - \frac{n}{6t} =\right)$ $1 - \frac{2n}{3t}$.

COROL. II.

If the ſurvivor, and the younger of the perſons to be ſurvived, are of equal ages; that is, if $A = B$; then the anſwer will become $\left(1 - \frac{m}{2m} - \frac{nn}{6mm} = 1 - \frac{1}{2} - \frac{nn}{6mm} =\right)$ $\frac{1}{2} - \frac{nn}{6mm}$; as in corol. 1. caſe 2.

Theſe ſurvivorſhips are inſerted in the following table, in the ſame manner as the former.

A TABLE of the probabilities of the ſurvivorſhip, of any one, out of three perſons; viz, *A*, the youngeſt; *B*, the ſecond; and *C*, the eldeſt.

Caſe	Perſons to be ſurvived.	Survivor	If the lives are	SOLUTION. Put *n*, for the complement of the eldeſt life; *m*, for that of the ſecond; and *t*, for that of the youngeſt.	Numeral anſ. for the ages 43, 54, and 66.
1	*A*, *B*,	*C*	all unequal	$\frac{nn}{3tm}$	0, 0968
2	*A*, *C*,	*B*	all unequal	$\frac{m}{2t} - \frac{nn}{6tm}$	0, 3237
3	*B*, *C*,	*A*	all unequal	$1 - \frac{m}{2t} - \frac{nn}{6tm}$	0, 5795

Caſe	Perſons to be ſurvived.	Survivor	If the lives are	The ſum of the above three probabilities, taken either litterally or numerally, is unity; and they are the ſame with thoſe given by Mr. *De Moivre*, in his treatiſe of annuities.	Numeral anſ. for the ages 43, 54, and 66.
4	*A*, *B*,	*C*	$A = B$	$\frac{nn}{3mm}$	0, 1302
5	*A*, *B*, *A*, *C*,	*C* *B*	$B = C$	$\frac{n}{3t}$	0, 1550
6	*A*, *C*, *B*, *C*,	*B* *A*	$A = B$	$\frac{1}{2} - \frac{nn}{6mm}$	0, 4349
7	*B*, *C*,	*A*	$B = C$	$1 - \frac{2n}{3t}$	0, 6900
8	*A*, *B*, *A*, *C*, *B*, *C*,	*C* *B* *A*	all equal	$\frac{1}{3}$	0 ,3333

QUESTION XLII.

The reſpective ages of three perſons, *A*, *B*, and C (whereof *A* is the youngeſt, and *C* the eldeſt) are given; *A* is poſſeſſed of an eſtate, worth $\mathfrak{P}$ pounds, which upon his deceaſe will devolve to *B*, if then living, fo his life only; after which *C* (if he ſurvives *B*) is to poſſeſs it, without farther limitation; but, if *B* dies befor *A*, then *C* is not to inherit at all; what is the preſen worth of *C*'s intereſt in that eſtate?

SOLUTION.

If t, m, and n, ſeverally, repreſent the complemen of life of *A*, *B*, and *C*; and $\mathfrak{R}$ denotes the value of a annuity (ſecured by land) for the life of *C*.

Then, becauſe $\left(\frac{nn}{6tm}\right) = \frac{1}{m} \times \frac{nn}{6t}$ is (by que 35.) the ſum of the ſeries of the annual probabilities, th the proper order of ſurvivorſhip will take place; therefor $\left(\overline{\frac{1}{m} \times \mathfrak{R} - \frac{n}{6r}} \times \frac{n}{2t} \text{ or }\right) \overline{\mathfrak{R} - \frac{n}{6r}} \times \frac{n}{2tm}$ will b the ſum of the ſeries of preſent worths, correſpondin thereto.

And, ſince $\mathfrak{P}$ is the value of the eſtate in queſtion, therefore $\overline{\mathfrak{R} - \frac{n}{6r}} \times \frac{n\mathfrak{P}}{2tm}$ will be the value o *C*'s intereſt therein; which was required.

EXAMPLE.

C (aged 66) will enjoy an eſtate of 1£. per annum, o a legacy of 25£, if *A* (aged 43) dies before *B* (aged 54 and *B* alſo dies before him (*C*); required the preſent v lue of his intereſt therein?

Her

Here $\overline{\mathfrak{R} - \frac{n}{6r}} = 4{,}468$; $n = 20$; $m = 32$; $t = 43$; and $\mathfrak{P} = 25$;

Therefore $\frac{4{,}468 \times 20 \times 25}{2 \times 43 \times 32} =)$ 0,812 will be the value required.

COROL. I.

If the poſſeſſor, and expectant, are of equal ages; then $t = m$; and the intereſt of the ſurvivor will become $\overline{\mathfrak{R} - \frac{n}{6r}} \times \frac{n\mathfrak{P}}{2mm} = 1{,}091$.

COROL. II.

If the expectant, and ſurvivor, are of equal ages; then $m = n$; and the intereſt of the ſurvivor will become,

$$\overline{\mathfrak{R} - \frac{n}{6r}} \times \frac{n\mathfrak{P}}{2tn} = \Big) \overline{\mathfrak{R} - \frac{n}{6r}} \times \frac{\mathfrak{P}}{2t} = 1{,}299.$$

COROL. III.

If the three perſons are of equal ages; then $t = m = n$; and the intereſt of the ſurvivor will become,

$$\overline{\mathfrak{R} - \frac{n}{6r}} \times \frac{n\mathfrak{P}}{2nn} = \Big) \overline{\mathfrak{R} - \frac{n}{6r}} \times \frac{\mathfrak{P}}{2n} = 2{,}7925.$$

QUESTION XLIII.

The ſame things being given, as in the laſt queſtion; ſuppoſe *B*, to be the poſſeſſor; and *A* (the youngeſt) to be the expectant; and, that the intereſt of *C* (the eldeſt) depending on *B*'s dying before *A*, and *A*'s dying before *C*, is required.

SOLU-

SOLUTION.

If the ſame ſymbols be retained as in the laſt queſtion; then $\left(\frac{nn}{6mt} \text{ or }\right) \frac{1}{t} \times \frac{nn}{6m}$ being (by queſt. 36.) the ſum of the ſeries of annual probabilities; therefore $\left(\frac{1}{t} \times \mathfrak{P} - \frac{n}{6r} \times \frac{n}{2m} \text{ or }\right) \mathfrak{P} - \frac{n}{6r} \times \frac{n}{2mt}$ will be the ſum of the correſponding ſeries of preſent-worths; and conſequently, $\mathfrak{P} - \frac{n}{6r} \times \frac{n\mathfrak{P}}{2mt}$ will be the value of *C*'s intereſt; the ſame as in the laſt queſtion.

COROL. I.

Hence, if the poſſeſſor and expectant are of equal ages; the anſwer will be $\mathfrak{P} - \frac{n}{6r} \times \frac{n\mathfrak{P}}{2mm}$; as in corol. 1. of the laſt queſtion.

COROL. II.

If the poſſeſſor and ſurvivor are of equal ages; the anſwer will be $\mathfrak{P} - \frac{n}{6r} \times \frac{\mathfrak{P}}{2t}$ as in corol. 2. of the laſt queſt.

QUESTION XLIV.

The ſame things being given, as in the two preceding queſtions; ſuppoſe *A* (the youngeſt) to be the poſſeſſor; C (the eldeſt) to be the expectant; and that the intereſt of

B,

B, in the eſtate (depending upon *A*'s dying before *C*, and *C*'s dying before *B*) be required.

SOLUTION.

The probability of this order of ſurvivorſhip, being in queſt. 37.) found by ſubtracting $1 - \frac{n}{2m} - \frac{n}{2t} + \frac{nn}{3mt}$ (the ſum of *n* terms of the ſeries of annual probabilities $\frac{2m-1}{2m} \times \frac{2t-1}{2t} \times \frac{1}{n} + \frac{2m-3}{2m} \times \frac{2t-3}{2t} \times \frac{1}{n}$ &c.) from $1 - \frac{n}{2m}$ (the ſum of *n* terms of the ſeries of annual probabilities $\frac{2m-1}{2m} \times \frac{1}{n} + \frac{2m-3}{2m} \times \frac{1}{n}$ &c.) Th. the value of the ſurvivorſhip muſt (by the ſcholiums annexed to queſt. 23. (caſe 2) and 32) be found by ſubtracting $\frac{^{n}\mathfrak{M}}{n} - {}^{n}\mathfrak{M} - \frac{nn}{6rm} \times \frac{1}{2t}$ (the ſum of *n* terms of the ſeries of preſent worths, correſponding to the former) from $\frac{^{n}\mathfrak{M}}{n}$ (the ſum of *n* terms of the ſeries of preſent worths, correſponding to the latter;) and therefore, if ${}^{n}\mathfrak{M} - \frac{nn}{6rm} \times \frac{1}{2t}$ (their difference) be multiplied by $\mathfrak{P}$ (the value of the eſtate) the product, ${}^{n}\mathfrak{M} - \frac{nn}{6rm} \times \frac{\mathfrak{P}}{2t}$, will be the value of *B*'s intereſt therein.

EXAMPLE.

B (aged 54) the ſecond wife of *C* (aged 66) will enjoy n eſtate of 1£. per annum, or a legacy of 25£. if *A* (aged 43) the daughter of *C*, ſhould die before her father, and ſhe (*B*) ſhould ſurvive him; required the preſent value of *B*'s intereſt therein.

Here ${}^{\bullet}\mathfrak{M} - \frac{nn}{6rm} = 7,888$ (by exa. to queſt. 7.) $\mathfrak{P} = 25$; and $t = 43$.

Then $\frac{7,888 \times 25}{86} = 2,293$ will be the value of *B*'s intereſt.

COROL. I.

If the poſſeſſor and ſurvivor are of equal ages; then $t = m$; and the value of *B*'s intereſt will become

$$ {}^{\bullet}\mathfrak{M} - \frac{nn}{6rm} \times \frac{\mathfrak{P}}{2m} = 3,081. $$

COROL. II.

If the expectant and the ſurvivor are of equal ages, then $m = n$; and ${}^{\bullet}\mathfrak{M} = \mathfrak{N}$; whence the value of *B*'s intereſt will become $\left(\mathfrak{N} - \frac{nn}{6rn} \times \frac{\mathfrak{P}}{2t} \text{ or}\right)$

$\mathfrak{N} - \frac{n}{6r} \times \frac{\mathfrak{P}}{2t}$ as in corol. 2. queſt. 42.

QUESTION XLV.

The ſame things being given, as in the preceding queſtions; ſuppoſe *C* (the eldeſt) to be the poſſeſſor; *A* (the youngeſt) to be the expectant; and that the intereſt of, *B*, in the eſtate (depending upon *C*'s dying before *A*, and *A*'s dying before *B*) be required.

SOLUTION.

Retaining the uſual ſymbols, the ſum of the ſeries of the annual probabilities of this order of ſurvivorſhip is (by queſt. 38. $\frac{m}{2t} - \frac{n}{2t} + \frac{nn}{6mt}$ or) $\frac{1}{t} \times \frac{m}{2} - \frac{n}{2} + \frac{nn}{6m}$; therefore the ſum of the correſponding ſeries of preſent worths will be $\frac{1}{t} \times \mathfrak{M} - \mathfrak{N} + \mathfrak{N} - \frac{n}{6r} \times \frac{n}{2m}$; and conſequently, the preſent value of *B's* intereſt in the eſtate will be $\frac{\mathfrak{P}}{t} \times \mathfrak{M} - \mathfrak{N} + \mathfrak{N} - \frac{n}{6r} \times \frac{n}{2m}$.

EXAMPLE.

C (aged 66) who is poſſeſſed of an eſtate of 1*£*. per annum, has by deed given it to his wife *A* (aged 43) if ſhe be alive at the time of his deceaſe; required the intereſt of *B*, aged 54 (who is brother, and heir at law, to the wife) in the eſtate, dependent on her ſurviving the huſband, and the brother's ſurviving her.

Here $\mathfrak{M} = 10,757$; $\mathfrak{N} = 7,673$; $\mathfrak{N} - \frac{n}{6r} \times \frac{n}{2m} = 1,397$ (by exa. 3. queſt. 4.) $\mathfrak{P} = 25$; and $t = 43$.
Then $10,757 - 7,673 + 1,397 = 4,481$; and $\left(\frac{4,481 \times 25}{43} =\right)$ 2,605 will be the value required.

COROL. I.

If the expectant and ſurvivor are of equal ages; then $t = m$; and the intereſt of *B* will become

$\mathfrak{P}$

$$\frac{\mathfrak{P}}{m} \times \overline{\mathfrak{M} - \mathfrak{N} + \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{n}{2m}} = 3{,}501.$$

COROL. II.

If the poſſeſſor and ſurvivor are of equal ages; then $m = n$, and $\mathfrak{M} = \mathfrak{N}$; therefore the intereſt of *B* will become $\left(\frac{\mathfrak{P}}{t} \times \overline{\mathfrak{N} - \mathfrak{N} + \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{n}{2n}} = \frac{\mathfrak{P}}{t} \times \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{1}{2} =\right) \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{\mathfrak{P}}{2t}$ as in corol. 2. queſt. 42.

QUESTION XLVI.

The ſame things being given, as in the preceding queſtions; ſuppoſe *B* to be the poſſeſſor; *C* (the eldeſt) to be the expectant; and that the intereſt of *A* (the youngeſt) in the eſtate (depending upon *B*'s dying before *C*, and *C*'s dying before *A*) be required.

SOLUTION.

The probability of this order of ſurvivorſhip being (in queſt. 39.) found by ſubtracting $1 - \frac{n}{2m} - \frac{n}{2t} + \frac{nn}{3mt}$, from $1 - \frac{n}{2t}$; therefore the value required muſt be found, by ſubtracting $\frac{{}^{n}\mathfrak{M}}{n} - \overline{{}^{n}\mathfrak{M} - \frac{nn}{6rm}} \times \frac{1}{2t}$, from $\frac{{}^{n}\mathfrak{F}}{n}$;

And therefore, if the remainder, $\frac{{}^n\mathfrak{F}}{n}-\frac{{}^n\mathfrak{M}}{n}+\overline{{}^n\mathfrak{M}-\frac{nn}{6rm}}\times\frac{1}{2t}$; be multiplied by, $\mathfrak{P}$, the value of the eſtate; the product, viz.

$$\mathfrak{P}\times\overline{\frac{{}^n\mathfrak{F}-{}^n\mathfrak{M}}{n}+\overline{{}^n\mathfrak{M}-\frac{nn}{6rm}}\times\frac{1}{2t}},$$

will be the preſent value of *A*'s intereſt therein.

EXAMPLE.

B (aged 54) is poſſeſſed of an eſtate of 1£. per annum; which he has, by deed, ſettled on his ſiſter *C* (aged 66) if ſhe ſurvive him; but if ſhe dies before him, her ſon *A* (aged 43) is not to inherit; required the preſent value of *A*'s expectation, dependent on *C*'s ſurviving *B*, and *A*'s ſurviving *C*.

Here ${}^n\mathfrak{F}=10{,}838$; and ${}^n\mathfrak{M}=9{,}891$ (by queſt. 2.) $\overline{{}^n\mathfrak{M}-\frac{nn}{6rm}}=7{,}888$ (by queſt. 7.) $n=20$; $t=43$; and $P=25$. Then $\frac{10{,}838-9{,}891}{20}=0{,}04735$; and $\frac{7{,}888}{2\times43}=0{,}09172$;

Therefore $(0{,}04735+0{,}09172\times25=)$ 3,477 will be the value required.

COROL. I.

If the poſſeſſor and ſurvivor are of equal ages; then ${}^n\mathfrak{F}={}^n\mathfrak{M}$, and $t=m$; whence the value of *A*'s intereſt will become

$$\left(\mathfrak{P}\times\overline{\frac{{}^n\mathfrak{M}-{}^n\mathscr{M}}{n}+\overline{{}^n\mathscr{M}-\frac{nn}{6rm}}\times\frac{1}{2m}}=\right)$$

$\overline{{}^n\mathscr{M}-\frac{nn}{6rm}}\times\frac{\mathfrak{P}}{2m}$ as in corol 1. queſt. 44.

COROL.

COROL. II.

If the poſſeſſor and expectant are of equal ages; then ${}^{n}\mathfrak{M} = \mathfrak{N}$, and $m = n$; whence the value of *A*'s intereſt will become $\big(\overline{\mathfrak{P} \times \frac{{}^{n}\mathfrak{F} - \mathfrak{N}}{n} + \overline{\mathfrak{N} - \frac{nn}{6rn}}} \times \frac{1}{2t}$

$$=\big)\, \overline{\mathfrak{P} \times \frac{{}^{n}\mathfrak{F} - \mathfrak{N}}{n} + \overline{\mathfrak{N} - \frac{n}{6r}}} \times \frac{1}{2t} = 5{,}255.$$

QUESTION XLVII.

The ſame things being given, as in the preceding queſtions; ſuppoſe *C* (the eldeſt) to be the poſſeſſor; *B*, to be the expectant; and that the intereſt of *A* (the youngeſt) in the eſtate (depending upon *C*'s dying before *B*, and *B*'s dying before *A*) be required.

SOLUTION.

The ſame ſymbols being retained as before; ſince $1 - \frac{m}{2t} - \frac{n}{2m} + \frac{nn}{6tm}$ is (by queſt. 40.) the ſum of the ſeries of the annual probabilities of this order of ſurvivorſhip; and ſince $\frac{{}^{m}\mathfrak{F}}{m}$, $\frac{\mathfrak{N}}{m}$, and $\overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{n}{2tm}$, are the ſums of the three ſeries of preſent worths, ſeverally, correſponding to the three ſeries of annual probabilities, whoſe ſums are, $1 - \frac{m}{2t}$, $\frac{n}{2m}$, and $\frac{nn}{6tm}$; Therefore $\big(\mathfrak{P} \times \overline{\frac{{}^{m}\mathfrak{F}}{m}, - \frac{\mathfrak{N}}{m} + \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{n}{2tm}}$

or) $\frac{\mathfrak{P}}{m}\times\overline{{}^m\mathfrak{F}-\mathfrak{N}+\overline{\mathfrak{N}-\frac{n}{6r}}\times\frac{n}{2t}}$ will be the value of *A*'s intereſt in the eſtate.

EXAMPLE.

C (aged 66) is poſſeſſed of an eſtate of 1 £. per annum, which will deſcend to his brother *B* (aged 54) if he be alive at the time of *C*'s deceaſe; it is required to determine, what intereſt *A* (the wife of *B*, aged 43) has in the eſtate, dependent on *B*'s ſurviving *C*, and *A*'s ſurviving *B*?

Here ${}^m\mathfrak{F}=12{,}578$; $\mathfrak{N}=7{,}673$; $\overline{\mathfrak{N}-\frac{n}{6r}}\times\frac{n}{2t}=1{,}039$, $\mathfrak{P}=25$; and $m=32$:

Therefore $(\overline{12{,}578-7{,}673+1{,}039}\times\frac{25}{32}=)$ 4,644 will be the value required.

COROL. I.

If the expectant and ſurvivor are of equal ages; then ${}^m\mathfrak{F}=\mathfrak{M}$, and $t=m$; therefore the intereſt of *A* will become $\frac{\mathfrak{P}}{m}\times\overline{\mathfrak{M}-\mathfrak{N}+\overline{\mathfrak{N}-\frac{n}{6r}}\times\frac{n}{2m}}$ as in corol. 1. queſt. 45.

COROL. II.

If the poſſeſſor and expectant are of equal ages; then $= n$; and the value of *A*'s intereſt in the eſtate will be

$$\left(\frac{\mathfrak{P}}{n}\times\overline{{}^n\mathfrak{F}-\mathfrak{N}+\overline{\mathfrak{N}-\frac{n}{6r}}\times\frac{n}{2t}}=\right)$$

$\mathfrak{P}\times\overline{\frac{{}^n\mathfrak{F}-\mathfrak{N}}{n}+\overline{\mathfrak{N}-\frac{n}{6r}}\times\frac{1}{2t}}$ as in corol. 2. queſt. 46.

QUES-

QUESTION XLVIII.

To find the preſent worth of the reverſion of an eſtate, worth $\mathfrak{P}$ pounds, depending on one perſon's ſurviving two others.

SOLUTION.

Let *A*, *B*, and *C*, ſeverally, repreſent the name of the younger, ſecond, and elder perſon; *t*, *m*, and *n*, their complements of life; $\mathfrak{M}$ and $\mathfrak{N}$ the values of annuities (ſecured by land) for the lives of *B* and *C*; ${}^{m}\mathfrak{F}$, ${}^{n}\mathfrak{F}$, and ${}^{n}\mathfrak{M}$, the values of ſuch annuities for *m*, and *n*, years certain, if *A* and *B* ſhould live ſo long.

CASE I.

If the ſurvivor be elder than the two perſons to be ſurvived; that is, if *C* is to ſurvive both *A* and *B*.

Then, if *A* be ſuppoſed to die firſt; and *B*, ſecond; the value of *C*'s intereſt in the eſtate will (by queſt. 42.) be $\mathfrak{N} - \frac{n}{6r} \times \frac{n\mathfrak{P}}{2tm}$; and if *B* be ſuppoſed to die firſt; and *A* ſecond; the value of *C*'s intereſt will alſo be $\mathfrak{N} - \frac{n}{6r} \times \frac{n\mathfrak{P}}{2tm}$; by queſt. 43.

Therefore their ſum, $\mathfrak{N} - \frac{n}{6r} \times \frac{n\mathfrak{P}}{tm}$, will be the value required.

EXAMPLE.

C (aged 66) will become poſſeſſed of an eſtate of 1*l.* per annum, if he ſurvives his two neices *A*, and *B*, of the reſpective ages of 43, and 54; required the preſent value of his intereſt in that eſtate.

Here $\overline{\mathfrak{P} - \frac{n}{6r}} \times \frac{n\mathfrak{P}}{2tm} = 0,812$ by example to queſt. 42.

Therefore $(0,812 \times 2 =)$ 1,624 will be the value required.

COROL. I.

If the perſons, to be ſurvived, are of equal ages.

Then $t = m$; and the value will become

$$\overline{\mathfrak{P} - \frac{n}{6r}} \times \frac{n\mathfrak{P}}{mm} = 2,182.$$

COROL. II.

If one of the perſons, to be ſurvived, be of the ſame ge with the ſurvivor; then $m = n$; and the value will become $\left(\overline{\mathfrak{P} - \frac{n}{6r}} \times \frac{n\mathfrak{P}}{tn} =\right) \overline{\mathfrak{P} - \frac{n}{6r}} \times \frac{\mathfrak{P}}{t} =$,598.

COROL. III.

If all the three perſons are of equal ages; then $t = m = n$, and the value required will be $\overline{\mathfrak{P} - \frac{n}{6r}} \times \frac{\mathfrak{P}}{n} = 5,585$.

CASE II.

If the ſurvivor be, younger than the one, and elder an the other, of the two perſons to be ſurvived; that is, *B* is to ſurvive *C* and *A*.

Then if *A*, be ſuppoſed to die firſt; and *C*, ſecond; e value of *B*'s intereſt will (by queſt. 44.) be

$\overline{{}^{n}\mathfrak{M}-\frac{nn}{6rm}}\times\frac{\mathfrak{P}}{2t}$: and if, *C* be ſuppoſed to die firſt; and *A*, ſecond; *B*'s intereſt will by queſt. 45. be worth $(\overline{\mathfrak{M}-\mathfrak{N}+\overline{\mathfrak{N}-\frac{n}{6r}}\times\frac{n}{2m}}\times\frac{\mathfrak{P}}{t}=)$

$$\overline{\overline{\mathfrak{M}-\mathfrak{N}}\times 2+\overline{\mathfrak{N}-\frac{n}{6r}}\times\frac{n}{m}}\times\frac{\mathfrak{P}}{2t}.$$

Therefore, their ſum

$$(\overline{\overline{\mathfrak{M}-\mathfrak{N}}\times 2+\overline{\mathfrak{N}-\frac{n}{6r}}\times\frac{n}{m}+\overline{{}^{n}\mathfrak{M}-\frac{nn}{6rm}}}\times\frac{\mathfrak{P}}{2t}$$

$$\text{or) }\overline{\overline{\mathfrak{M}-\mathfrak{N}}\times 2+{}^{n}\mathfrak{M}+\overline{\mathfrak{N}-\frac{2n}{6r}}\times\frac{n}{m}}\times\frac{\mathfrak{P}}{2t}$$

will be the value required.

EXAMPLE.

C (aged 66) is poſſeſſed of an eſtate of 1 £. per annum, which will deſcend to his daughter, *A* (aged 43); but after the deceaſe of both of them, will become the property of *B*, the brother of *C*, aged 54, if he be then living; required the value of *B*'s intereſt in that eſtate.

Here $\mathfrak{M}=10,757$; $\mathfrak{N}=7,673$; ${}^{n}\mathfrak{M}=9,891$; $\frac{n}{6r}=3,205$; $n=20$; $m=32$; $\mathfrak{P}=25$; and $t=43$;

Then $\overline{10,757-7,673}\times 2=6,168$; $3,205\times 2=6,410$; $7,673-6,410=1,263$; $\overline{\mathfrak{N}-\frac{2n}{6r}}\times\frac{n}{m}=\frac{1,263\times 20}{32}=0,789$; and $6,168+9,891+0,789=16,848$.

Therefore $(\frac{16,848\times 25}{2\times 43}=)$ 4,898 will be the value required.

COROL. I.

If the ſurvivor and the younger perſon, to be ſurvived, are of the ſame age; then $t = m$; and the value required will be,

$$\overline{\overline{\mathfrak{M}-\mathfrak{N}}\times 2+{}^{n}\mathfrak{M}+\overline{\mathfrak{N}-\frac{2n}{6r}}}\times\frac{n}{m}\times\frac{\mathfrak{P}}{2m}$$

$= 6{,}581.$

COROL. II.

If the ſurvivor and the elder perſon, to be ſurvived, re of the ſame ages; then $\mathfrak{M} = {}^{n}\mathfrak{M} = \mathfrak{N}$, and $n = n$; whence the value required will be

$$\overline{\overline{\mathfrak{N}-\mathfrak{N}}\times 2+\mathfrak{N}+\overline{\mathfrak{N}-\frac{2n}{6r}}}\times\frac{n}{n}\times\frac{\mathfrak{P}}{2t};$$

That is $\left(\overline{2\,\mathfrak{N}-\frac{2n}{6r}}\times\frac{\mathfrak{P}}{2t}\text{ or}\right)\ \overline{\mathfrak{N}-\frac{n}{6r}}\times\frac{\mathfrak{P}}{t}$ s in corol. 2. caſe 1.

CASE III.

If the ſurvivor be younger, than either of the perſons, o be ſurvived; that is, if A is to ſurvive B and C.

Then, if B be ſuppoſed to die firſt, and C ſecond; the alue of A's intereſt will (by queſt. 46.) be

$$\overline{\frac{{}^{n}\mathfrak{F}-{}^{n}\mathfrak{M}}{n}+\overline{{}^{n}\mathfrak{M}-\frac{nn}{6rm}}}\times\frac{1}{2t}\times\mathfrak{P};$$

And, if C be ſuppoſed to die firſt, and B ſecond; then, e value of A's intereſt will (by queſt. 47.) be

$$\overline{\frac{{}^{m}\mathfrak{F}-\mathfrak{N}}{m}+\overline{\mathfrak{N}-\frac{n}{6r}}}\times\frac{n}{2tm}\times\mathfrak{P}.$$

Therefore, their ſum

$$\left.\begin{array}{l}\frac{{}^{n}\mathfrak{F}-{}^{n}\mathfrak{M}}{n}+\frac{{}^{m}\mathfrak{F}-\mathfrak{N}}{m}\\ +\overline{{}^{n}\mathfrak{M}-\frac{nn}{6rm}}\times\frac{1}{2t}+\overline{\mathfrak{N}-\frac{n}{6r}}\times\frac{n}{2tm}\end{array}\right\}\times\mathfrak{P}$$

$$\text{or)}\left.\begin{array}{l}\frac{{}^{n}\mathfrak{F}-{}^{n}\mathfrak{M}}{n}+\frac{{}^{m}\mathfrak{F}-\mathfrak{N}}{m}\\ +\overline{{}^{n}\mathfrak{M}+\mathfrak{N}-\frac{2n}{6r}}\times\frac{n}{m}\times\frac{1}{2t}\end{array}\right\}\times\mathfrak{P}$$ will be the value required.

EXAMPLE.

C (aged 66) is poſſeſſed of an eſtate of 1£. per annum, in which his wife *B* (aged 54) is jointured; which eſtate will deſcend to his ſon *A* (aged 43) if he ſurvives them both; what is the preſent value of *A*'s intereſt in that eſtate?

Here ${}^{n}\mathfrak{F}=10{,}838$; ${}^{n}\mathfrak{M}=9{,}891$; ${}^{m}\mathfrak{F}=12{,}578$; $\mathfrak{N}=7{,}673$; $n=20$; $m=32$; $t=43$; $\mathfrak{P}=25$,

and $\overline{\mathfrak{N}-\frac{2n}{6r}}\times\frac{n}{m}=0{,}789$ (by exam. caſe 2.)

Then $\frac{10{,}838-9{,}891}{20}=0{,}04735$; $\frac{12{,}578-7{,}673}{32}=0{,}15328$;

And $\frac{9{,}891+0{,}789}{2\times43}=0{,}12419$:

Therefore $(\overline{0{,}04735+0{,}15328+0{,}12419}\times25=$ 8,1205 will be the value required.

COROL. I.

If the ſurvivor and the younger of the perſons, to be ſurvived, are of equal ages; then ${}^{n}\mathfrak{F}={}^{n}\mathfrak{M}$, ${}^{m}\mathfrak{F}=\mathfrak{M}$, and $t=m$; therefore

$$\left(\overline{\frac{\mathfrak{M}-\mathfrak{N}}{m} + \overline{{}^{n}\mathfrak{M} + \overline{\mathfrak{N} - \frac{2n}{6r}} \times \frac{n}{m}} \times \frac{1}{2m}} \times \mathfrak{P} =\right)$$

$$\overline{\overline{\overline{\mathfrak{M}-\mathfrak{N}}} \times 2 + {}^{n}\mathfrak{M} + \overline{\mathfrak{N} - \frac{2n}{6r}} \times \frac{n}{m}} \times \frac{\mathfrak{P}}{2m}$$

(as in corol. 1. caſe 2.) will be the value required.

COROL. II.

If the two perſons, to be ſurvived, are of equal ages; then $m = n$. ${}^{m}\mathfrak{F} = {}^{n}\mathfrak{F}$, and ${}^{n}\mathfrak{M} = \mathfrak{N}$; therefore

$$\left(\overline{\frac{\overline{\overline{{}^{n}\mathfrak{F}-\mathfrak{N}}} \times 2}{n} + \overline{\mathfrak{N} + \overline{\overline{\mathfrak{N} - \frac{2n}{6r}}}} \times \frac{1}{2t}} \times \mathfrak{P} \text{ or}\right)$$

$$\overline{\frac{\overline{{}^{n}\mathfrak{F}-\mathfrak{N}} \times 2}{n} + \overline{\mathfrak{N} - \frac{n}{6r}}} \times \frac{1}{t} \times \mathfrak{P}$$ will be the value required.

QUESTION XLIX.

A perſon who is intituled to an abſolute eſtate, in fee ſimple, if he ſurvives the preſent poſſeſſor, grants the reverſion of an annuity of 1£. per annum, to be ſecured thereon, to a third perſon, for the remainder of his (the third perſon's) life after the deceaſe of the preſent poſſeſſor; but ſo, that if the granter doth not live to come into poſſeſſion, the annuity is not to take place: what is the preſent value of that reverſion, ſo depending on the ſurvivorſhip?

SOLUTION.

Let *A*, *B*, and *C*, repreſent the names of the youngeſt, ſecond, and eldeſt, of the three perſons; and let *t*, *m*, and *n*, be their reſpective complements of life.

CASE I.

If the poſſeſſor, be the youngeſt; the expectant, ſecond; and the annuitant, the eldeſt, of the three perſons.

Then the annual probabilities of the expectant's coming into poſſeſſion will (by queſt. 21) be $\frac{2m-1}{2m} \times \frac{1}{t}$, $\frac{2m-3}{2m} \times \frac{1}{t}$, $\frac{2m-5}{2m} \times \frac{1}{t}$, &c. that is $\frac{1}{t} - \frac{1}{2mt}$, $\frac{1}{t} - \frac{3}{2mt}$, $\frac{1}{t} - \frac{5}{2mt}$, &c.

Now, if the ſurvivorſhip ſhould take place in the firſt year, the annuitant will be entituled to the annuity for his whole life, *viz.* to $\frac{2n-1}{2nr} + \frac{2n-3}{2nr^2} + \frac{2n-5}{2nr^3}\,(n)$;

Therefore if, $\frac{1}{t} - \frac{1}{2mt}$, the probability of the ſurvivorſhip's taking place therein, be multiplied thereby: the product, *viz.* $\overline{\frac{1}{t} - \frac{1}{2mt}} \times \overline{\frac{2n-1}{2nr} + \frac{2n-3}{2nr^2} + \frac{2n-5}{2nr^3}}\,(n)$, will be the annuitant's expectation for that year.

But, if the ſurvivorſhip fails in the firſt, and takes place in the ſecond year, then the annuitant will be entituled to the whole annuity for his life, except the firſt annual payment (which he will not receive, becauſe the granter will not be then in poſſeſſion) *viz.* $\frac{2n-3}{2nr^2} + \frac{2n-5}{2nr^3} + \frac{2n-7}{2nr^4}\,(n-1)$; which, being multiplied by, $\frac{1}{t} - \frac{3}{2mt}$, the probability of the ſurvivorſhip's, ſo, taking place, will produce,

$\overline{\frac{1}{t} - \frac{3}{2mt}} \times \overline{\frac{2n-3}{2nr^2} + \frac{2n-5}{2nr^3} + \frac{2n-7}{2nr^4}}\,(n-1)$, the annuitant's expectation for the ſecond year.

In like manner, it may be ſhewn that the annuitants expectation, for the third year, will be $\overline{\frac{1}{t}-\frac{5}{2mt}}\times\overline{\frac{2n-5}{2nr^3}+\frac{2n-7}{2nr^4}+\frac{2n-9}{2nr^5}}\,(n-2)$, &c.

Now if theſe expectations be expanded by multiplication, and the affirmative terms, in each product, be ſeparated from the negative, they will appear as below.

$$\begin{array}{lllll}
+\frac{2n-1}{2nr}\times\frac{1}{t} & +\frac{2n-3}{2nr^2}\times\frac{1}{t} & +\frac{2n-5}{2nr^3}\times\frac{1}{t} & +\frac{2n-7}{2nr^4}\times\frac{1}{t} & \&c.\\
 & +\frac{2n-3}{2nr^2}\times\frac{1}{t} & +\frac{2n-5}{2nr^3}\times\frac{1}{t} & +\frac{2n-7}{2nr^4}\times\frac{1}{t} & \&c.\\
 & & +\frac{2n-5}{2nr^3}\times\frac{1}{t} & +\frac{2n-7}{2nr^4}\times\frac{1}{t} & \&c.\\
 & & & +\frac{2n-7}{2nr^4}\times\frac{1}{t} & \&c.\\
-\frac{2n-1}{2nr}\times\frac{1}{2mt} & -\frac{2n-3}{2nr^2}\times\frac{1}{2mt} & -\frac{2n-5}{2nr^3}\times\frac{1}{2mt} & -\frac{2n-7}{2nr^4}\times\frac{1}{2mt} & \&c.\\
 & -\frac{2n-3}{2nr^2}\times\frac{3}{2mt} & -\frac{2n-5}{2nr^3}\times\frac{3}{2mt} & -\frac{2n-7}{2nr^4}\times\frac{3}{2mt} & \&c.\\
 & & -\frac{2n-5}{2nr^3}\times\frac{5}{2mt} & -\frac{2n-7}{2nr^4}\times\frac{5}{2mt} & \&c.\\
 & & & -\frac{2n-7}{2nr^4}\times\frac{7}{2mt} & \&c.
\end{array}$$

Where the affirmative ſerieſes, being added, produce the ſeries, $\frac{2n-1}{2nr} \times \frac{1}{t} + \frac{2n-3}{2nr^2} \times \frac{2}{t} + \frac{2n-5}{2nr^3} \times \frac{3}{t} + \frac{2n-7}{2nr^4} \times \frac{4}{t}\ (n)$;

And the negative ſerieſes, being added, produce the ſeries $\left(-\frac{2n-1}{2nr} \times \frac{1}{2mt} - \frac{2n-3}{2nr^2} \times \frac{4}{2mt} - \frac{2n-5}{2nr^3} \times \frac{9}{2mt} - \frac{2n-7}{2nr^4} \times \frac{16}{2mt}\ (n);\right.$

$\left.\text{or}\right)\ \frac{2n-1}{2nr} \times \frac{1}{2m} \times \frac{1}{t} - \frac{2n-3}{2nr^2} \times \frac{2}{2m} \times \frac{2}{t} - \frac{2n-5}{2nr^3} \times \frac{3}{2m} \times \frac{3}{t} - \frac{2n-7}{2nr^4} \times \frac{4}{2m} \times \frac{4}{t}\ (n)$:
The firſt, of which may be ſummed, by queſt. 21. vol. 2, and the latter, by queſt. 22. vol. 2.

Now firſt, in order to find the ſum of the affirmative ſeries; it is known that the ſum of $\frac{2n-1}{2nr} + \frac{2n-3}{2nr^2}\ (n)$ is $\mathfrak{P}$ and its common difference $+\frac{1}{nr}$; and the ſum of $\frac{1}{t} + \frac{2}{t}\ (n)$ is $\frac{nn+n}{2t}$, and its common difference $-\frac{1}{t}$:

Whence (by queſt. 21.vol. 2.) the ſum of n terms of the ſeries of products will be

$$\left(\frac{\mathfrak{P} \times \frac{nn+n}{2t}}{n} - \frac{\overline{n+1}\,.\,n\,.\,\overline{n-1}}{2\,.\,2\,.\,3} \times \frac{1}{ntr}\ \text{or}\right)$$

$$\frac{\overline{n+1} \times \mathfrak{P}}{2t} - \frac{\overline{n+1} \times \overline{n-1}}{2t \times 6r};$$

That is $\overline{\mathfrak{P} - \frac{n-1}{6r}} \times \frac{n+1}{2t}$; will be the ſum of the affirmative ſeries.

Secondly, to find the ſum of the negative ſeries;

The ſum of $\frac{2n-1}{2nr}+\frac{2n-3}{2nr^2}$ (n) is $\mathfrak{P}$, its firſt term $\frac{2n-1}{2nr}$, and its common difference $\frac{1}{nr}$;

The ſum of $\frac{1}{t}+\frac{2}{t}$ (n) is $\frac{nn+n}{2t}$, its firſt term $\frac{1}{t}$, and its common difference $-\frac{1}{t}$;

The ſum of $\frac{1}{2m}+\frac{2}{2m}$ (n) is $\frac{nn+n}{4m}$, its firſt term $\frac{1}{2m}$, and its common difference $-\frac{1}{2m}$;

Whence the ſum of the ſeries of products will (by qu. 22. vol. 2.) be

$$\left(\frac{\mathfrak{P}\times\frac{nn+n}{2t}\times\frac{nn+n}{4m}}{nn}+\frac{\overline{n+1}\cdot n\cdot\overline{n-1}}{2\cdot 2\cdot 3}\times\overline{\frac{2n-1}{4nmtr}-\frac{1}{2\,nmtr}-\frac{1}{2\,nmtr}}\right.$$

$$\left.-\frac{\overline{n+1}\cdot n\cdot\overline{n-1}^{\,2}}{2\cdot 2\cdot 2}\times\frac{1}{2\,nmtr}\text{ or}\right)$$

$$\mathfrak{P}\times\frac{n+1}{2t}\times\frac{n+1}{4m}+\frac{\overline{n+1}\cdot\overline{n-1}}{2\cdot 6r}\times\frac{2n-5}{4mt}$$

$$\left(-\frac{\overline{n+1}\cdot\overline{n-1}^{\,2}}{8mt\times 2r};\right.$$

That is $\mathfrak{P}\times\frac{n+1}{2t}\times\frac{n+1}{4m}+\frac{\overline{n+1}\cdot\overline{n-1}}{8mt}\times\frac{2n-5}{6r}$

$$\left(-\frac{\overline{n+1}\cdot\overline{n-1}}{8mt}\times\frac{3n-3}{6r}:\right.$$

But $\frac{2n-5}{6r}-\frac{3n-3}{6r}=-\frac{n+2}{6r}$; whence we ſhall have ($\mathfrak{P} \times \frac{n+1}{2t} \times \frac{n+1}{4m} - \frac{\overline{n+1} \,.\, \overline{n-1}}{8mt} \times \frac{n+1}{6r}$

Or) $\mathfrak{P} \times \frac{\overline{n+1} \,.\, \overline{n+1}}{8mt} - \frac{\overline{n+1} \,.\, \overline{n+2}}{8mt} \times \frac{n-1}{6r}$;

Now, if we write, $\frac{\overline{n+1} \times \overline{n+1}}{8mt} \times \frac{n-1}{6r}$, for $\frac{\overline{n+1} \times \overline{n+2}}{8mt} \times \frac{n-1}{6r}$ (which differ by $\frac{\overline{n+1} \times \overline{n-1}}{48mtr}$ which is leſs than $\frac{1}{48r}$) the above expreſſion will become ($\mathfrak{P} \times \frac{\overline{n+1}^2}{8mt} - \frac{\overline{n+1}^2}{8mt} \times \frac{n-1}{6r}$

or) $\overline{\mathfrak{P} - \frac{n-1}{6r}} \times \frac{\overline{n+1}^2}{8mt}$; which is the ſum of the negative ſeries.

Now, if from, $\overline{\mathfrak{P} - \frac{n-1}{6r}} \times \frac{n+1}{2t}$, the ſum of the affirmative ſeries, we take, $\overline{\mathfrak{P} - \frac{n-1}{6r}} \times \frac{\overline{n+1}^2}{8mt}$, the ſum of the negative ſeries; then, the remainder

($\overline{\mathfrak{P} - \frac{n-1}{6r}} \times \overline{\frac{n+1}{2t} - \frac{\overline{n+1}^2}{8mt}}$

or) $\overline{\mathfrak{P} - \frac{n-1}{6r}} \times \overline{1 - \frac{n+1}{4m}} \times \frac{n+1}{2t}$ will be the value of the annuitants whole expectation; that is, of the reverſion depending on the ſurvivorſhip.

EXAMPLE.

B (aged 54) who is heir to an eſtate, upon the death of his neice *A* (aged 43) if he be then alive, grants an annuity of 1£. per annum to *C* (aged 66) for his life, ſecured on, and iſſuable out of the eſtate, to commence at the time, when he (*B*) enters into poſſeſſion thereof; what preſent money ought *C* to pay, for the ſame?

Here $\mathfrak{P} = 7{,}673$; $\frac{n-1}{6r} = 3{,}045$; $\frac{n+1}{4m} = \left(\frac{21}{4 \cdot 32} =\right) \frac{21}{128}$; $1 - \frac{21}{128} = \left(\frac{128 - 21}{128} =\right) \frac{107}{128}$; $\frac{n+1}{2t} = \left(\frac{21}{2 \cdot 43} =\right) \frac{21}{86}$; and $7{,}673 - 3{,}045 = 4{,}628$.

Then $\left(\frac{4{,}628 \times 107 \times 21}{128 \times 86} =\right) 0{,}945$ will be the value required.

COROL. I.

If the poſſeſſor and expectant are of equal ages; then $t = m$; and the value of the annuitant's expectation will become, $\overline{\mathfrak{P} - \frac{n-1}{6r}} \times \overline{1 - \frac{n+1}{4m}} \times \frac{n+1}{2m}$.

COROL. II.

If the expectant and annuitant are of equal ages; then $m = n$; and the value required will be

$$\left(\overline{\mathfrak{P} - \frac{n-1}{6r}} \times \overline{1 - \frac{n+1}{4n}} \times \frac{n+1}{2t} \text{ or}\right)$$

$$\overline{\mathfrak{P} - \frac{n-1}{6r}} \times \frac{3n-1}{4n} \times \frac{n+1}{2t}$$

COROL. III.

If the three perſons be of equal ages; then,

$\overline{\mathfrak{P} - \frac{n-1}{6r}} \times \frac{3n-1}{4n} \times \frac{n+1}{2n}$, will be the value required.

CASE II.

If the expectant, be the youngeſt; the poſſeſſor, the ſecond; and the annuitant, the eldeſt, of the three perſons.

Then (by writing m for t, and t for m, in the ſolution of the laſt caſe) we ſhall have,

$\overline{\mathfrak{P} - \frac{n-1}{6r}} \times \overline{1 - \frac{n+1}{4t}} \times \frac{n+1}{2m}$, for the value required.

COROL. I.

Hence if $t = m$, the value will become,

$\overline{\mathfrak{P} - \frac{n-1}{6r}} \times \overline{1 - \frac{n+1}{4m}} \times \frac{n+1}{2m}$, as in corol. 1. caſe 1.

COROL. II.

If $m = n$; then $\overline{\mathfrak{P} - \frac{n-1}{6r}} \times \overline{1 - \frac{n+1}{4t}} \times \frac{n+1}{2n}$ will be the value required.

CASE III.

If the poſſeſſor, be the youngeſt; the annuitant, the ſecond; and the expectant, the eldeſt, of the three perſons

Then, the annual probabilities of the expectant's coming into poſſeſſion will be $\frac{2n-1}{2n} \times \frac{1}{t}$, $\frac{2n-3}{2n} \times \frac{1}{t}$, c, continued only to n terms; which probabilities may as in the laſt caſe) be wrote $\frac{1}{t} - \frac{1}{2nt}$, $\frac{1}{t} - \frac{3}{2nt}$ &c. nd conſequently, if the ſurvivorſhip takes place in the rſt, ſecond, third, &c. year the annuitant's expectaions will be,

$$\overline{- \frac{1}{2nt}} \times \overline{\frac{2m-1}{2mr} + \frac{2m-3}{2mr^2} + \frac{2m-5}{2mr^3}}\ (m),$$

$$\overline{- \frac{3}{2nt}} \times \overline{\frac{2m-3}{2mr^2}.} \overline{+ \frac{2m-5}{2mr^3} + \frac{2m-7}{2mr^4}}\ (m-1),$$

$$\overline{- \frac{5}{2nt}} \times \overline{\frac{2m-5}{2mr^3} + \frac{2m-7}{2mr^4}} \overline{+ \frac{2m-9}{2mr^5}}\ (m-2), \text{ \&c.}$$

Of which ſerieſes the number will be only n; becauſe e probability of the expectant's ſurviving the poſſeſſor eaſes, at the end of n years.

Let now theſe ſerieſes be ſuppoſed to be expanded by ultiplication, and ranged as in the firſt caſe; and, let s examine the conſequence of expounding the ſymbol, , by the numbers 2, 3, and 4, reſpectively.

If $n = 2$; then, only, the firſt and ſecond products ill be to be added together; and the reſult will be reprented by $+ \frac{2m-1}{2mr} \times \frac{1}{t} + \frac{2m-3}{2mr^2} \times \frac{2}{t} + \frac{2m-5}{2mr^3} \times \frac{2}{t}$ $\frac{2m-7}{2mr^4} \times \frac{2}{t}\ (m)$, $- \frac{2m-1}{2mr} \times \frac{1}{2mt} - \frac{2m-3}{2mr^2} \times \frac{4}{2nt}$ $\frac{2m-5}{2mr^3} \times \frac{4}{2nt} - \frac{2m-7}{2mr^4} \times \frac{4}{2nt}\ (m)$.

here the terms following the 2 firſt, in each ſeries, conſiſt ſuch expreſſions as are found in the value of the ſingle e of the annuitant, multiplied into the conſtant factors, $\frac{2}{t}$

$\frac{2}{t}$ and $\frac{4}{2nt}$; which (becauſe they ariſe from the ſuppoſition of the equality of 2 and n) may be wrote $\frac{n}{t}$ and $\left(\frac{nn}{2nt}\text{ or}\right)\frac{n}{2t}$; and conſequently (in this caſe) the negative ſeries will, after the ſecond term, be juſt half the affirmative.

Again, if $n = 3$; then three of the ſerieſes, produced by the multiplications, will be to be added together; and the reſult will be repreſented by

$$+\frac{2m-1}{2mr}\times\frac{1}{t}+\frac{2m-3}{2mr^2}\times\frac{2}{t}+\frac{2m-5}{2mr^3}\times\frac{3}{t}$$

$$\left(+\frac{2m-7}{2mr^4}\times\frac{3}{t}+\frac{2m-9}{2mr^5}\times\frac{3}{t}\right.(m),$$

$$-\frac{2m-1}{2mr}\times\frac{1}{2nt}-\frac{2m-3}{2mr^2}\times\frac{4}{2nt}-\frac{2m-5}{2mr^3}\times\frac{9}{2nt}$$

$$\left(-\frac{2m-7}{2mr^4}\times\frac{9}{2nt}-\frac{2m-9}{2mr^5}\times\frac{9}{2nt}\right.(m);$$

wherein, after the three firſt terms, the factors (which are multiplied into the terms of the ſingle life of the annuitant) become invariably $\frac{3}{t}$ and $\frac{9}{2nt}$; which (becauſe $n = 3$) may be wrote $\frac{n}{t}$ and $\left(\frac{nn}{2nt}\text{ or}\right)\frac{n}{2t}$ as before.

And, if we proceed to make $n = 4$, we ſhall find the ſame things occur after four terms; and conſequently the value of the annuity required will be repreſented by

$$\frac{2m-1}{2mr}\times\frac{1}{t}+\frac{2m-3}{2mr^2}\times\frac{2}{t}+\frac{2m-5}{2mr^3}\times\frac{3}{t}(n)$$

$$\left(+\frac{2m-2n-1}{2mr^{n+1}}\times\frac{n}{t}+\frac{2m-2n-3}{2mr^{n+2}}\times\frac{n}{t}\right.(m-n);$$

$-\,2m-1$

$$-\frac{2m-1}{2mr} \times \frac{1}{2nt} - \frac{2m-3}{2mr^2} \times \frac{4}{2nt}$$

$$-\frac{2m-5}{2mr^3} \times \frac{9}{2nt}\ (n) - \frac{2m-2n-1}{2mr^{n+1}} \times \frac{nn}{2tn}$$

$$\left(- \frac{2m-2n-3}{2mr^{n+2}} \times \frac{nn}{2tn}\ (m-n);\right.$$

Now, becauſe $\frac{2m-1}{2mr} + \frac{2m-3}{2mr^2} + \frac{2m-5}{2mr^3}\ (n) = {}^{n}\mathfrak{M}$, therefore $\frac{2m-2n-1}{2mr^{n+1}} + \frac{2m-2n-3}{2mr^{n+2}}\ (m-n)$ $= \mathfrak{M} - {}^{n}\mathfrak{M}$; and; becauſe $\frac{n}{t} - \frac{n}{2t} = \frac{n}{2t}$; therefore, the difference of thoſe parts of the affirmative and negative ſeries, which have invariable factors, will be

$$\overline{\mathfrak{M} - {}^{n}\mathfrak{M}} \times \frac{n}{2t}.$$

It remains therefore, only, to find the ſum of thoſe n terms of each ſeries, which have variable factors.

Firſt, in order to find the ſum of, $\frac{2m-1}{2mr} \times \frac{1}{t}$ $+ \frac{2m-3}{2mr^2} \times \frac{2}{t}\ (n)$, it is known, that $\frac{2m-1}{2mr} + \frac{2m-3}{2mr^2}\ (n) = {}^{n}\mathfrak{M}$; the common difference thereof being $\frac{1}{mr}$;

Alſo $\frac{1}{t} + \frac{2}{t}\ (n) = \frac{nn+n}{2t}$, and the common difference $-\frac{1}{t}$; therefore the ſum of n terms of the ſeries of products will (by queſt. 21. vol. 2.) be

$({}^{n}\mathfrak{M} \times \frac{nn+n}{2t} \times \frac{1}{n} - \frac{\overline{n+1}\,.\,n\,.\,\overline{n-1}}{2\,.\,2\,.\,3} \times \frac{1}{mtr}$

or) $\mathfrak{M} \times \frac{n+1}{2t} - \frac{n\,.\,\overline{n-1}}{6rm} \times \frac{n+1}{2t}$; that is

$\overline{{}^{n}\mathfrak{M} - \frac{n\,.\,\overline{n-1}}{6rm}} \times \frac{n+1}{2t}$ will be the ſum of the affirmative ſeries.

Again to find the ſum of the ſeries, $\frac{2m-1}{2mr} \times \frac{1}{2nt} + \frac{2m-3}{2mr^2} \times \frac{4}{2nt}$ (n);

The ſum of $\frac{2m-1}{2mr} + \frac{2m-3}{2mr^2}$ (n) is ${}^{n}\mathfrak{M}$; its firſt term $\frac{2m-1}{2mr}$; and common difference $\frac{1}{mr}$:.

The ſum of $\frac{1}{t} + \frac{2}{t}$ (n) is $\frac{nn+n}{2t}$; its firſt term $\frac{1}{t}$; and common difference $-\frac{1}{t}$:

The ſum of $\frac{1}{2n} + \frac{2}{2n}$ (n) is $\left(\frac{nn+n}{4n} =\right) \frac{n+1}{4}$; its firſt term $\frac{1}{2n}$; and common difference $-\frac{1}{2n}$:

Whence, the ſum of the ſeries of products will (by queſt. 22. vol. 2.) be $\Big({}^{n}\mathfrak{M} \times \frac{nn+n}{2t} \times \frac{n+1}{4} \times \frac{1}{nn} + \frac{\overline{n+1}\,.\,n\,.\,\overline{n-1}}{2\,.\,2\,.\,3} \times \overline{\frac{2m \quad 1}{4mntr} - \frac{1}{2\,mntr} - \frac{1}{2\,mntr}}$ $- \frac{\overline{n+1}\,.\,n\,.\,\overline{n-1}^{2}}{2\,.\,2\,.\,2} \times \frac{1}{2mntr}$ or$\Big)$

$^n\mathfrak{M}\times\frac{n+1}{2t}\times\frac{n+1}{4n}+\frac{\overline{n+1}\cdot\overline{n-1}}{8mt}\times\frac{2m-5}{6r}$

$\Big(-\frac{\overline{n+1}\cdot\overline{n-1}}{8mt}\times\frac{3n-3}{6r}$;

That is ($^n\mathfrak{P}\times\frac{n+1}{4n}\times\frac{n+1}{2t}+\frac{n-1}{6r}\times\frac{2m-3n-2}{4m}\times\frac{n+1}{2t}$ or)

$\overline{^n\mathfrak{M}\times\frac{n+1}{4n}+\frac{n-1}{6r}\times\frac{2m-3n-2}{4m}}\times\frac{n+1}{2t}$, will be the ſum of the negative ſeries.

And the difference of the two ſeries, which have variable factors, will be

($^n\mathfrak{P}-\frac{n\cdot\overline{n-1}}{6rm}-{}^n\mathfrak{M}\times\frac{n+1}{4n}-\frac{n-1}{6r}\times\frac{2m-3n-2}{4m}\times\frac{n+1}{2t}$ or)

$\overline{\overline{1-\frac{n+1}{4n}\times{}^n\mathfrak{M}-\frac{n-1}{6r}\times\frac{n}{m}+\frac{2m-3n-2}{4m}}}\times\frac{n+1}{2t}$; that is,

$\overline{\frac{3n-1}{4n}\times{}^n\mathfrak{P}-\frac{n-1}{6r}\times\frac{2m+n-2}{4m}}\times\frac{n+1}{2t}$, will be the difference of the two ſerieſes, which have variable factors.

To which, if, $\mathfrak{P}-{}^n\mathfrak{P}\times\frac{n}{2t}$, the difference of thoſe, which have invariable factors, be added, the ſum

$3n-1$

$$\left(\overline{\frac{3n-1}{4n}\times {}^{n}\mathfrak{M}-\frac{n-1}{6r}\times\frac{2m+n-2}{4m}}\times\frac{n+1}{2t}+\overline{\mathfrak{M}-{}^{n}\mathfrak{M}}\times\frac{n}{2t}\text{ or}\right)$$

$$\overline{\frac{3n-1}{4n}\times {}^{n}\mathfrak{M}-\frac{n-1}{6r}\times\frac{2m+n-2}{4m}}\times\overline{n+1}+\overline{\mathfrak{M}-{}^{n}\mathfrak{M}}\times n\times\frac{1}{2t},$$ will be the value of the annuitant's expectation.

EXAMPLE.

A (aged 43) is poſſeſſed of an eſtate, which on her deceaſe will devolve to her uncle *C* (aged 66) if he be then alive; what is the preſent worth of the reverſion of an annuity (to be ſecured on that eſtate) for the remainder of the life of *B* (aged 54) to commence when *C* becomes poſſeſſed of the eſtate?

Here $\mathfrak{M}=10{,}757$; ${}^{n}\mathfrak{M}=9{,}891$; $\frac{n-1}{6r}=3{,}045$; $m=32$; $n=20$; $t=43$; $\frac{3n-1}{4n}=\frac{59}{80}$; $n+1=21$; and $\frac{2m+n-2}{4m}=\left(\frac{64+20-2}{4\cdot 32}=\right)\frac{82}{128}$:

Then $9{,}891\times\frac{59}{80}=7{,}295$: $3{,}045\times\frac{82}{128}=1{,}951$; $7{,}295-1{,}951=5{,}344$:

And 5, 344 × 21 = 112,224;
Again 10,757 — 9,891 = 0,866; and 0,866×20 =17,320:

Now 112,224 + 17,320 = 129,544; and $\frac{129,544}{2 . 43}$ = 1,506 the value required.

COROL. I.

If the poſſeſſor and annuitant are of equal ages, then =m, and the value of the reverſion will become

$$\left.\begin{matrix}\overline{\frac{n-1}{4n}\,{}^{n}\mathfrak{N}-\frac{n-1}{6r}\times\frac{2m+n-2}{4m}\times \overline{n+1}}\\ \overline{+\mathfrak{M}-{}^{n}\mathfrak{N}\times n}\end{matrix}\right\}\times\frac{1}{2m}:$$

COROL. II.

If the expectant and annuitant are of equal ages; then =n, and $\mathfrak{M} = {}^{n}\mathfrak{N} = \mathfrak{N}$; whence the value reuired will be $\overline{\frac{3n-1}{4n}\mathfrak{N}-\frac{n-1}{6r}\times\frac{3n-2}{4n}}\times\frac{n+1}{2t}$;

hat is (if we write $3n - 1$, for $3n - 2$)

$\overline{\mathfrak{N}-\frac{n-1}{6r}}\times\frac{3n-1}{4n}\times\frac{n+1}{2t}$ will be the value reuired, as in corol. 2. caſe 1.

CASE IV.

If the poſſeſſor, be the eldeſt; the annuitant, the ſeond; and the expectant the youngeſt, of the three perons.

Then the annual probabilities of the expectant's comg into poſſeſſion will be $\frac{2t-1}{2t}\times\frac{1}{n}$; $\frac{2t-3}{2t}\times\frac{1}{n}$ &c.

which may (as before) be otherwiſe wrote $\frac{1}{n} - \frac{1}{2nt}$; $\frac{1}{n} - \frac{3}{2nt}$ &c. and, the proceſs being continued in th beforegoing manner, we ſhall find that the value of th reverſion of the annuity required will be expreſſed, by th difference of the two following ſeries,

$$\frac{2m-1}{2mr} \times \frac{1}{n} + \frac{2m-3}{2mr^2} \times \frac{2}{n} + \frac{2m-5}{2mr^3} \times \frac{3}{n}\,(n)$$

$$+ \frac{2m-2n-1}{2mr^{n+1}} \times \frac{n}{n} + \frac{2m-2n-3}{2mr^{n+2}} \times \frac{n}{n}\,(m-n);\text{an}$$

$$- \frac{2m-1}{2mr} \times \frac{1}{2nt} - \frac{2m-3}{2mr^2} \times \frac{4}{2nt} - \frac{2m-5}{2mr^3} \times \frac{9}{2nt}$$

$$- \frac{2m-2n-1}{2mr^{n+1}} \times \frac{nn}{2nt} - \frac{2m-2n-3}{2mr^{n+2}} \times \frac{nn}{2nt}\,(m-n):$$

where (becauſe $\frac{n}{n} = 1$, and $\frac{nn}{2nt} = \frac{n}{2t}$) the differen of thoſe parts of the two ſeries, which have invariabl factors, will be $\mathfrak{M} - {}^{n}\mathfrak{M} \times 1 - \frac{n}{2t}$.

In the affirmative ſeries, having variable factors, viz. $\frac{2m-1}{2mr} \times \frac{1}{n} + \frac{2m-3}{2mr^2} \times \frac{2}{n}\,(n)$; $\frac{2m-1}{2mr} + \frac{2m-3}{2mr^2}\,(n)$ $= {}^{n}\mathfrak{M}$, and the common difference $= \frac{1}{mr}$; $\frac{1}{n} + \frac{2}{n}\,(n) = \left(\frac{nn+n}{2n} =\right) \frac{n+1}{2}$, and the common differ-ence is $-\frac{1}{n}$; therefore the ſum of the ſeries of product will be $\left({}^{n}\mathfrak{M} \times \frac{n+1}{2} \times \frac{1}{n} - \frac{\overline{n+1} \,.\, n \,.\, \overline{n-1}}{2 \,.\, 2 \,.\, 3} \times \frac{1}{mr} \right.$ or

$$) \; {}^{n}\mathfrak{M} \times \frac{n+1}{2n} - \frac{\overline{n+1} \, . \, \overline{n-1}}{2m \, . \, 6r}.$$

The negative ſeries, having variable factors, is the me with that before exhibited; and conſequently its ſum ill be

$$\overline{\mathfrak{M} \times \frac{n+1}{4n} + \frac{n-1}{6r} \times \frac{2m-3n-2}{4m}} \times \frac{n+1}{2t}:$$

his, being ſubtracted, from the above found ſum of e affirmative ſeries, having variable factors, will leave

$$ {}^{n}\mathfrak{M} \times \frac{n+1}{2n} - \frac{\overline{n+1} \, . \, \overline{n-1}}{2m \, . \, 6r} -$$

$$\overline{{}^{n}\mathfrak{M} \times \frac{n+1}{4n} + \frac{n-1}{6r} \times \frac{2m-3n-2}{4m}} \times \frac{n+1}{2t} \text{ or)}$$

$$ {}^{n}\mathfrak{M} \times \frac{n+1}{2n} - \frac{\overline{n+1} \, . \, \overline{n-1}}{2m \, . \, 6r} - {}^{n}\mathfrak{M} \times \frac{n+1}{2n} \times \frac{n+1}{4t}$$

$$\left(- \frac{n-1}{6r} \times \frac{2m-3n-2}{4m} \times \frac{n+1}{2t}; \right.$$

hat is (${}^{n}\mathfrak{M} \times \overline{1 - \frac{n+1}{4t}} \times \frac{n+1}{2n}$

$$\left(- \frac{n-1}{6r} \times \overline{\frac{n+1}{2m} + \frac{2m-3n-2}{4m} \times \frac{n+1}{2t}} \right.$$

r) ${}^{n}\mathfrak{M} \times \overline{1 - \frac{n+1}{4t}} \times \frac{n+1}{2n}$

$$\left(- \frac{n-1}{6r} \times \overline{\frac{n+1}{2m} + \frac{2m-3n-2}{4t} \times \frac{n+1}{2m}}, \right.$$

hat is, ${}^{n}\mathfrak{M} \times \overline{1 - \frac{n+1}{4t}} \times \frac{n+1}{2n}$

$$\left(- \frac{n-1}{6r} \times \overline{1 + \frac{2m-3n-2}{4t}} \times \frac{n+1}{2m} \right.$$

ill be the difference of the two ſerieſes, which have va-able factors.

To which, if, $\overline{\mathfrak{M} - {}^{n}\mathfrak{M}} \times \overline{1 - \frac{n}{2t}}$, the differen of thoſe parts of each ſeries, which have invariable factors, be added; the ſum, ${}^{n}\mathfrak{M} \times \overline{1 - \frac{n+1}{4t}} \times \frac{n+1}{2n}$ $- \frac{n-1}{6r} \times \overline{1 + \frac{2m-3n-2}{4t}} \times \frac{n+1}{2m}$ $+ \overline{\mathfrak{M} - {}^{n}\mathfrak{M}} \times \overline{1 - \frac{n}{2t}}$, will be the value of the annuitant's expectation.

EXAMPLE.

A (aged 43) who will inherit an eſtate, upon the deceaſe of his father *C* (aged 66), grants an annuity (on the ſecurity thereof) to *B* (aged 54) for the remainder of his (*B*'s) life, after he (*A*) ſhall come into poſſeſſion: what is the preſent worth thereof?

Here $\mathfrak{M} = 10{,}757$; ${}^{n}\mathfrak{M} = 9{,}891$; $\frac{n-1}{6r} = 3{,}045$; $\frac{n+1}{4t} = \left(\frac{21}{4 \cdot 43} =\right) \frac{21}{172}$; $1 - \frac{21}{172} = \left(\frac{172-21}{172} =\right) \frac{151}{172}$; $\frac{n+1}{2n} = \frac{21}{40}$; $\frac{2m-3n-2}{4t} = \left(\frac{64-60-2}{4 \cdot 43} =\right) \frac{2}{172}$; $1 + \frac{2}{172} = \left(\frac{174}{172} =\right) \frac{87}{86}$; $\frac{n+1}{2m} = \frac{21}{64}$; $\mathfrak{M} - {}^{n}\mathfrak{M} = 0{,}866$; $1 - \frac{n}{2t} = \left(1 - \frac{20}{86} = \frac{66}{86} =\right) \frac{33}{43}$.

Then $9{,}891 \times \frac{151}{172} \times \frac{21}{40} = 4{,}559$;

$3{,}045 \times \frac{174}{172} \times \frac{21}{64} = 1{,}011$;

Alſo

lſo $0{,}866 \times \frac{33}{43} = 0{,}665$; and $(4{,}559 - 1{,}011 +$ $665 =)$ $4{,}213$ will be the value of the annuitant's expectation.

COROL. I.

If the expectant and the annuitant are of equal ages; en $t=m$; and the annuitant's expectation will become

$${}^{n}\mathfrak{M} \times \overline{1 - \frac{n+1}{4m}} \times \frac{n+1}{2n} - \frac{-1}{6r} \times \frac{6m-3n-2}{4m} \times \frac{n+1}{2m} + \overline{\mathcal{M} - {}^{n}\mathfrak{M}} \times \overline{1 - \frac{n}{2m}}.$$

COROL. II.

If the annuitant and the poſſeſſor are of equal ages; en $m=n$, and $\mathcal{M} = {}^{n}\mathfrak{M} = \mathfrak{N}$; whence the value quired will be

$$\mathfrak{N} \times \overline{1 - \frac{n+1}{4t}} \times \frac{n+1}{2n} - \frac{-1}{6r} \times \overline{1 - \frac{n+2}{4t}} \times \frac{n+1}{2n};$$

which (by writing $\frac{+1}{4t}$ for $\frac{n+2}{4t}$) will become

$$\mathfrak{N} \times \overline{1 - \frac{n+1}{4t}} \times \frac{n+1}{2n} - \frac{n-1}{6r} \times \overline{1 - \frac{n+1}{4t}} \times \frac{n+1}{2n}$$

$$) \; \overline{\mathfrak{N} - \frac{n-1}{6r}} \times \overline{1 - \frac{n+1}{4t}} \times \frac{n+1}{2n}.$$

CASE V.

If the annuitant, be the youngeſt; the poſſeſſor, ſecond; and the expectant, the eldeſt, of the three perſons, Then the annual probabilities of the expectant's co ing into poſſeſſion will be $\left(\frac{2n-1}{2n}\times\frac{1}{m},\ \frac{2n-3}{2n}\times\frac{1}{m},\right.$ &c. or) $\frac{1}{m}-\frac{1}{2nm},\ \frac{1}{m}-\frac{3}{3nm},$ &c. and theſe, bei multiplied into the proper terms of the ſeries, $\frac{2t-1}{2tr}+\frac{2t-3}{2tr^2}$ &c. expreſſing the value of the annuitant's life, will produce the two following ſeries,

$$\frac{2t-1}{2tr}\times\frac{1}{m}+\frac{2t-3}{2tr^2}\times\frac{2}{m}+\frac{2t-5}{2tr^3}\times\frac{3}{m}\ (n)$$

$$+\frac{2t-2n-1}{2tr^{n+1}}\times\frac{n}{m}+\frac{2t-2n-3}{2tr^{n+2}}\times\frac{n}{m}\ (t-n);$$ a

$$-\frac{2t-1}{2tr}\times\frac{1}{2nm}-\frac{2t-3}{2tr^2}\times\frac{4}{2nm}-\frac{2t-5}{2tr^3}\times\frac{9}{2nm}\ ($$

$$-\frac{2t-2n-1}{2tr^{n+1}}\times\frac{nn}{2nm}-\frac{2t-2n-3}{2tr^{n+2}}\times\frac{nn}{2nm}\ (t-n);$$

and their difference will be the value of the annuitant' expectation.

If theſe ſerieſes be compared, with thoſe in the third caſe; it will appear, that if we write, $\mathfrak{F}$ for $\mathfrak{M}$, ${}^{n}\mathfrak{F}$ fo ${}^{n}\mathfrak{M}$, *t* for *m*, and *m* for *t*, in the ſolution of that caſe; the ſolution of this caſe will be, thereby, obtained: therefore

$$\left.\begin{array}{l}\overline{\frac{3n-1}{4n}\times{}^{n}\mathfrak{F}-\frac{n-1}{6r}\times\frac{2t+n-2}{4t}}\times\overline{n+1}\\ +\overline{\mathfrak{F}-{}^{n}\mathfrak{F}}\times n\end{array}\right\}\times\frac{1}{2n}$$

will be the value of the annuitant's expectation.

EXAMPLE.

C (aged 66) who will come into poſſeſſion of an eſtate, t the deceaſe of B (aged 54) if he (C) be then alive, would grant an annuity (ſecured on that eſtate) to A (aged 43) which is to commence, when C takes poſſeſſion of he eſtate, and continue during the remainder of A's life: he preſent worth thereof is required.

Here $\mathfrak{F} = 12{,}920$; ${}^{n}\mathfrak{F} = 10{,}838$; $\frac{n-1}{6r} = 3{,}045$; $\frac{3n-1}{4n} = \frac{59}{80}$; $\frac{2t+n-2}{4t} = \left(\frac{86+20-2}{4\times 43} = \frac{104}{4\times 43} =\right) \frac{26}{43}$; $n+1 = 21$; $\mathfrak{F} - {}^{n}\mathfrak{F} = 2{,}082$; $n = 20$; and $m = 32$.

Then $12{,}920 \times \frac{59}{80} = 9{,}528$; $3{,}045 \times \frac{26}{43} = 1{,}841$; $9{,}528 - 1{,}841 = 7{,}687$; $7{,}687 \times 21 = 161{,}427$; $2{,}082 \times 20 = 41{,}640$; and $161{,}427 + 41{,}640 = 203{,}067$.

Whence $\left(\frac{203{,}067}{2\times 32} =\right)$ 3,173 will be the value required.

COROL. I.

If the annuitant and poſſeſſor be of equal ages; then $\mathfrak{F} = \mathfrak{M}$, ${}^{n}\mathfrak{F} = {}^{n}\mathfrak{M}$, and $t = m$: whence the value of he annuitant's expectation will become,

$$\left.\begin{array}{l}\overline{\frac{3n-1}{4n}\times {}^{n}\mathfrak{M} - \frac{n-1}{6r}\times\frac{2m+n-2}{4m}}\times\overline{n+1}\\ +\overline{\mathfrak{M} - {}^{n}\mathfrak{M}}\times n\end{array}\right\}\times\frac{1}{2m},$$

s in Corol. 1. caſe 3.

COROL. II.

If the poſſeſſor and expectant are of equal ages; then $m = n$; and the value required will become,

$$\left.\begin{matrix}\overline{\frac{3n-1}{4n}\times{}^{n}\mathfrak{N}-\frac{n-1}{6n}\times\frac{2t+n-2}{4t}\times\overline{n+1}}\\ +\overline{\mathfrak{N}-{}^{n}\mathfrak{N}}\times n\end{matrix}\right\}\times\frac{1}{2t}$$

CASE VI.

If the annuitant, be the youngeſt; the expectant, ſecond; and the poſſeſſor, the eldeſt, of the three perſons.

Then the annual probabilities of the expectant's poſſeſſing the eſtate will be $\left(\frac{2m-1}{2m}\times\frac{1}{n},\ \frac{2m-3}{2m}\times\frac{1}{n}\right.$, &c. or) $\frac{1}{n}-\frac{1}{2nm},\ \frac{1}{n}-\frac{3}{2nm}$, &c. which (multiplied into the proper terms of the ſeries, expreſſing the value of the annuitant's life) will produce the two following ſeries,

$$\frac{2t-1}{2tr}\times\frac{1}{n}+\frac{2t-3}{2tr^2}\times\frac{2}{n}+\frac{2t-5}{2tr^3}\times\frac{3}{n}\ (n)$$

$$+\frac{2t-2n-1}{2tr^{n+1}}\times\frac{n}{n}+\frac{2t-2n-3}{2tr^{n+2}}\times\frac{n}{n}\ (t-n);\ \text{and}$$

$$-\frac{2t-1}{2tr}\times\frac{1}{2nm}-\frac{2t-3}{2tr^2}\times\frac{4}{2nm}-\frac{2t-5}{2tr^3}\times\frac{9}{2nm}(n)$$

$$-\frac{2t-2n-1}{2tr^{n+1}}\times\frac{nn}{2nm}-\frac{2t-2n-3}{2tr^{n+2}}\times\frac{nn}{2nm}\ (t-n);$$

whoſe difference, will be the value of the annuitant's expectation.

If theſe ſerieſes be compared with thoſe, in the fourth caſe, it will appear, that if we write $\mathfrak{N}$ for $\mathfrak{M}$, ${}^{n}\mathfrak{N}$ for ${}^{n}\mathfrak{M}$, t for m, and m for t, in the ſolution of that caſe;

then the reſult ${}^{n}\mathfrak{F} \times \overline{1 - \frac{n+1}{4m}} \times \frac{n+1}{2n} -$ $\frac{n-1}{6r} \times \overline{1 + \frac{2t-3n-2}{4m}} \times \overline{\frac{n+1}{2t}} + \mathfrak{F} - {}^{n}\mathfrak{F} \times \overline{1 - \frac{n}{2m}}$ will be the value of the annuitant's expectation, in this caſe.

EXAMPLE.

B (aged 54) who will poſſeſs an eſtate, on the death f his brother C (aged 66) if he (B) be the ſurvivor, would rant an annuity (ſecured on that eſtate) to A (aged 43) which is to commence when B comes into poſſeſſion, and to continue during the remainder of A's life: what is the preſent worth thereof?

Here $\mathfrak{F} = 12,920$; ${}^{n}\mathfrak{F} = 10,838$; $\frac{n+1}{4m} = \left(\frac{21}{4\cdot 32} =\right) \frac{21}{128}$; $1 - \frac{21}{128} = \frac{107}{128}$; $\frac{n+1}{2n} = \frac{21}{40}$; $\frac{n-1}{6r} = 3,045$; $\frac{2t-3n-2}{4m} = \left(\frac{86-60-2}{4\cdot 32} = \frac{24}{4\cdot 32} =\right) \frac{3}{16}$; $1 + \frac{2t-3n-2}{4m} = \left(1 + \frac{3}{16} =\right) \frac{19}{16}$; $\frac{n+1}{2t} = \frac{21}{86}$; $\mathfrak{F} - {}^{n}\mathfrak{F} = 2,082$; $1 - \frac{n}{2m} = \left(\frac{44}{64} =\right) \frac{11}{16}$:

Then $10,838 \times \frac{107}{128} \times \frac{21}{40} = 4,756$; $3,045 \times \frac{19}{16} \times \frac{21}{86} = 0,883$; $2,082 \times \frac{11}{16} = 1,431$: and $(4,756 - 0,883 + 1,431 =)$ 5,304 will be the value required.

COROL. I.

If the annuitant and expectant are of equal ages; th $\mathfrak{F} = \mathfrak{M}$, ${}^{n}\mathfrak{F} = {}^{n}\mathfrak{M}$, and $t = m$; whence the val required will become, ${}^{n}\mathfrak{M} \times \overline{1 - \frac{n+1}{4m}} \times \frac{n+1}{2n}$

$$- \frac{n-1}{6r} \times \frac{6m-3n-2}{4m} \times \frac{n+1}{2m} + \overline{\mathfrak{M} - {}^{n}\mathfrak{M}} \times 1 - \frac{1}{2n}$$

as in corol. 1. Case 4.

COROL. II.

If the expectant and possessor are of equal ages; th $m = n$; and the required value will become

$$({}^{n}\mathfrak{F} \times \frac{3n-1}{4n} \times \frac{n+1}{2n} - \frac{n-1}{6r} \times \overline{1 + \frac{2t-3n-2}{4n}}$$

$$\Big(\frac{n+1}{2t} + \overline{\mathfrak{F} - {}^{n}\mathfrak{F}} \times \overline{1 - \frac{1}{2t}}$$

Or)

$$ {}^{n}\mathfrak{F} \times \frac{3n-1}{4n} \times \frac{n+1}{2n} - \frac{n-1}{6r} \times \frac{2t+n-2}{4n} \times \frac{n+}{2t}$$

$$(+ \overline{\mathfrak{F} - {}^{n}\mathfrak{F}} \times$$

That is

$$({}^{n}\mathfrak{F} \times \frac{3n-1}{4n} \times \frac{n+1}{2n} - \frac{n-1}{6r} \times \frac{2t+n-2}{4t} \times \frac{n+}{2t}$$

$$(+ \overline{\mathfrak{F} - {}^{n}\mathfrak{F}} \times$$

Or)

$$\overline{{}^{n}\mathfrak{F} \times \frac{3n-1}{4n} - \frac{n-1}{6r} \times \frac{2t+n-2}{4t}} \times \frac{n+1}{2n}$$

$$(+ \overline{\mathfrak{F} - {}^{n}\mathfrak{F}} \times \frac{n}{2t}$$

The

Therefore, the value required may be expreſſed by,

$$\left.\begin{array}{l}\overline{n\frac{p}{r} \times \frac{3n-1}{4n} - \frac{n-1}{6r} \times \frac{2t+n-2}{4t} \times n+1} \\ +\overline{\frac{p}{r} - n\frac{p}{r}} \times n\end{array}\right\} \times \frac{1}{2n},$$

s in corol. 2. caſe 5.

QUESTION L.

The reſpective ages of 4 perſons, *A* the youngeſt; *B*, he ſecond; *C*, the third; and *D*, the eldeſt, being given. It is required to determine the probability of their dying, in the following order; *viz.* that *A* and *B*, the two youngeſt, ſhall die before either of the two eldeſt; that *C*, the next younger, ſhall die next; and that *D*, the eldeſt, ſhall ſurvive the other three: and it is farther required, to find the preſent value of *D*'s intereſt in an eſtate, of 1£. per ann. or a legacy of 25£. dependent on this order of ſurvivorſhip's taking place?

SOLUTION.

Let the complements of *A*, *B*, *C*, and *D*, be ſeverally denoted by t, m, n, and v; then (ſince $\frac{2t-1}{2t}$ and $\frac{2m-1}{2m}$ are the ſeparate expectations of life, of *A*, and *B*, for the firſt year) $1-\frac{2t-1}{2t}$, and $1-\frac{2m-1}{2m}$, will be the ſeparate expectations of their death, at the beginning, or early in that year; whence ($\overline{1-\frac{2t-1}{2t}} \times \overline{1-\frac{2m-1}{2m}}$ or) $1-\frac{2t-1}{2t}-\frac{2m-1}{2m}+\frac{2t-1}{2t}\times\frac{2m-1}{2m}$ will be the expectation, that both of them will die at, or about that time; if therefore this expectation be multiplied by, $\frac{2v-1}{2v} \times \frac{1}{n}$, the probability of *D's* ſurviving *C*, for that year, then the product

($\overline{1 - \frac{2t-1}{2t} - \frac{2m-1}{2m} + \frac{2t-1}{2t} \times \frac{2m-1}{2m}} \times \frac{2v-1}{2v} \times \frac{1}{n}$ or) $\frac{2v-1}{2v} \times \frac{1}{n} - \frac{2t-1}{2t} \times \frac{2v-1}{2v} \times \frac{1}{n} - \frac{2m-1}{2m} \times \frac{2v-1}{2v} \times \frac{1}{n} + \frac{2t-1}{2t} \times \frac{2m-1}{2m} \times \frac{2v-1}{2v} \times \frac{1}{n}$ will be the probability of the propoſed order of ſurvivorſhip's taking place, in the firſt year: and (by arguing in the ſame manner) $\frac{2v-3}{2v} \times \frac{1}{n} - \frac{2t-3}{2t} \times \frac{2v-3}{2v} \times \frac{1}{n} - \frac{2m-3}{2m} \times \frac{2v-3}{2v} \times \frac{1}{n} + \frac{2t-3}{2t} \times \frac{2m-3}{2m} \times \frac{2v-3}{2v} \times \frac{1}{n}$, $\frac{2v-5}{2v} \times \frac{1}{n} - \frac{2t-5}{2t} \times \frac{2v-5}{2v} \times \frac{1}{n} - \frac{2m-5}{2m} \times \frac{2v-5}{2v} \times \frac{1}{n} + \frac{2t-5}{2t} \times \frac{2m-5}{2m} \times \frac{2v-5}{2v} \times \frac{1}{n}$, &c. will be the probabilities of the propoſed order of ſurvivorſhip's taking place, in the ſecond, third, &c. years: where the ſum of the firſt affirmative ſeries $\frac{2v-1}{2v} \times \frac{1}{n} + \frac{2v-3}{2v} \times \frac{1}{n} + \frac{2v-5}{2v} \times \frac{1}{n}$ (v) is the prob. of *D*'s ſurviving C, by queſt. 21. the ſums of the 2 negative ſeries *viz.* $\frac{2t-1}{2t} \times \frac{2v-1}{2v} \times \frac{1}{n} + \frac{2t-3}{2t} \times \frac{2v-3}{2v} \times \frac{1}{n}$ (v)

and

nd $\frac{2m-1}{2m}\times\frac{2v-1}{2v}\times\frac{1}{n}+\frac{2m-3}{2m}\times\frac{2v-3}{2v}\times\frac{1}{n}\,(v)$ re ſeverally, the probabilities, that both *A*, and *D*, will ſurvive *C*, and that both *B*, and *D*, will ſurvive *C*, by caſe 1. queſt. 31. and the ſum of the laſt affirmaive ſeries $\left(\frac{2t-1}{2t}\times\frac{2m-1}{2m}\times\frac{2v-1}{2v}\times\frac{1}{n}+\frac{2t-3}{2t}\times\frac{2m-3}{2m}\times\frac{2v-3}{2v}\times\frac{1}{n}\,(v)\right)$ is the probability that all the three perſons, *A*, *B*, and *D* will ſurvive *C*, by caſe 1. queſt. 33.

Hence, if we call (as before) the two perſons, whoſe deaths are required to happen firſt in order, the poſſeſſors; *the perſon, upon whoſe death the ſurvivorſhip will take lace, the* expectant; *and the fourth perſon, the* ſurvivor: *Then, from the probability of the ſurvivor's outliving the xpectant, take the probabilities, that both the elder poſſeſſor nd ſurvivor, and alſo that both the younger poſſeſſor and urvivor, ſhall, ſeverally, outlive the expectant; then, to the remainder, add the probability that the two poſſeſſors nd ſurvivor ſhall, all three of them, outlive the expectant; o ſhall the reſult be the probability, that the two poſſeſſors hall both die before, either the expectant or ſurvivor; and, hat the expectant will, alſo, die before the ſurvivor; which probability was required.*

In this caſe, the probability of

$$\left.\begin{array}{r} D\text{'s} \\ \text{both } A, \,\&\, D\text{'s} \\ \text{both } B, \,\&\, D\text{'s} \\ A,\, B,\, \&\, D\text{'s} \end{array}\right\} \begin{array}{l} \text{ſurviv-} \\ \text{ing } C, \\ \text{is} \end{array} \left\{\begin{array}{l} \frac{v}{2} \times \frac{1}{n} \\ \frac{v}{2} \times \frac{1}{n} - \frac{vv}{6t} \times \frac{1}{n} \\ \frac{v}{2} \times \frac{1}{n} - \frac{vv}{6m} \times \frac{1}{n} \\ \frac{v}{2} \times \frac{1}{n} - \frac{vv}{6t} \times \frac{1}{n} - \frac{vv}{6m} \times \frac{1}{n} + \frac{v^3}{12tm} \times \frac{1}{n} \end{array}\right\} \text{by queſt.} \left\{\begin{array}{l} 21; \\ 3^1; \\ 3^1; \\ 33; \end{array}\right.$$

Then (by proceeding to ſubtract, and add, in the manner above directed) the probability required will appear to be $\frac{v^3}{12tm} \times \frac{1}{n}$.

Again, ſince, $\mathfrak{D} - \frac{v}{6r} \times \frac{vv}{4mt}$, is the ſum of the ſeries of preſent worths, which correſponds with the ſeries of probabilities, whoſe ſum is $\frac{v^3}{12tm}$;

Therefore, the intereſt of *D* in an eſtate, or legacy, worth $\mathfrak{P}$ pounds, will be $\left(\mathfrak{D} - \frac{v}{6r} \times \frac{vv}{4mt} \times \frac{\mathfrak{P}}{n} =\right)$ $\overline{\mathfrak{D} - \frac{v}{6r}} \times \frac{vv\mathfrak{P}}{4mtn}$.

EXAMPLE.

If *A* (aged 43) and *B* (aged 54) are poſſeſſed of an eſtate of 1£. per annum; which is to deſcend to *C* (aged 66) if he ſurvives them both, and will after that come into the poſſeſſion of *D* (aged 76) if he be alive at the death of *C*; required the probability that he (*D*) has of becoming poſſeſſed of the eſtate, and the preſent worth of his intereſt therein?

Here $v = 10$; $n = 20$; $m = 32$; $t = 43$; whence the probability of his becoming poſſeſſed of the eſtate will be $\left(\frac{10 \times 10 \times 10}{12 \times 43 \times 32 \times 20} =\right)$ 0,00303; again $\mathfrak{D} - \frac{v}{6r} =$ 2,715, and $\mathfrak{P} = 25$; whence, the preſent worth of his intereſt in the eſtate will be $\left(\frac{2,715 \times 10 \times 10 \times 25}{4 \times 43 \times 32 \times 20} =\right)$ 0,0617.

COROL. I.

If the two poſſeſſors, are of equal ages; then $t = m$; therefore the anſwers to the queſtion will be $\frac{v^3}{12mmn}$, and $\overline{\mathfrak{P} - \frac{v}{6r}} \times \frac{vv\mathfrak{P}}{4mmn}$.

COROL. II.

If the elder poſſeſſor, and expectant, are of equal ages; then $m = n$; and the anſwers to the queſtion will be $\frac{v^3}{12tnn}$, and $\overline{\mathfrak{P} - \frac{v}{6r}} \times \frac{vv\mathfrak{P}}{4tnn}$.

COROL. III.

If the expectant, and ſurvivor, are of equal ages; then $n = v$; and the anſwers to the queſtion will be $\frac{vv}{12tm}$, and $\overline{\mathfrak{P} - \frac{v}{6r}} \times \frac{v\mathfrak{P}}{4mt}$.

COROL. IV.

If the two poſſeſſors, and the expectant, are of equal ages; then $t = m = n$; therefore the anſwers to the queſtion will be $\frac{v^3}{12n^3}$, and $\overline{\mathfrak{P} - \frac{v}{6r}} \times \frac{vv\mathfrak{P}}{4n^3}$.

COROL. V.

If the elder poſſeſſor, the expectant, and the ſurvivor, are of equal ages; then $m = n = v$; and the anſwers to the queſtion will be $\frac{v}{12t}$, and $\mathfrak{P} - \frac{v}{6r} \times \frac{\mathfrak{P}}{4t}$.

COROL.

COROL. VI.

If the four lives be of equal ages; then $t = m = n = v$; and the anſwers to the queſtion will become $\frac{1}{12}$, and $\overline{\mathfrak{P} - \frac{v}{6r}} \times \frac{\mathfrak{P}}{4v}$.

Note. As each of the following eleven queſtions, are of the ſame nature with this, and admit of ſeveral corollaries; they will (for the ſake of brevity) be propoſed by making uſe of the letters *A*, *B*, *C*, and *D*, inſtead of the names, and *t*, *m*, *n*, and *v*, inſtead of the complements of the youngeſt, ſecond, third, and eldeſt; the ſolutions (ſo far as regard the probabilities of ſurvivorſhip) will proceed according to the rule printed in Italics in page 151; and the preſent worths will be found, either directly from thoſe reſults, or by a ſimilar proceſs, as the nature of the caſe may require.

QUESTION LI.

The probability, that *A* and *C*, conſidered as poſſeſſors, ſhall both die before, either *B*, or *D*; and that *B*, conſidered as the expectant, ſhall die before *D*, the ſurvivor: alſo, the preſent worth of *D*'s intereſt, in an eſtate, or legacy, worth $\mathfrak{P}$ pounds, dependent on this order of ſurvivorſhip; are required.

SOLUTION.

In this caſe, the probability, that

$$\left.\begin{array}{r} D \\ \text{both } A, \,\&\, D \\ \text{both } C, \,\&\, D \\ A, C, \,\&\, D \end{array}\right\} \begin{array}{l} \text{will ſur-} \\ \text{vive} B, \\ \text{is} \end{array} \left\{\begin{array}{l} \frac{v}{2} \times \frac{1}{m}. \\ \frac{v}{2} \times \frac{1}{m} - \frac{vv}{6t} \times \frac{1}{m} \\ \frac{v}{2} \times \frac{1}{m} - \frac{vv}{6n} \times \frac{1}{m} \\ \frac{v}{2} \times \frac{1}{m} - \frac{vv}{6t} \times \frac{1}{m} - \frac{vv}{6n} \times \frac{1}{m} + \frac{v^3}{12tn} \times \frac{1}{m} \end{array}\right\}. \text{by queſt.} \left\{\begin{array}{l} 21; \\ 31; \\ 31; \\ 33; \end{array}\right.$$

which probabilities (being ſubtracted, and added, according to the manner preſcribed) will give, $\frac{v^3}{12tnm}$, for the probability required; and conſequently,

$\mathfrak{P} - \frac{v}{6r} \times \frac{vv\mathfrak{P}}{4mtn}$, for the preſent worth of *D*'s intereſt, as in the laſt queſtion.

COROL.

Theſe anſwers (being the ſame with thoſe to the laſt ueſtion) all the varieties (occaſioned, by ſuppoſing two, r three, of the given lives to be of the ſame ages) will ave the ſame reſults, as in the corollaries to the laſt ueſtion; interchanging, only, the names of *poſſeſſor*, and *xpectant*, in their application to the lives, denoted by *B*, and *C*.

QUESTION LII.

The probability, that *B* and *C*, conſidered as poſſeſſors, ſhall both die before, either *A*, or *D*; and that *A*, onſidered as the expectant, ſhall die before *D*, the ſurvivor; alſo the preſent worth of *D*'s intereſt, in an eſtate, or legacy, worth $\mathfrak{P}$ pounds, dependent on this order of ſurvivorſhip; are required.

SOLU-

SOLUTION.

Here, the probability, that

$$\left.\begin{array}{r} D \\ \text{both } B, \,\&\, D \\ \text{both } C, \,\&\, D \\ B, C, \,\&\, D \end{array}\right\} \text{will ſurvive } A, \text{ is } \left\{\begin{array}{l} \frac{v}{2}\times\frac{1}{t} \\ \frac{v}{2}\times\frac{1}{t}-\frac{vv}{6m}\times\frac{1}{t} \\ \frac{v}{2}\times\frac{1}{t}-\frac{vv}{6n}\times\frac{1}{t} \\ \frac{v}{2}\times\frac{1}{t}-\frac{vv}{6m}\times\frac{1}{t}-\frac{vv}{6n}\times\frac{1}{t}+\frac{v^3}{12mn}\times\frac{1}{t} \end{array}\right\} \text{by queſtion} \left\{\begin{array}{l} 21; \\ 31; \\ 31; \\ 33; \end{array}\right.$$

whic

which probabilities (being ſubtracted, and added, according to the manner preſcribed) will again, give, $\frac{v^3}{12nmt}$, for the probability required; and conſequently, $\overline{\mathfrak{P} - \frac{v}{6r}} \times \frac{vv\mathfrak{P}}{4nmt}$, for the preſent worth of *D*'s intereſt, as in the two laſt queſtions.

COROL.

Hence the reſults of all the varieties (occaſioned by ſuppoſing two, or three, of the given lives to be of equal ages) will be the ſame as in the corollaries to queſt. 50; interchanging only the names of *poſſeſſor* and *expectant*, in their application to the lives, denoted by the letters *A*, and *C*.

QUESTION LIII.

The probability, that *A* and *B*, conſidered as poſſeſſors, ſhall both die before, either *D* or *C*; and that *D*, conſidered as the expectant, ſhall die before *C*, the ſurvivor; alſo the preſent worth of *C*'s intereſt, in an eſtate, or legacy, worth $\mathfrak{P}$ pounds, dependent on this order of ſurvivorſhip; are required.

SOLUTION.

Here the probabilities, that *D* ſhall be ſurvived, by

$$\left.\begin{array}{rl} C, \text{ is} & 1-\frac{v}{2n} \\ A, \,\&\, C, \text{ is} & 1-\frac{v}{2n}-\frac{v}{2t}+\frac{vv}{3nt} \\ B \,\&\, C, \text{ is} & 1-\frac{v}{2n}-\frac{v}{2m}+\frac{vv}{3nm} \\ A, B \,\&\, C, \text{ is} & 1-\frac{v}{2n}-\frac{v}{2m}-\frac{v}{2t}+\frac{vv}{3nt}+\frac{vv}{3nm}+\frac{vv}{3mt}-\frac{v^3}{4tmn} \end{array}\right\} \text{by queſt.} \left\{\begin{array}{l} 22; \\ 31; \\ 31; \\ 33; \end{array}\right.$$

which probabilities (being ſubtracted, and added, according to the directions above mentioned) will give $\left(\frac{vv}{3mt} - \frac{v^3}{4mtn} \text{ or }\right) \overline{\frac{1}{3} - \frac{v}{4n}} \times \frac{vv}{mt}$, for the probability required.

Now the preſent worths, correſponding to the probabilities (above ſubtracted and added) are

$$\left.\begin{array}{l}
\frac{{}^v\mathfrak{P}}{v} \\
\frac{{}^v\mathfrak{P}}{v} - \overline{{}^v\mathfrak{P} - \frac{vv}{6rn}} \times \frac{1}{2t} \\
\frac{{}^v\mathfrak{P}}{v} - \overline{{}^v\mathfrak{P} - \frac{vv}{6rn}} \times \frac{1}{2m} \\
\frac{{}^v\mathfrak{P}}{v} - \overline{\overline{{}^v\mathfrak{P} - \frac{vv}{6rn}} \times \overline{m+t} \times 2 - v - \frac{vv}{6rn} \times \overline{n-v} \times 2} \times \frac{1}{4mt}
\end{array}\right\} \text{by ſcholium to queſtion} \left\{\begin{array}{l}
23; \\
32; \\
32; \\
34;
\end{array}\right.$$

which preſent worths, are to be ſubtracted, and added, in the ſame manner, as the beforegoing probabilities.

Now $\overline{v\boldsymbol{\mathfrak{P}} - \frac{vv}{6rn}} \times \overline{\frac{1}{2t} + \frac{1}{2m}} =$

$\overline{v\boldsymbol{\mathfrak{P}} - \frac{vv}{6rn}} \times \frac{\overline{m+t} \times 2}{4mt}$;

And $\overline{v\mathfrak{P} - \frac{vv}{6rn}} \times \frac{\overline{m+t} \times 2 - v}{4mt}$, =

$\overline{v\boldsymbol{\mathfrak{P}} - \frac{vv}{6rn}} \times \overline{\frac{\overline{m+t} \times 2}{4mt} - \frac{v}{4mt}}$;

Their difference is, therefore, $v\mathfrak{P} + \overline{\frac{vv}{6rn} \times \frac{v}{4mt}}$:

Whence, the reſult will be

($\overline{\overline{v\mathfrak{P} - \frac{vv}{6rn}} \times v + \frac{vv}{6rn} \times \overline{n - v}} \times 2 \times \frac{1}{4mt}$;

or) $\overline{v\mathfrak{P}v + \frac{2vvn}{6rn} - \frac{3v^3}{6rn}} \times \frac{1}{4mt}$; that is,

($\overline{v\mathfrak{P}v + \frac{2n-3v}{6rn} \times vv} \times \frac{1}{4mt}$

or) $\overline{v\boldsymbol{\mathfrak{P}} + \frac{2n-3v}{6rn} \times v} \times \frac{v}{4mt}$; and therefore,

$\overline{v\boldsymbol{\mathfrak{P}} + \frac{2n-3v}{6rn} \times v} \times \frac{v\mathfrak{P}}{4mt}$, will be the preſent worth required.

EXAMPLE.

If *A* (aged 43) and *B* (aged 54) are poſſeſſed of an eſtate of 1£. per annum; which is to deſcend to *D* (aged 76) if he ſurvives them both; and will, after that, come into the poſſeſſion of *C* (aged 66) if he be alive at the death of *D*; required the probability that he (*C*) has of becoming poſſeſſed of the eſtate; and the preſent worth of his intereſt therein.

Here ${}^{v}\mathfrak{P} = 6,215$; $n = 20$; $v = 10$; $r = 1,04$; $\mathfrak{P} = 25$; $m = 32$; $t = 43$.

Then $\frac{1}{3} = 0,3333$; $\frac{10}{4.20} = 0,125$; $0,3333 - 0,125 = 0,20833$ and $\left(\frac{0,20833 \times 10 \times 10}{32 \times 43} =\right)$ $0,01514$, the probability of *C*'s becoming poſſeſſed of the eſtate: alſo $2 \times 20 - 3 \times 10 = 10$; $\frac{10 \times 10}{6 \times 1,04 \times 20} = 0,801$; and $6,215 + 0,801 = 7,016$;

Then $\left(\frac{7,016 \times 10 \times 25}{4 \times 32 \times 43} =\right)$ $0,31875$ will be the preſent worth of *C*'s intereſt, in the eſtate.

COROL. I.

If *A* and *B* are of equal ages; then $m = t$; and the anſwers will be $\overline{\frac{1}{3} - \frac{v}{4n}} \times \frac{vv}{mm}$; and $\overline{{}^{v}\mathfrak{P} + \frac{2n - 3v}{6rn}} \times v \times \frac{v\mathfrak{P}}{4mm}$.

COROL. II.

If *B* and *C* are of equal ages; then $m = n$; and the anſwers will become $\overline{\frac{1}{3} - \frac{v}{4n}} \times \frac{vv}{nt}$, and $\overline{{}^{v}\mathfrak{P} + \frac{2n - 3v}{6rn}} \times v \times \frac{v\mathfrak{P}}{4nt}$.

COROL. III.

If *C*, and *D*, are of equal ages; then $v\mathfrak{Q} = \mathfrak{P}$, and $n = v$; whence the anſwers will become

$$\left(\overline{\frac{1}{3} - \frac{1}{4}} \times \frac{vv}{mt} \text{ or}\right) \frac{vv}{12mt}, \text{ and } \overline{\mathfrak{P} - \frac{v}{6r}} \times \frac{v\mathfrak{P}}{4mt}.$$

COROL. IV.

[If *A*, *B*, and *C*, are of equal ages; then $t = m = n$; and the anſwers will be $\overline{\frac{1}{3} - \frac{v}{4n}} \times \frac{vv}{nn}$, and

$$\overline{v\mathfrak{Q} + \frac{2n - 3v}{6rn}} \times v \times \frac{v\mathfrak{P}}{4nn}.$$

COROL. V.

If *B*, *C*, and *D*, are of equal ages; then $v\mathfrak{Q} = \mathfrak{P}$, and $m = n = v$; whence the anſwers will be

$$\left(\overline{\frac{1}{3} - \frac{1}{4}} \times \frac{v}{t} \text{ or}\right) \frac{v}{12t}, \text{ and } \overline{\mathfrak{P} - \frac{v}{6r}} \times \frac{\mathfrak{P}}{4t}$$

QUESTION LIV.

The probability, that *A* and *D*, conſidered as poſſeſſors, ſhall both die before, either B, or *C*; and that *B*, conſidered as expectant, ſhall die before *C*, the ſurvivor; alſo the preſent worth of *C*'s intereſt, in an eſtate, or legacy, worth $\mathfrak{P}$ pounds, dependent upon this order of ſurvivorſhip; are required.

SOLU-

SOLUTION.

Here the probabiliey, that *B* ſhall be ſurvived, by

$$\left.\begin{array}{r l} C\text{, is} & \frac{n}{2m} \\ A\text{, \& } C\text{, is} & \frac{n}{2m} - \frac{nn}{6tm} \\ D\text{, \& } C\text{, is} & \frac{v}{2m} - \frac{vv}{6nm} \\ A, D\text{, \& } C\text{, is} & \frac{v}{2m} - \frac{vv}{6nm} - \frac{vv}{6tm} + \frac{v^3}{12ntm} \end{array}\right\} \underbrace{\text{by queſtion}} \left\{\begin{array}{l} 21; \\ 31; \\ 31; \\ 33; \end{array}\right.$$

which probabilities (being ſubtracted, and added, in the manner above directed) will give

$$\left(\frac{nn}{6tm} - \frac{vv}{6tm} + \frac{v^3}{12ntm} \text{ or}\right) \overline{nn - vv + \frac{v^3}{2n}} \times \frac{1}{6tm},$$

for the probability required; and conſequently

$$\left(\overline{\mathfrak{R} - \frac{n}{6r}} \times \frac{n\mathfrak{P}}{2tm} - \overline{\mathfrak{D} - \frac{v}{6r}} \times \frac{v\mathfrak{P}}{2tm} + \overline{\mathfrak{D} - \frac{v}{6r}} \times \frac{vv\mathfrak{P}}{4mtn} \text{ or}\right)$$

$$\overline{\overline{\mathfrak{R} - \frac{n}{6r}} \times n - \overline{\mathfrak{D} - \frac{v}{6r}} \times \overline{1 - \frac{v}{2n}} \times v} \times \frac{\mathfrak{P}}{2mt}$$

will be the preſent worth of *C*'s intereſt in that eſtate.

EXAMPLE.

D (aged 76) and his daughter *A* (aged 43) are poſſeſſed of an eſtate of 1£. per ann. which, after both their deaths, will belong to *B*, the brother of *D* (aged 54) if he be then alive: *B* has a wife *C* (aged 66) who expects that her huſband will, if he comes into poſſeſſion, leave the eſtate to her: what probability has ſhe of poſſeſſing the eſtate, and what is the preſent worth of her intereſt therein?

Here $n = 20$; $v = 10$; $t = 43$; $m = 32$; $\mathfrak{R} = 7{,}673$; $\mathfrak{D} = 4{,}318$; and $\mathfrak{P} = 25$; then $20 \times 20 - 10 \times 10 = 300$; $\frac{10^3}{2 \times 20} = \left(\frac{100}{4} =\right) 25$; and $300 + 25 = 325$;

Therefore $\frac{325}{6 \times 43 \times 32} =)\ 0.03937$, will be the probability of her coming into poſſeſſion of the eſtate.

Also $\mathfrak{R} - \frac{n}{6r} = 4{,}468$; $\mathfrak{D} - \frac{v}{6r} = 2{,}715$; and

$\mathfrak{P} = 25$; then $4{,}460 \times 20 = 89{,}36$; $2{,}715 \times \overline{1 - \frac{10}{2.20}} \times 10 = (2{,}715 \times \frac{30}{4} =)\ 20{,}3625$; $89{,}36 - 20{,}3625 = 68{,}9975$; laſtly $\left(\frac{68{,}9975 \times 25}{2.32.43} =\right)$ $0{,}6268$ will be the preſent value of her intereſt therein.

COROL. I.

If *A*, and *B*, are of equal ages; then $m = t$; and the anſwers will be $\overline{nn - vv + \frac{v^3}{2n}} \times \frac{1}{6mm}$, and $\overline{\overline{\mathfrak{P} - \frac{n}{6r}} \times n - \overline{\mathfrak{D} - \frac{v}{6r}} \times \overline{1 - \frac{v}{2n}}} \times v \times \frac{\mathfrak{P}}{2mm}$.

COROL. II.

If *B*, and *C*, are of equal ages; then $m = n$; and the anſwers will become $\overline{nn - vv + \frac{v^3}{2n}} \times \frac{1}{6tn}$, and $\overline{\overline{\mathfrak{P} - \frac{n}{6r}} \times n - \overline{\mathfrak{D} - \frac{v}{6r}} \times \overline{1 - \frac{v}{2n}}} \times v \times \frac{\mathfrak{D}}{2tn}$.

COROL. III.

If *C*, and *D*, are of equal ages; then $n = v$, and $\mathfrak{N} = \mathfrak{D}$; whence the anſwers will become $\frac{vv}{12tm}$, and $(\overline{\overline{\mathfrak{D} - \frac{v}{6r}} \times v - \overline{\mathfrak{D} - \frac{v}{6r}} \times v \times \overline{1 - \frac{1}{2}}} \times \frac{\mathfrak{P}}{2mt}$, or $\overline{\mathfrak{D} - \frac{v}{6r}} \times \frac{v}{2} \times \frac{\mathfrak{P}}{2mt} =)\ \overline{\mathfrak{D} - \frac{v}{6r}} \times \frac{v\mathfrak{P}}{4mt}$.

COROL. IV.

If *A*, *B*, and *C*, are of equal ages; then $t = m =$ and the anſwers will be $nn - vv + \frac{v^3}{2n} \times \frac{1}{6nn}$, an

$$\overline{\overline{\mathfrak{Q} - \frac{n}{6r}} \times n - \overline{\mathfrak{D} - \frac{v}{6r}} \times \overline{1 - \frac{v}{2n}} \times v \times \frac{\mathfrak{P}}{2nn}}.$$

COROL. V.

If *B*, *C*, and *D*, are of equal ages; then $m = n =$ and $\mathfrak{Q} = \mathfrak{D}$; whence the anſwers will be, $\frac{v}{12t}$ and

$$\overline{\mathfrak{D} - \frac{v}{6r}} \times \frac{\mathfrak{P}}{4t}.$$

QUESTION LV.

The probability, that *B* and *D*, conſidered as poſſeſſors, ſhall both die before either *A* or *C*; and that *A*, conſidered as expectant, ſhall die before *C*, the ſurvivor; alſo the preſent worth of *C*'s intereſt, in an eſtate or legacy, worth $\mathfrak{P}$ pounds, dependent upon this order of ſurvivorſhip; are required.

SOLU

SOLUTION.

Here the probability that *A* ſhall be ſurvived, by

$$\left.\begin{array}{ll} C, & \text{is } \frac{n}{2t} \\ B, \& C, & \text{is } \frac{n}{2t} - \frac{nn}{6mt} \\ D, \& C, & \text{is } \frac{v}{2t} - \frac{vv}{6nt} \\ B, D, \& C, & \text{is } \frac{v}{2t} - \frac{vv}{6nt} - \frac{vv}{6mt} + \frac{v^3}{12ntm} \end{array}\right\} \text{by queſt.} \left\{\begin{array}{l} 21; \\ 31; \\ 31; \\ 33; \end{array}\right.$$

Which probabilities, being ſubtracted and added, in the manner above directed, will give

$$\left(\frac{nn}{6tm} - \frac{vv}{6tm} + \frac{v^3}{12ntm} \text{ or }\right) \overline{nn - vv + \frac{v^3}{2n}} \times \frac{1}{6tm}$$

for the probability required; and conſequently

$$(\overline{\mathfrak{N}-\frac{n}{6r}}\times\frac{n\mathfrak{P}}{2tm}-\overline{\mathfrak{V}-\frac{v}{6r}}\times\frac{v\mathfrak{P}}{2tm}$$

$$\left(+\overline{\mathfrak{V}-\frac{v}{6r}}\times\frac{vv\mathfrak{P}}{4mtn},\right.$$

$$\text{or)}\ \overline{\overline{\mathfrak{N}-\frac{n}{6r}}\times n-\overline{\mathfrak{V}-\frac{v}{6r}}\times\overline{1-\frac{v}{2n}}\times v}\times\frac{\mathfrak{P}}{2mt},$$

will be the preſent worth of *C*'s intereſt in that eſtate.

COROL.

Since the reſult, above found, is exactly the ſame, with that of the laſt queſtion; therefore, the numeral proceſs, and the anſwers to all the varieties, occaſioned by ſuppoſing two or three of the given lives to be of equal ages, will be the ſame, as in the example and corollaries, to that queſtion.

QUESTION LVI.

The probability that *A* and *C*, conſidered as poſſeſſors, ſhall both die before either *D* or *B*; and that *D*, conſidered as expectant, ſhall die before *B*, the ſurvivor; alſo the preſent worth of *B*'s intereſt, in an eſtate or legacy, worth $\mathfrak{P}$ pounds, dependent upon this order of ſurvivorſhip; are required.

SOLU-

SOLUTION.

Here the probability, that *D* ſhall be ſurvived, by

$$\left.\begin{array}{l} B,\text{ is } 1-\frac{v}{2m} \\ A,\ \&\ B,\text{ is } 1-\frac{v}{2m}-\frac{v}{2t}+\frac{vv}{3mt} \\ C,\ \&\ B,\text{ is } 1-\frac{v}{2n}-\frac{v}{2m}+\frac{vv}{3nm} \\ A,C,\ \&\ B,\text{ is } 1-\frac{v}{2n}-\frac{v}{2m}-\frac{v}{2t}+\frac{vv}{3nt}+\frac{vv}{3nm}+\frac{vv}{3mt}-\frac{v^3}{4tmn} \end{array}\right\}\text{by queſt.}\left\{\begin{array}{l}22;\\31;\\31;\\33;\end{array}\right.$$

Which probabilities, being ſubtracted, and added, according to the directions above-mentioned, will give $\left(\frac{vv}{3nt}-\frac{v^3}{4mtn}=\right)\frac{1}{3}-\frac{v}{4m}\times\frac{vv}{nt}$, for the probability required.

Now the preſent worths, correſponding to the above probabilities are

$$\left.\begin{array}{l}
\frac{v\mathfrak{M}}{v} \\[1ex]
\frac{v\mathfrak{M}}{v} - \overline{v\mathfrak{M} - \frac{vv}{6rm}} \times \frac{1}{2t} \\[1ex]
\frac{v\mathfrak{N}}{v} - \overline{v\mathfrak{N} - \frac{vv}{6rn}} \times \frac{1}{2m} \\[1ex]
\frac{v\mathfrak{N}}{v} - \overline{\overline{v\mathfrak{N} - \frac{vv}{6rn}} \times \overline{m+t} \times \overline{2-v-\frac{vv}{6rn}} \times \overline{n-v} \times 2} \times \frac{1}{4mt}
\end{array}\right\} \text{by ſchol. to queſtion} \left\{\begin{array}{l} 23; \\[1ex] 32; \\[1ex] 32; \\[1ex] 34; \end{array}\right.$$

which preſent worths are to be ſubtracted and added, in the ſame manner, as the before going probabilities; and the reſult will be

$$\overline{v\mathfrak{M} - \frac{vv}{6rm}} \times \frac{1}{2t} -$$

$$\overline{\overline{v\mathfrak{P} - \frac{vv}{6rn}} \times \overline{2m-v} - \frac{vv}{6rn} \times \overline{n-v} \times 2} \times \frac{1}{4mt},$$

Or)

$$\left.\begin{array}{l} \overline{v\mathfrak{M} - \frac{vv}{6rm}} \times 2m - \overline{v\mathfrak{P} - \frac{vv}{6rn}} \times \overline{2m-v} \\ + \frac{vv}{6rn} \times \overline{n-v} \times 2 \end{array}\right\} \times \frac{1}{4mt}:$$

And therefore,

$$\left.\begin{array}{l} \overline{v\mathfrak{M} - \frac{vv}{6rm}} \times 2m - \overline{v\mathfrak{P} - \frac{vv}{6rn}} \times \overline{2m-v} \\ + \frac{vv}{6rn} \times \overline{n-v} \times 2 \end{array}\right\} \times \frac{\mathfrak{P}}{4mt},$$

will be the preſent worth required.

EXAMPLE.

C (aged 66) is poſſeſſed of an eſtate of 1£. per annum, in which his wife *A* aged 43 is jointured; after their deceaſe, it will devolve to *D* (aged 76) the uncle of *C*, if he be then alive; now *D* has a daughter *B* (aged 54) who will ſucceed to the ſame after *D*'s deceaſe, if ſhe be then alive; required the probability that *B* has of becoming poſſeſſed of the eſtate, and the preſent worth of her intereſt therein.

Here $v\mathfrak{M}=6{,}925$; $v\mathfrak{N}=6{,}215$; $v=10$; $n=20$; $m=32$; $t=43$; $r=1{,}04$; $\mathfrak{P}=25$.

Then $\frac{1}{3}=0{,}3333$; $\frac{10}{4\cdot32}=0{,}0781$;

And $0{,}3333-0{,}0781=0{,}2552$; therefore ($\frac{0.2552\times10\times10}{20\times43}=$) $0{,}02967$ will be the probability required.

Alſo $\frac{10\cdot10}{6.1{,}04.20}=0{,}801$; $\frac{10\cdot10}{6.1{,}04.32}=0{,}501$; $6{,}925-0{,}501=6{,}424$; $2m-v=54$; $6{,}215-0{,}801=5{,}414$; $6{,}424\times2\times32=411{,}136$; $5{,}414\times54=292{,}356$; $0{,}801\times\overline{20-10}\times2=16{,}02$; and $411{,}136-292{,}356+16{,}02=134{,}8$;

Therefore ($\frac{134{,}8\times25}{4\times32\times43}=$) $0{,}6126$ will be the preſent worth required.

COROL. I.

If *A*, and *B*, are of equal ages; then $t=m$; and the anſwers will be $\overline{\frac{1}{3}-\frac{v}{4m}}\times\frac{vv}{nm}$ and

$$\left.\begin{array}{l}\overline{v\mathfrak{M}-\frac{vv}{6rm}}\times2m-\overline{v\mathfrak{N}-\frac{vv}{6rn}}\times\overline{2m-v}\\+\frac{vv}{6rn}\times\overline{n-v}\times2\end{array}\right\}\times\frac{\mathfrak{P}}{4mm}.$$

COROL. II.

If *B* and *C* are of equal ages; then $m=n$; and $v\mathfrak{M}=v\mathfrak{N}$ whence the anſwers will become $\overline{\frac{1}{3}-\frac{v}{4n}}\times\frac{vv}{nt}$

and ($\overline{\overline{v\mathfrak{N} - \frac{vv}{6rn}} \times v + \frac{vv}{6rn} \times \overline{n-v} \times 2} \times \frac{\mathfrak{P}}{4nt}$, or)

$\overline{\overline{v\mathfrak{N} - \frac{vv}{6rn}} + \frac{2v}{6rn} \times \overline{n-v}} \times \frac{v\mathfrak{P}}{4nt}$;

That is ($\overline{v\mathfrak{N} - \frac{vv}{6rn} + \frac{2nv-2vv}{6rn}} \times \frac{v\mathfrak{P}}{4nt}$, or)

$\overline{v\mathfrak{N} + \frac{2n-3v}{6rn} \times v} \times \frac{v\mathfrak{P}}{4nt}$.

COROL. III.

If *C* and *D*, are of equal ages; then $n = v$, and $v\mathfrak{N} = \mathfrak{P}$; whence the anſwers will become

$\overline{\frac{1}{3} - \frac{v}{4m}} \times \frac{v}{t}$, and

($\overline{\overline{v\mathfrak{M} - \frac{vv}{6rm}} \times 2m - \overline{\mathfrak{P} - \frac{v}{6r}} \times \overline{2m-v}} \times \frac{\mathfrak{P}}{4mt}$,

or) $\overline{v\mathfrak{M} - \frac{vv}{6rm} - \overline{\mathfrak{P} - \frac{v}{6r}} \times \frac{2m-v}{2m}} \times \frac{\mathfrak{P}}{2t}$.

COROL. IV.

If *A*, *B*, and *C*, are of equal ages; then $t = m = n$, and $v\mathfrak{M} = v\mathfrak{N}$; whence the anſwers will become,

$\overline{\frac{1}{3} - \frac{v}{4n}} \times \frac{vv}{nn}$, and

($\overline{\overline{v\mathfrak{N} - \frac{vv}{6rn}} + \frac{2v}{6rn} \times \overline{n-v}} \times \frac{v\mathfrak{P}}{4nn}$,

or) $\overline{v\mathfrak{N} + \frac{2n-3v}{6rn} \times v} \times \frac{v\mathfrak{P}}{4nn}$.

COROL. V.

If *B*, *C*, and *D*, are of equal ages; then $m = n = v$, and $v\mathfrak{M} = v\mathfrak{N} = \mathfrak{D}$; whence the anſwers will become $\left(\overline{\frac{1}{3}-\frac{v}{4v}}\times\frac{v}{t}\text{ or}\right)\frac{v}{12t}$, and

$$\overline{\left(\overline{\mathfrak{D}-\frac{v}{6r}}\times 2v-\overline{\mathfrak{D}-\frac{v}{6r}}\times\overline{2v-v}\right.}\times\frac{\mathfrak{P}}{4vt},\text{ or)}$$

$$\overline{\mathfrak{D}-\frac{v}{6r}}\times\frac{\mathfrak{P}}{4t}.$$

QUESTION LVII.

The probability, that *A* and *D*, conſidered as poſſeſſors, ſhall both die before either *C* or *B*; and that *C*, conſidered as expectant, ſhall die before *B*, the ſurvivor; alſo the preſent worth of *B*'s intereſt, in an eſtate or legacy, worth $\mathfrak{P}$ pounds, dependent upon this order of ſurvivorſhip; are required.

SOLUTION.

Here the probability that *C* ſhall be ſurvived, by

B, is $1 - \frac{n}{2m}$	by queſtion	22;
A, & *B*, is $1 - \frac{n}{2m} - \frac{n}{2t} + \frac{nn}{3mt}$	by queſtion	31;
D, & *B*, is $\frac{v}{2n} - \frac{vv}{6mn}$	by queſtion	31;
A, *D*, & *B*, is $\frac{v}{2n} - \frac{vv}{6mn} - \frac{vv}{6tn} + \frac{v^3}{12tmn}$	by queſtion	33;

Which probabilities, being ſubtracted and added, according to the abovementitoned directions, will give $+ \frac{n}{2t} - \frac{nn}{3tm} - \frac{vv}{6tn} + \frac{v^3}{12tnm}$ or $\frac{1}{2} - \frac{n}{3m} \times n - 1 - \frac{v}{2m} \times \frac{vv}{6n} \times \frac{1}{t}$, for the probability required.

Now the preſent worths correſponding to the abov
probabilities are

$$\left.\begin{array}{l} \dfrac{{}^{n}\mathfrak{M}}{n} \\[2ex] \dfrac{{}^{n}\mathfrak{M}}{n} - \overline{{}^{n}\mathfrak{M} - \dfrac{nn}{6rm}} \times \dfrac{1}{2t} \\[2ex] \dfrac{\mathfrak{A}}{n} - \overline{\mathfrak{B} - \dfrac{v}{6r}} \times \dfrac{v}{2mn} \\[2ex] \dfrac{\mathfrak{B}}{n} - \overline{\mathfrak{B} - \dfrac{v}{6r}} \times \dfrac{\overline{\overline{m+t} \times 2 - v} \times v}{4mtn} \end{array}\right\} \text{by queſtion} \left\{\begin{array}{l} 23; \\[2ex] 32; \\[2ex] 32; \\[2ex] 34; \end{array}\right.$$

Theſe preſent worths, being ſubtracted and added, in th
manner as the beforegoing probabilities, will give

$$\overline{{}^{n}\mathfrak{M} - \frac{nn}{6rm}} \times \frac{1}{2t} + \overline{\mathfrak{B} - \frac{v}{6r}} \times \frac{v}{2mn} -$$

$$\overline{\mathfrak{B} - \frac{v}{6r}} \times \frac{\overline{\overline{m+t} \times 2 - v} \times v}{4mtn},$$ for the reſult.

But ($\overline{\mathfrak{A} - \frac{v}{6r}} \times \frac{v}{2mn} - \overline{\mathfrak{B} - \frac{v}{6r}} \times \frac{\overline{\overline{m+t} \times 2 - v} \times v}{4mtn}$, or

$\overline{\mathfrak{B} - \frac{v}{6r}} \times \frac{v}{2mn} - \overline{\mathfrak{A} - \frac{v}{6r}} \times \frac{v}{2mn} \times \frac{2m+2t-v}{2t}$,

will (becauſe $1 - \frac{2m-2t-v}{2t} = -\frac{2m-v}{2t}$) be equa

to $(-\overline{\mathfrak{B} - \frac{v}{6r}} \times \frac{v}{2mn} \times \frac{2m-v}{2t} =)$

$-\overline{\mathfrak{A} - \frac{v}{6r}} \times \frac{\overline{2m-v} \times v}{4mtn}$; and conſequently, the re
reſult will become,

$$\overline{{}^{n}\mathfrak{M} - \frac{nn}{6rm}} \times \frac{1}{2t} - \overline{\mathfrak{B} - \frac{v}{6r}} \times \frac{\overline{2m-v} \times v}{4mtn},$$ or)

${}^{n}\mathfrak{M}$

$$\overline{n\mathfrak{M}-\frac{nn}{6rm}-\overline{\mathfrak{P}-\frac{v}{6r}}\times\frac{\overline{2m-v}\times v}{2mn}}\times\frac{1}{2t};$$

therefore the required preſent worth will be,

$$\overline{n\mathfrak{M}-\frac{nn}{6rm}-\overline{\mathfrak{P}-\frac{v}{6r}}\times\frac{\overline{2m-v}\times v}{2mn}}\times\frac{\mathfrak{P}}{2t}.$$

EXAMPLE.

There are two brothers *D* (aged 76) and *C* (aged 66) *D*, the elder brother, is poſſeſſed of a paternal eſtate of 1 £. per annum, and hath a daughter *A* (aged 43) who is to poſſeſs the ſame, during her life, if ſhe ſurvives her father; but, after their deceaſe, if *C*, the younger brother, be then alive, he will become poſſeſſed thereof, and may leave it to his wife *B* (aged 54), the probability of *B*'s becoming poſſeſſed of the eſtate, together with the preſent worth of her intereſt therein, are required.

Here $\overline{n\mathfrak{M}-\frac{nn}{6rm}}=7{,}888$; $\overline{\mathfrak{P}-\frac{v}{6r}}=2{,}715$; $v=10$; $n=20$; $m=32$; $t=43$; $2m-v=54$; and $2mn=1280$;

Then $\frac{20}{3.32}=0{,}2083$; and $0{,}5-0{,}2083=0{,}2916$; alſo $0{,}2916\times 20=5{,}833$;

Again $\frac{10}{2.32}=0{,}15625$; and $1-0{,}15625=0{,}84375$; alſo $\frac{0{,}84375\times 10\times 10}{6.\ 20}=0{,}703$;

Whence ($5{,}833-0{,}703$ or) $5{,}130\times\frac{1}{43}=0{,}1193$, will be the probability required.

Now $\frac{2{,}715\times 54\times 10}{1280}=1{,}145$; and $7{,}888-1{,}145=6{,}743$;

Whence $\left(\frac{6{,}743\times 25}{2\times 43}=\right)$ 1,960 will be the preſent worth required.

COROL. I.

If *A*, and *B*, are of equal ages; then $n = m$; and the anſwers will become

$$\overline{\overline{\tfrac{1}{2}-\frac{n}{3m}}\times\overline{n-1-\frac{v}{2m}}\times\frac{vv}{6n}}\times\frac{1}{m},\ \text{and}$$

$$\overline{{}^{n}\mathfrak{M}-\frac{nn}{6rm}-\overline{\mathfrak{D}-\frac{v}{6r}}\times\frac{\overline{2m-v}\times v}{2mn}}\times\frac{\mathfrak{P}}{2m}.$$

COROL. II.

If *B*, and *C*, are of equal ages; then $m = n$, and ${}^{n}\mathfrak{M} = \mathfrak{N}$; whence $(\overline{\overline{\tfrac{1}{2}-\tfrac{1}{3}}\times\overline{n-1-\frac{v}{2n}}\times\frac{vv}{6n}}\times\frac{1}{t}$ or)

$$\overline{\frac{n}{6}-\overline{1-\frac{v}{2n}}\times\frac{vv}{6n}}\times\frac{1}{t},$$

That is $\overline{\overline{n-1-\frac{v}{2n}}\times\frac{vv}{n}}\times\frac{1}{6t}$ will be the probability; and,

$$\overline{\overline{\mathfrak{N}-\frac{n}{6r}}-\overline{\mathfrak{D}-\frac{v}{6r}}\times\frac{\overline{2n-v}\times v}{2nn}}\times\frac{\mathfrak{P}}{2t},\ \text{will be}$$

the preſent worth required.

COROL. III.

If *C*, and *D*, are of equal ages; then $n = v$; and

$$(\overline{\overline{\tfrac{1}{2}-\frac{v}{3m}}\times\overline{v-1-\frac{v}{2m}}\times\frac{v}{6}}\times\frac{1}{t},\ \text{or})$$

$$\overline{\overline{3-\frac{2v}{m}}-\overline{1-\frac{v}{2m}}}\times\frac{v}{6t},\ \text{that is}$$

$(\overline{2-\frac{3v}{2m}}\times\frac{v}{6t}$ or$)\ \overline{\frac{1}{3}-\frac{v}{4m}}\times\frac{v}{t}$, will be the probability; and, $\overline{{}^{v}\mathfrak{M}-\frac{vv}{6rm}-\overline{\mathfrak{D}-\frac{v}{6r}}\times\frac{2m-v}{2m}}\times\frac{\mathfrak{P}}{2t}$, will be the preſent worth required.

COROL. IV.

If *A*, *B*, and *C*, are of equal ages; then $t=m=n$, and ${}^{n}\mathfrak{M}={}^{t}\mathfrak{P}$; whence

$(\overline{\overline{n-1-\frac{v}{2n}}\times\frac{vv}{n}}\times\frac{1}{6n}$, or$)$

$\frac{1}{6}-\overline{1-\frac{v}{2n}}\times\frac{vv}{6nn}$, will be the probability; and

$\overline{\overline{\mathfrak{P}-\frac{n}{6r}}-\overline{\mathfrak{D}-\frac{v}{6r}}\times\frac{\overline{2n-v}\times v}{2nn}}\times\frac{\mathfrak{P}}{2n}$, will be the preſent worth required.

COROL. V.

If *B*, *C*, and *D*, are of equal ages; the $m=n=v$, and ${}^{n}\mathfrak{M}=\mathfrak{C}$; whence $(2-\frac{3}{2}\times\frac{v}{6t}$ or$)\frac{v}{12t}$, will be the probability, and

$(\overline{\mathfrak{D}-\frac{v}{6r}}-\overline{\mathfrak{D}-\frac{v}{6r}}\times\overline{\frac{2v-v}{2v}}\times\frac{\mathfrak{P}}{2t}=)$

$\overline{\mathfrak{D}-\frac{v}{6r}}\times\frac{\mathfrak{P}}{4t}$, will be the preſent worth required.

QUESTION

QUESTION LVIII.

The probability, that *C* and *D*, conſidered as poſſeſſors, ſhall both die before either *A* or *B*, and that *A*, conſidered as expectant, ſhall die before *B*, the ſurvivor; alſo the preſent worth of an eſtate or legacy, worth *p*, pounds, dependent upon this order of ſurvivorſhip; are required.

SOLUTION.

Here the probability, that *A* ſhall be ſurvived, by

$$\left.\begin{array}{l} B\text{, is } \dfrac{m}{2t} \\ C, \& \ B\text{, is } \dfrac{n}{2t} - \dfrac{nn}{6mt} \\ D, \& \ B\text{, is } \dfrac{v}{2t} - \dfrac{vv}{6mt} \\ C,D,\& \ B\text{, is } \dfrac{v}{2t} - \dfrac{vv}{6mt} - \dfrac{vv}{6nt} + \dfrac{v^3}{12mnt} \end{array}\right\} \text{by question} \left\{\begin{array}{l} 21; \\ 31; \\ 31; \\ 33; \end{array}\right.$$

Which probabilities, being ſubtracted and added, according to the directions before mentioned, will give $\left(\frac{m}{2t} - \frac{n}{2t} + \frac{nn}{6mt} - \frac{vv}{6nt} + \frac{v^3}{12mnt}\text{, or}\right) \frac{m}{2t} - \overline{1 - \frac{n}{3m}} \times \frac{n}{2t} - \overline{1 - \frac{v}{2m}} \times \frac{vv}{6nt}$;

That is, $\overline{m-1-\frac{n}{3m}}\times\overline{n-1-\frac{v}{2m}}\times\frac{vv}{3n}\times\frac{1}{2t}$, for the probability required.

And the preſent worths correſponding to thoſe probabilities will be $\frac{\mathfrak{M}}{t}-\frac{\mathfrak{N}}{t}+\overline{\mathfrak{N}-\frac{n}{6r}}\times\frac{n}{2mt}-\overline{\mathfrak{D}-\frac{v}{6r}}\times\frac{v}{2nt}+\overline{\mathfrak{D}-\frac{v}{6r}}\times\frac{vv}{4nmt}$.

But, $\overline{\mathfrak{D}-\frac{v}{6r}}\times\frac{v}{2nt}-\overline{\mathfrak{D}-\frac{v}{6r}}\times\frac{v}{2nt}\times\frac{v}{2m}$

$=\overline{\mathfrak{D}-\frac{v}{6r}}\times\overline{1-\frac{v}{2m}}\times\frac{v}{2nt}$;

Whence the above expreſſion will become

$\Big(\frac{\mathfrak{M}}{t}-\frac{\mathfrak{N}}{t}+\overline{\mathfrak{N}-\frac{n}{6r}}\times\frac{n}{2mt}-$

$\overline{\mathfrak{D}-\frac{v}{6r}}\times\overline{1-\frac{v}{2m}}\times\frac{v}{2nt}$, or$\Big)$

$$\left.\begin{array}{l}\mathfrak{M}-\mathfrak{N}+\\ \overline{\overline{\mathfrak{N}-\frac{n}{6r}}\times n-\overline{\mathfrak{D}-\frac{v}{6r}}\times\frac{2m-v}{2n}\times v\times\frac{1}{2m}}\end{array}\right\}\times\frac{1}{t};$$

(therefore the) preſent worth of *B*'s intereſt in the eſtate will be,

$$\left.\begin{array}{l}\mathfrak{M}-\mathfrak{N}+\\ \overline{\overline{\mathfrak{N}-\frac{n}{6r}}\times n-\overline{\mathfrak{D}-\frac{v}{6r}}\times\frac{2m-v}{2n}\times v\times\frac{1}{2m}}\end{array}\right\}\times\frac{\mathfrak{P}}{t}.$$

EXAMPLE

EXAMPLE.

D (aged 76) is poſſeſſed of an eſtate of 1 £. per annum; which (on his deceaſe) will deſcend to his brother *C* (aged 66); and, after both their deaths, to *A*, the ſon of *C* (aged 43) if he be then alive, who may leave it to *B*, his wiſe, aged 54: the probability of her coming into poſſeſſion of, and the preſent value of her intereſt in, the eſtate, are required.

Here $v = 10$; $n = 20$; $m = 32$; $r = 43$; $\mathfrak{M} = 10{,}757$; $\mathfrak{N} = 7{,}673$; $\mathfrak{N} - \frac{n}{6r} = 4{,}468$; $\mathfrak{P} - \frac{v}{6r} = 2{,}715$; and $\mathfrak{P} = 25$.

Then $\frac{20}{3 \times 32} = 0{,}2083$; $1 - 0{,}2083 = 0{,}7917$; $0{,}7917 \times 20 = 15{,}834$;

And $\frac{10}{2 \times 32} = 0{,}15625$; $1 - 0{,}15625 = 0{,}84375$; $0{,}84375 \times \frac{10 \times 10}{3 \times 20} = 1{,}406$:

Therefore ($32 - 15{,}834 - 1{,}406$ or) $14{,}76 \times \frac{1}{2 \times 43} = 0{,}172$, will be the probability required.

Again $4{,}468 \times 20 = 89{,}36$; $\frac{2m - v}{2n} = \left(\frac{54}{40} = \frac{27}{20} =\right) 1{,}35$; $2{,}715 \times 1{,}35 \times 10 = 36{,}6525$; $89{,}36 - 36{,}6525 = 52{,}7075$; and $\frac{52{,}7075}{2 \times 32} = 0{,}824$.

Then $10{,}757 - 7{,}673 + 0{,}824 = 3{,}908$: therefore $\left(\frac{3{,}908 \times 25}{43} =\right)$ 2,272 will be the preſent worth required.

COROL. I.

If *A*, and *B*, are of equal ages; then $t = m$; and the anſwers will become,

$$\overline{-1-\frac{n}{3m}} \times \overline{n-1-\frac{v}{2m}} \times \overline{\frac{vv}{3n}} \times \frac{1}{2m}, \text{ and}$$

$$\left.\begin{array}{l} \mathfrak{N}-\mathfrak{N}+\overline{\mathfrak{N}-\frac{n}{6r}} \times \frac{n}{2m} - \\ \overline{-\frac{v}{6r}} \times \frac{2m-v}{2n} \times v \times \frac{1}{2m} \end{array}\right\} \times \frac{\mathfrak{P}}{m}.$$

COROL. II.

If *B*, and *C*, are of equal ages; then $m = n$, and $\mathfrak{M} = \mathfrak{N}$;

Whence ($\overline{n-1-\frac{1}{3}} \times \overline{n-1-\frac{v}{2n}} \times \frac{vv}{3n} \times \frac{1}{2t}$, or)

$$\overline{-1-\frac{v}{2n}} \times \frac{vv}{3n} \times \frac{1}{2t}; \text{ that is}$$

$$\overline{-1-\frac{v}{2n}} \times \frac{vv}{n} \times \frac{1}{6t}, \text{ will be the probabili-}$$

ty; and

$$\overline{\overline{\mathfrak{N}-\frac{n}{6r}} \times n - \overline{\mathfrak{D}-\frac{v}{6r}} \times \frac{2n-v}{2n} \times v} \times \frac{\mathfrak{P}}{2nt},$$

(Or) $\overline{\mathfrak{N}-\frac{n}{6r}-\overline{\mathfrak{A}-\frac{v}{6r}} \times \frac{\overline{2n-v}\times v}{2nn}} \times \frac{\mathfrak{P}}{2t}$, will be the preſent worth required.

COROL.

COROL. III.

If *C*, and *D*, are of equal ages; then $n = v$, and $\mathfrak{N} = \mathfrak{V}$; whence

$$\left(\overline{m-\overline{1-\frac{v}{3m}}\times v-\overline{1-\frac{v}{2m}}}\times\frac{v}{3}\times\frac{1}{2t},\ \text{or}\right)$$

$$\overline{m-\overline{4-\frac{3v}{2m}}}\times\frac{v}{3}\times\frac{1}{2t},$$ will be the probability; and

$$\left.\begin{matrix}\mathfrak{M}-\mathfrak{V}+\overline{\mathfrak{T}-\frac{v}{6r}}\times\frac{v}{2m}-\\ \overline{\mathfrak{T}-\frac{v}{6r}}\times v\times\frac{2m-v}{2v}\times\frac{1}{2m}\end{matrix}\right\}\times\frac{\mathfrak{P}}{t}$$

$$\text{or)}\ \overline{\mathfrak{M}-\mathfrak{T}+\overline{\mathfrak{T}-\frac{v}{6r}}\times\frac{\overline{3v-2m}}{4m}}\times\frac{\mathfrak{P}}{t}$$ will be the preſent worth required.

COROL. IV.

If *A*, *B*, and *C*, are of equal ages; then $t = m = n$, and $\mathfrak{M} = \mathfrak{N}$; whence $\left(\overline{n-1-\frac{v}{2n}}\times\frac{vv}{n}\times\frac{1}{6n},\ \text{or}\right)\ \frac{1}{6}$

$$\overline{-1-\frac{v}{2n}}\times\frac{vv}{6nn},$$ will be the probability; and

$$\overline{\mathfrak{N}-\frac{n}{6r}-\overline{\mathfrak{V}-\frac{v}{6r}}\times\frac{\overline{2n-v}\times v}{2nn}}\times\frac{\mathfrak{P}}{2n}$$ will be the preſent worth required.

COROL.

COROL. V.

If *B*, *C*, and *D*, are of equal ages; then $m = n = v$, and $\mathfrak{M} = \mathfrak{N} = \mathfrak{Q}$; whence $\left(\overline{v-4} = \frac{3}{4} \times \frac{v}{3} \times \frac{1}{2t}\right.$, or$\left.\right)$ $\frac{v}{12t}$ will be the probability; and $\left(\overline{\mathfrak{P} - \frac{v}{6r}} \times \frac{1}{4} \times \frac{\mathfrak{P}}{t}\right.$; or$\left.\right)$ $\overline{\mathfrak{P} - \frac{v}{6r}} \times \frac{\mathfrak{P}}{4t}$, will be the preſent worth required.

QUESTION LIX.

The probability, that *B* and *C*, conſidered as poſſeſſors, ſhall both die before either *D* or *A*; and that *D*, conſidered as expectant, ſhall die before *A*, the ſurvivor; alſo the preſent worth of an eſtate, or legacy, worth $\mathfrak{P}$ pounds, dependent upon this order of ſurvivorſhip; are required.

SOLUTION.

Here the probability that *D* ſhall be ſurvived, by

$$\left.\begin{array}{rl} A, \text{ is } & 1-\frac{v}{2t} \\ C, \ \&\ A, \text{ is } & 1-\frac{v}{2n}-\frac{v}{2t}+\frac{vv}{3nt} \\ B, \ \&\ A, \text{ is } & 1-\frac{v}{2m}-\frac{v}{2t}+\frac{vv}{3mt} \\ C,\ B, \ \&\ A, \text{ is } & 1-\frac{v}{2n}-\frac{v}{2m}-\frac{v}{2t}+\frac{vv}{3nt}+\frac{vv}{3mn}+\frac{vv}{3mt}-\frac{v^3}{4tmn} \end{array}\right\} \text{by queſt.} \left\{\begin{array}{l} 22; \\ 31; \\ 31; \\ 33; \end{array}\right.$$

Which probabilities, being ſubtracted and added, according to the directions abovementioned, will give $\left(\frac{vv}{3mn}-\frac{v^3}{4tnm}\text{ or}\right)\ \frac{1}{3}-\frac{v}{4t}\times\frac{vv}{nm}$, for the probability required.

Now the preſent worths, correſponding to the above probabilities are

$$\left.\begin{array}{l} \frac{v\mathfrak{F}}{v} \\ \frac{v\mathfrak{N}}{v} - \overline{v\mathfrak{N} - \frac{vv}{6rn}} \times \frac{1}{2t} \\ \frac{v\mathfrak{M}}{v} - \overline{v\mathfrak{M} - \frac{vv}{6rm}} \times \frac{1}{2t} \\ \frac{v\mathfrak{N}}{v} - \overline{\overline{v\mathfrak{N} - \frac{vv}{6rn}} \times \overline{m+t} \times \overline{2 - v - \frac{vv}{6rn}} \times \overline{n-v}} \times 2 \times \frac{1}{4mt} \end{array}\right\} \text{by schol. to question} \left\{\begin{array}{l} 23; \\ 32; \\ 32; \\ 34; \end{array}\right.$$

Which preſent worths, being ſubtracted and added, in the ſame manner, as the foregoing probabilities; the reſult will be $\left(\frac{v\mathfrak{F}}{v} - \frac{v\mathfrak{M}}{v} + \overline{v\mathfrak{M} - \frac{vv}{6rm}} \times \frac{1}{2t} - v\mathfrak{N}\right.$

$$\overline{v\mathfrak{N}-\frac{vv}{6rn}\times\overline{2t-v}-\frac{vv}{6rn}\times\overline{n-v}}\times 2\times\frac{1}{4mt},\ \text{or}$$

$$\left.\begin{array}{l}\dfrac{v\mathfrak{F}}{v}-\dfrac{v\mathfrak{M}}{v}+\\ \hline v\mathfrak{M}-\dfrac{vv}{6rm}\times\overline{2m-}\\ \hline v\mathfrak{N}-\dfrac{vv}{6rm}\times\overline{2t-v}+\dfrac{2vv}{6rn}\times\overline{n-v}\end{array}\right\}\times\frac{1}{4mt};$$

Therefore the preſent worth of *A*'s intereſt, in the eſtate, will be

$$\mathfrak{P}\times\left\{\begin{array}{l}\dfrac{v\mathfrak{F}-v\mathfrak{M}}{v}+\\ \hline v\mathfrak{M}-\dfrac{vv}{6rm}\times\overline{2m-}\\ \hline v\mathfrak{N}-\dfrac{vv}{6rn}\times\overline{2t-v}+\dfrac{2vv}{6rn}\times\overline{n-v}\end{array}\right\}\times\frac{1}{4m}$$

EXAMPLE.

Two ſiſters *C* (aged 66) and *B* (aged 54) are jointly poſſeſſed of an eſtate of 1£. per ann. which, upon the deceaſe of the ſurvivor of them, is to deſcend to *D*, their uncle (aged 76) if he be then alive; *D*, if he comes into poſſeſſion, may leave it to his wife *A*, (aged 43) the probability that *A* has of becoming poſſeſſed of the eſtate, and the preſent worth of her intereſt therein, are required.

Here $\mathfrak{P}=25$; $v\mathfrak{F}=7{,}228$; $v\mathfrak{M}=6{,}925$; $v\mathfrak{N}=6{,}215$; $t=43$; $m=32$; $n=20$; $v=10$; $\frac{vv}{6rn}=0{,}8015$; $\frac{vv}{6rm}=0{,}501$; $\frac{7{,}228-6{,}925}{10}=0{,}0303$;

Then $\frac{10}{4\times 43}=0{,}0581$; $0{,}3333-0{,}0581=0{,}2752$;

And $\left(\frac{0,2752 \times 10 \times 10}{20 \times 32} =\right)$ 0,043 will be the probability required.

Alſo $6,925 - 0,501 = 6,424$; $6,424 \times 2 \times 32 = 411,136$;

$6,215 - 0,8015 = 5,4135$; $5,4135 \times \overline{2 \times 43 - 10} = 411,426$;

$0,8015 \times \overline{20 - 10} \times 2 = 16,03$; $411,136 - 411,426 + 16,030 = 15,740$;

$\frac{15,740}{4 \times 32 \times 43} = 0,0029$; $0,0303 + 0,0029 = 0,0332$;

And $0,0332 \times 25 =)$ 0,830 will be the preſent worth required.

COROL. I.

If *A*, and *B*, are of equal ages; then $r = m$, and $v\mathfrak{R} = v\mathfrak{M}$; whence the anſwers will become

$\overline{\frac{1}{3} - \frac{v}{4m}} \times \frac{vv}{nm}$, and

$$\left.\begin{array}{l} \overline{v\mathfrak{M} - \frac{vv}{6rm}} \times 2m - \\ \overline{v\mathfrak{N} - \frac{vv}{6rn}} \times \overline{2m - v} + \\ \frac{vv}{6rn} \times \overline{n - v} \times 2 \end{array}\right\} \times \frac{\mathfrak{P}}{4mm}.$$

COROL.

COROL. II.

If B, and C, are of equal ages; then $m=n$, and $v\mathfrak{M}=v\mathfrak{N}$; whence the anſwers will become $\overline{\frac{1}{3}-\frac{v}{4t}}\times\frac{vv}{nn}$, and

$$\mathfrak{P}\times\overline{\frac{v\mathfrak{F}-v\mathfrak{N}}{v}+\overline{v\mathfrak{N}-\frac{vv}{6rn}}\times\overline{2n+v-2t}+\overline{\frac{vv}{6rn}\times\overline{n-v}\times 2}\times\frac{1}{4nt}}.$$

COROL. III.

If C, and D, are of equal ages; then $n=v$, and $v\mathfrak{N}=\mathfrak{D}$; whence the anſwers will become $\overline{\frac{1}{3}-\frac{v}{4t}}\times\frac{v}{m}$, and

$$\mathfrak{P}\times\overline{\frac{v\mathfrak{F}-v\mathfrak{M}}{v}+\overline{v\mathfrak{M}-\frac{vv}{6rm}}\times 2m-\overline{]-\frac{v}{6r}}\times\overline{2t-v}\times\frac{1}{4mt}}.$$

COROL. IV.

If *A*, *B*, and *C*, are of equal ages; then $t = m = n$, and $v\mathfrak{F} = v\mathfrak{M} = v\mathfrak{N}$; whence the anſwers will become $\overline{\frac{1}{3} - \frac{v}{4n}} \times \frac{vv}{nn}$, and

$$\left(\overline{\overline{v\mathfrak{N} - \frac{vv}{6rn}} \times v + \frac{vv}{6rn} \times \overline{n - v} \times 2} \times \frac{\mathfrak{P}}{4nn}, \text{ or}\right)$$

$$\overline{v\mathfrak{N} + \frac{2n - 3v}{6rn}} \times v \times \frac{v\mathfrak{P}}{4nn}.$$

COROL. V.

If *B*, *C*, and *D*, are of equal ages; then $m = n = v$, and $v\mathfrak{M} = v\mathfrak{N} = \mathfrak{D}$; whence the anſwers will become $\frac{1}{3} - \frac{v}{4t}$, and

$$\left(\mathfrak{P} \times \overline{\frac{v\mathfrak{F} - \mathfrak{D}}{v} + \overline{\mathfrak{D} - \frac{v}{6r}} \times \overline{3v - 2t}} \times \frac{1}{4vt}, \text{ or}\right)$$

$$\overline{v\mathfrak{F} - \mathfrak{D} + \overline{\mathfrak{D} - \frac{v}{6r}} \times \frac{3v - 2t}{4t}} \times \frac{\mathfrak{P}}{v}.$$

QUESTION LX.

The probability, that *B* and *D*, conſidered as poſſeſſors, ſhall both die before either *C* or *A*; and that *C*, conſidered as expectant, ſhall die before *A*, the ſurvivor; alſo the preſent worth of an eſtate, or legacy, worth $\mathfrak{P}$ pounds, dependent upon this order of ſurvivorſhip; are required.

SOLUTION.

Here the probability that C ſhall be ſurvived, by

$$\left.\begin{array}{ll} A,\text{ is} & 1-\frac{n}{2t} \\ B,\ \&\ A,\text{ is} & 1-\frac{n}{2t}-\frac{n}{2m}+\frac{nn}{3mt} \\ D,\ \&\ A,\text{ is} & \frac{v}{2n}-\frac{vv}{6tn} \\ B,\ D,\ \&\ A,\text{ is} & \frac{v}{2n}-\frac{vv}{6mn}-\frac{vv}{6tn}+\frac{v^3}{12tmn} \end{array}\right\}\text{by queſtion}\left\{\begin{array}{l} 22; \\ 31; \\ 31; \\ 33; \end{array}\right.$$

Which probabilities, (being ſubtracted and added, as above directed) will give $\left(\frac{n}{2m}-\frac{nn}{3mt}-\frac{vv}{6mn}+\frac{v^3}{12tmn}\right.$ or) $\overline{\tfrac{1}{2}-\frac{n}{3t}}\times\frac{n}{m}-\overline{1-\frac{v}{2t}}\times\frac{vv}{6nm}$; That is, $\overline{\overline{\tfrac{1}{2}-\frac{n}{3t}}\times n-\overline{1-\frac{v}{2t}}\times\frac{vv}{6n}}\times\frac{1}{m}$, for the probability required.

Now the preſent worths correſponding to the above robabilities are

$$\left.\begin{array}{l}
\dfrac{{}^{n}\mathfrak{F}}{n} \\[2ex]
\dfrac{{}^{n}\mathfrak{M}}{n} - \overline{{}^{n}\mathfrak{M} - \dfrac{nn}{6rm}} \times \dfrac{1}{2t} \\[2ex]
- \overline{\mathfrak{D} - \dfrac{v}{6r}} \times \dfrac{v}{2tn} \\[2ex]
- \overline{\mathfrak{D} - \dfrac{v}{6r}} \times \dfrac{\overline{m+t} \times \overline{2 - v} \times v}{4mtn}
\end{array}\right\} \text{by question} \left\{\begin{array}{l}
23; \\[2ex]
32; \\[2ex]
32; \\[2ex]
34;
\end{array}\right.$$

$$\left.\begin{array}{l}
\text{Now } \overline{\mathfrak{D} - \dfrac{v}{6r}} - \\[2ex]
\overline{\mathfrak{D} - \dfrac{v}{6r}} \times \dfrac{\overline{m+t} \times \overline{2 - v}}{2m}
\end{array}\right\} \times \dfrac{v}{2tn}, \text{ will be equal to}$$

$$\overline{\mathfrak{D} - \dfrac{v}{6r}} \times \overline{1 - \dfrac{2m + 2t - v}{2m}} \times \dfrac{v}{2tn} \text{ or)}$$

$$- \overline{\mathfrak{D} - \dfrac{v}{6r}} \times \dfrac{2t - v}{2m} \times \dfrac{v}{2tn},$$

Therefore, if the above preſent worths be ſubtracted and added (as uſual) the reſult will become,

$$\left(\dfrac{{}^{n}\mathfrak{F}}{n} - \dfrac{{}^{n}\mathfrak{H}}{n} + \overline{{}^{n}\mathfrak{M} - \dfrac{nn}{6rm}} \times \dfrac{1}{2t}\right.$$

$$- \overline{\mathfrak{D} - \dfrac{v}{6r}} \times \dfrac{2t - v}{2m} \times \dfrac{v}{2tn}, \text{ or)}$$

$$\dfrac{{}^{n}\mathfrak{F} - {}^{n}\mathfrak{H}}{n} + \overline{{}^{n}\mathfrak{H} - \dfrac{nn}{6rm} - \overline{\mathfrak{D} - \dfrac{v}{6r}} \times \dfrac{\overline{2t - v} \times v}{2mn}} \times \dfrac{1}{2t};$$

Therefore the preſent worth of an eſtate, worth $\mathfrak{P}$ pounds, will be

$$\mathfrak{P} \times \left\{ \begin{array}{l} \dfrac{{}^n\mathfrak{F} - {}^n\mathfrak{M}}{n} + \\ \overline{{}^n\mathfrak{M} - \dfrac{nn}{6rm} - \overline{\mathfrak{P} - \dfrac{v}{6r}} \times \dfrac{\overline{2t - v} \times v}{2mn}} \times \dfrac{1}{2t}. \end{array} \right.$$

EXAMPLE.

D (aged 76) and his daughter *B* (aged 54) are in poſſeſſion of an eſtate (worth $\mathfrak{P}$ pounds) which, after both are dead, will devolve to *C*, the brother of *D* (aged 66) if he be then living; *D* has a wife *A* (aged 43) who will, if ſhe ſurvives him, inherit after her huſband; what is the preſent worth of *A*'s intereſt in the eſtate?

Here ${}^n\mathfrak{F} = 10,838$; ${}^n\mathfrak{M} = 9,891$; $n = 20$; $\overline{{}^n\mathfrak{M} - \frac{nn}{6rm}} = 7,888$; $\overline{\mathfrak{P} - \frac{v}{6r}} = 2,715$; $t = 43$; $v = 10$; $m = 32$; $\frac{\overline{2t - v} \times v}{2mn} = \left(\frac{76 \times 10}{2 . 32 . 20} = \right) \frac{19}{32}$:

Then $10,838 - 9,891 = 0,947$; $\frac{0,947}{20} = 0,04735$; $2,715 \times \frac{19}{32} = 1,612$; $7888 - 1,612 = 6,276$; $\frac{6,276}{86} = 0,07298$; $0,04735 + 0,07298 = 0,12033$; and $(0,12033 \times 25 =)\ 3,00825$, will be the preſent worth required.

If the probability, of *A*'s becoming poſſeſſed of the eſtate, had been required; then $\frac{n}{3t} = \left(\frac{20}{3 \cdot 43} = \right) \frac{20}{129}$; $\frac{v}{2t} = \frac{10}{86}$; $\frac{1}{2} - \frac{20}{129} = \frac{89}{258}$; $1 - \frac{10}{86} = \frac{76}{86}$; and $\frac{vv}{6n} = \left(\frac{10 . 10}{6 . 20} = \right) \frac{5}{6}$:

Therefore $\frac{80}{258} \times 20 = 6{,}899$; $\frac{76}{86} \times \frac{5}{6} = 0{,}736$; $6{,}899 - 0{,}736 = 6{,}163$;

And $\left(\frac{6{,}163}{32} =\right) 0{,}193$ will be the probability required.

COROL. I.

If *A*, and *B*, are of equal ages; then ${}^{n}\mathfrak{F} = {}^{n}\mathfrak{M}$, and $t = m$; whence the anſwers will become

$$\overline{\frac{1}{2} - \frac{n}{3m}} \times n - \overline{1 - \frac{v}{2m}} \times \frac{vv}{6n} \times \frac{1}{m}, \text{ and}$$

$${}^{n}\mathfrak{M} - \frac{nn}{6rm} - \mathfrak{D} - \frac{v}{6r} \times \frac{\overline{2m - v} \times v}{2mn} \times \frac{\mathfrak{P}}{2m}.$$

COROL. II.

If *B*, and *C*, are of equal ages; then ${}^{n}\mathfrak{M} = \mathfrak{N}$, and $n = n$; whence the anſwers will become

$$\left(\overline{\frac{1}{2} - \frac{n}{3t}} \times n - \overline{1 - \frac{v}{2t}} \times \frac{vv}{6n} \times \frac{1}{n}, \text{ or,}\right)$$

$$\frac{1}{2} - \frac{n}{3t} - \overline{1 - \frac{v}{2t}} \times \frac{vv}{6nn}, \text{ and}$$

$$\mathfrak{P} \times \left\{ \frac{\frac{{}^{n}\mathfrak{F} - \mathfrak{N}}{n} +}{\mathfrak{N} - \frac{n}{6r} - \overline{\mathfrak{D} - \frac{v}{6r}} \times \frac{\overline{2t - v} \times v}{2nn} \times \frac{1}{2t}} \right.$$

COROL. III.

If *C*, and *D*, are of equal ages; then $n = v$; and the anſwers will become

$$(\overline{\tfrac{1}{2} - \frac{v}{3t}} \times v - \overline{1 - \frac{v}{2t}} \times \frac{v}{6} \times \frac{1}{m}, \text{ or})$$

$$\overline{2 - \frac{3v}{2t}} \times \frac{v}{6m}, \text{ and}$$

$$\mathfrak{P} \times \left\{ \frac{\frac{v\mathfrak{F} - v\mathfrak{M}}{v} + \overline{v\mathfrak{M} - \frac{vv}{6rm} - \mathfrak{D} - \frac{v}{6r}} \times \frac{2t - v}{2m} \times \frac{1}{2t}}{} \right.$$

COROL. IV.

If *A*, *B*, and *C*, are of equal ages; then $t = m = v$, and $n\mathfrak{F} = n\mathfrak{M} = \mathfrak{P}$; whence the anſwers will become

$$(\overline{\tfrac{1}{2} - \tfrac{1}{3}} \times n - \overline{1 - \frac{v}{2n}} \times \frac{vv}{6n} \times \frac{1}{n} \text{ or})$$

$$\tfrac{1}{6} - \overline{1 - \frac{v}{2n}} \times \frac{vv}{6nn}, \text{ and}$$

$$\overline{\mathfrak{P} - \frac{n}{6r} - \mathfrak{D} - \frac{v}{6r}} \times \frac{2n - v \times v}{2nn} \times \frac{\mathfrak{P}}{2n}.$$

COROL. V.

If *B*, *C*, and *D*, are of equal ages; then $m = n = v$, and $n\mathfrak{N} = \mathfrak{P} = \mathfrak{D}$; whence the anſwers will become,

$$\frac{1}{3} - \frac{v}{4t}, \text{ and}$$

$$\left(\mathfrak{P}\times\left\{\begin{array}{l}\dfrac{v\mathfrak{F}-\mathfrak{D}}{v}+\\ \overline{\overline{\mathfrak{D}-\dfrac{v}{6r}-\overline{\mathfrak{D}-\dfrac{v}{6r}}\times\dfrac{2t-v}{2v}}}\times\dfrac{1}{2t}\text{, or}\end{array}\right.\right)$$

$$\mathfrak{P}\times\overline{\frac{v\mathfrak{F}-\mathfrak{D}}{v}+\overline{\mathfrak{D}-\frac{v}{6r}}\times\overline{1-\frac{2t-v}{2v}}\times\frac{1}{2t}};$$

That is,

$$\left(\mathfrak{P}\times\overline{\frac{v\mathfrak{F}-\mathfrak{D}}{v}+\overline{\mathfrak{D}-\frac{v}{6r}}+\frac{3v-2t}{2v}\times}\frac{1}{2t}\text{, or}\right)$$

$$\overline{v\mathfrak{F}-\mathfrak{D}+\overline{\mathfrak{D}-\frac{v}{6r}}\times\frac{3v-2t}{4t}}\times\frac{\mathfrak{P}}{v}.$$

QUESTION LXI.

The probability, that *C* and *D*, conſidered as poſſeſſors, ſhall both die before either *B* or *A*; and that *B*, conſidered as expectant, ſhall die before *A*, the ſurvivor; alſo the preſent worth of an eſtate, or legacy, worth $\mathfrak{P}$ pounds, dependent upon this order of ſurvivorſhip; are required.

SOLUTION.

Here, the probability, that *B* ſhall be ſurvived, by

A, is, $1 - \frac{m}{2t}$

C & *A*, is, $\frac{n}{2m} - \frac{mn}{6tm}$

D & *A*, is, $\frac{v}{2m} - \frac{vv}{6tm}$

C, *D* & *A*, is, $\frac{v}{2m} - \frac{vv}{6nm} - \frac{vv}{6tm} + \frac{v^3}{12ntm}$

$\left.\begin{matrix} \\ \\ \\ \\ \end{matrix}\right\}$ by queſt. $\left\{\begin{matrix} 22; \\ 31; \\ 31; \\ 33; \end{matrix}\right.$

Which probabilities (being ſubtracted and added, as before) will give $(1 - \frac{m}{2t} - \frac{n}{2m} + \frac{mn}{6tm} - \frac{vv}{6nm} + \frac{v^3}{12ntm}$, or) $1 - \frac{m}{2t} - \overline{1 - \frac{n}{3t}} \times \frac{n}{2m} - \overline{1 - \frac{v}{2t}} \times \frac{vv}{6nm}$, for the probability required.

Now ſince the preſent worths

$$\frac{m\mathfrak{F}}{m},\ \frac{\mathfrak{P}}{m},\ \overline{\mathfrak{P}-\frac{n}{6r}}\times\frac{n}{2tm},\ \overline{\mathfrak{A}-\frac{v}{6r}}\times\frac{v}{2nm},$$

and $\overline{\mathfrak{A}-\frac{v}{6r}}\times\frac{vv}{4mtn}$, ſeverally, correſpond to the probabilities $1-\frac{m}{2t}$, $\frac{n}{2m}$, $\frac{nn}{6tm}$, $\frac{vv}{6nm}$, and $\frac{v^3}{12mtn}$; therefore thoſe preſent worths, being connected by the proper ſigns, viz.

$$\left(\frac{m\mathfrak{F}}{m}-\frac{\mathfrak{P}}{m}+\overline{\mathfrak{P}-\frac{n}{6r}}\times\frac{n}{2tm}-\overline{\mathfrak{A}-\frac{v}{6r}}\times\frac{v}{2nm}+\overline{\mathfrak{A}-\frac{v}{6r}}\times\frac{vv}{4mtn},\right.$$

$$\left.\text{or}\right)\ \frac{m\mathfrak{F}}{m}-\frac{\mathfrak{P}}{m}+\overline{\mathfrak{P}-\frac{n}{6r}}\times\frac{n}{2tm}-\overline{\mathfrak{A}-\frac{v}{6r}}\times\overline{1-\frac{v}{2t}}\times\frac{v}{2nm},$$

will be the preſent worth, correſponding to that probability.

And, becauſe m is a conſtant diviſor in every term thereof, the preſent worth of an eſtate, worth $\mathfrak{P}$ pounds, will be,

$$\left.\begin{array}{l} m\mathfrak{F}-\mathfrak{P}+\overline{\mathfrak{P}-\frac{n}{6r}}\times\frac{n}{2t}- \\ \overline{\mathfrak{A}-\frac{v}{6r}}\times\overline{1-\frac{v}{2t}}\times\frac{v}{2n} \end{array}\right\}\times\frac{\mathfrak{P}}{m},$$

which was required.

EXAMPLE.

D (aged 76) has jointured his ſecond wife *C* (aged 66) in an eſtate of 1£. per annum; which, after both their deaths, will deſcend to *D*'s ſon *B* (aged 54) if he be then alive; *B* has, by deed, ſettled the ſame on his wife *A*, aged 43; required the probability of her becoming poſſeſſed of the eſtate, and the preſent worth of her intereſt therein.

Here ${}^{m}\mathfrak{F} = 12,578$; $\mathfrak{M} = 7,673$; $\overline{\mathfrak{N} - \frac{n}{6r} \times \frac{n}{2t}}$ $= 1,039$ (by queſt. 4); $\overline{\mathfrak{P} - \frac{v}{6r}} = 2,715$; $\frac{v}{2t} =$ $\frac{10}{86}$; $1 - \frac{10}{86} = (\frac{76}{86} =) \frac{38}{43}$; $\frac{v}{2n} = (\frac{10}{40} =) \frac{1}{4}$; $\mathfrak{P} =$ 25; and $m = 32$. Then $2,715 \times \frac{38}{43} \times \frac{1}{4} = 0,600$; $12,578 - 7,673 + 1,039 - 0,600 = 5,344$; and $(5,344 \times \frac{25}{32} =) 4,175$, will be the preſent value of *A*'s intereſt in the eſtate.

Alſo $\frac{m}{2t} = 0,372$; $\frac{n}{3t} = \frac{20}{129}$; $1 - \frac{20}{129} = \frac{109}{129}$; $\frac{n}{2m} = (\frac{20}{64} =) \frac{5}{16}$; $\frac{v}{2t} = (\frac{10}{86} =) \frac{5}{43}$; $1 - \frac{5}{43} = \frac{38}{43}$ and $\frac{vv}{6nm} = \left(\frac{10 \times 10}{6 \cdot 20 \cdot 32} =\right) \frac{5}{192}$:

Then $\frac{109}{129} \times \frac{5}{16} = 0,264$; and $\frac{38}{43} \times \frac{5}{192} = 0,0230$

Therefore $(1 - 0,372 - 0,264 - 0,023 =) 0,341$ will be the probability of *A*'s becoming poſſeſſed of the eſtate.

COROL. I.

If *A* and *B*, are of equal ages; then ${}^{m}\mathfrak{F} = \mathfrak{M}$, and $t = m$: whence the anſwers will become

$$\frac{1}{2} - \overline{1 - \frac{n}{3m}} \times \frac{n}{2m} - \overline{1 - \frac{v}{2m}} \times \frac{vv}{6nm},$$

$$\left.\begin{array}{l} \text{And } \mathfrak{M} - \mathfrak{N} + \\ \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{n}{2m} - \overline{\mathfrak{P} - \frac{v}{6r}} \times \overline{1 - \frac{v}{2m}} \times \frac{v}{2n} \end{array}\right\} \times \frac{\mathfrak{P}}{m}$$

COROL. II.

If *B*, and *C*, are of equal ages; then $m = n$; and the anſwer will become $(1 - \frac{n}{2t} - \overline{1 - \frac{n}{3t}} \times \frac{1}{2} - \overline{1 - \frac{v}{2t}} \times \frac{vv}{6nn}$ or) $\frac{1}{2} - \frac{n}{3t} - \overline{1 - \frac{v}{2t}} \times \frac{vv}{6nn}$,

And

$$\left.\begin{array}{l} {}^{n}\mathfrak{F} - \mathfrak{N} + \\ \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{n}{2t} - \overline{\mathfrak{D} - \frac{v}{6r}} \times \overline{1 - \frac{v}{2t}} \times \frac{v}{2n} \end{array}\right\} \times \frac{\mathfrak{P}}{n}.$$

COROL. III.

If *C*, and *D*, are of equal ages; then $\mathfrak{N} = \mathfrak{D}$, and $n = v$; whence the probability will become $(1 - \frac{m}{2t} - \overline{1 - \frac{v}{3t}} \times \frac{v}{2m} - \overline{1 - \frac{v}{2t}} \times \frac{v}{6m}$, or) $1 - \frac{m}{2t} - \overline{1 - \frac{v}{3t}} \times \frac{3v}{6m} - \overline{1 - \frac{v}{2t}} \times \frac{v}{6m}$; that is $(1 - \frac{m}{2t} - \overline{3 - \frac{3v}{3t}} \times \frac{v}{6m} - \overline{1 - \frac{v}{2t}} \times \frac{v}{6m}$, or) $1 - \frac{m}{2t} - \overline{4 - \frac{3v}{2t}} \times \frac{v}{6m}$; that is $1 - \frac{m}{2t} - \overline{\frac{2}{3} - \frac{v}{4t}} \times \frac{v}{m}$:

And the preſent worth will be

$$\left.\begin{array}{l} {}^{m}\mathfrak{F} - \mathfrak{D} + \\ \overline{\mathfrak{D} - \frac{v}{6r}} \times \overline{\frac{v}{2t} - \overline{1 - \frac{v}{2t}} \times \frac{1}{2}} \end{array}\right\} \times \frac{\mathfrak{P}}{m}, \text{ or)}$$

$$\overline{{}^{m}\mathfrak{F} - \mathfrak{D} + \overline{\mathfrak{D} - \frac{v}{6r}} \times \frac{3v - 2t}{4t}} \times \frac{\mathfrak{P}}{m}.$$

COROL. IV.

If *A*, *B*, and *C*, are of equal ages; then $\mathfrak{Z} = \mathfrak{P}$, and $t = m = n$; whence the answers will become

$\left(\frac{1}{2} - \frac{1}{3} - \overline{1 - \frac{v}{2n}} \times \frac{vv}{6nn}\right.$, or) $\frac{1}{6} - \overline{1 - \frac{v}{2n}} \times \frac{vv}{6nn}$,

And

$\left(\overline{\overline{\mathfrak{M} - \frac{n}{6r}} \times \frac{1}{2} - \overline{\mathfrak{Q} - \frac{v}{6r}} \times \overline{1 - \frac{v}{2n}} \times \frac{v}{2n}} \times \frac{\mathfrak{P}}{n}\right.$, or)

$\overline{\mathfrak{M} - \frac{n}{6r} - \overline{\mathfrak{Q} - \frac{v}{6r}} \times \frac{\overline{2n - v} \times v}{2nn}} \times \frac{\mathfrak{P}}{2n}$.

COROL. V.

If *B*, *C*, and *D*, are of equal ages; then $\mathfrak{P} = \mathfrak{P}$, and $m = n = v$: whence the answers will become

$\left(\frac{1}{2} - \frac{v}{3t} - \overline{1 - \frac{v}{2t}} \times \frac{1}{6}\right.$, or) $\frac{1}{3} - \frac{v}{4t}$, and

$\overline{\mathfrak{Z} - \mathfrak{Q} + \overline{\mathfrak{Q} - \frac{v}{6r}} \times \frac{3v - 2t}{4t}} \times \frac{\mathfrak{P}}{v}$.

A TABLE, ſhewing the probabilities of ſurvivorſhip, among four perſons of unequal ages; viz. that any two of them, conſidered as poſſeſſors, ſhall both die before either of the other two; and that either of the laſt named two perſons, conſidered as expectant, ſhall die before the other, who is to be conſidered as the ſurvivor.

SOLUTION.

If *A*, *B*, *C*, and *D*, ſeverally repreſent the names of the youngeſt, ſecond, third, and eldeſt, of the four perſons; and *t*, *m*, *n*, and *v*, denote their reſpective complements of life; then,

Caſe	Poſſeſſors	Expectant	Survivor	
1	*A*, *B*	*C*	*L*	$+\frac{v^3}{12tnm}$
2	*A*, *C*	*B*	*D*	$+\frac{v^3}{12tnm}$
3	*B*, *C*	*A*	*D*	$+\frac{v^3}{12tnm}$
4	*A*, *B*	*D*	*C*	$+\frac{vv}{3mt}-\frac{v^3}{4tnm}$
5	*A*, *D*	*B*	*C*	$+\frac{nn}{6tm}-\frac{vv}{6mt}+\frac{v^3}{12tnm}$
6	*B*, *D*	*A*	*C*	$+\frac{nn}{6tm}-\frac{vv}{6mt}+\frac{v^3}{12tnm}$
7	*A*, *C*	*D*	*B*	$+\frac{vv}{3tn}-\frac{v^3}{4tnm}$

8

The Table of the probabilities of ſurvivorſhip continued.

SOLUTION.

Caſe	Poſſeſſors	Expectant	Survivor	
8	*A*, *L*	*C*	*B*	$+\frac{n}{2t}-\frac{nn}{3tm}-\frac{vv}{6tn}+\frac{v^3}{4tnm}$
9	*C*, *D*	*A*	*B*	$+\frac{m}{2t}-\frac{n}{2t}+\frac{nn}{6tm}-\frac{vv}{6tn}+\frac{v^3}{12tnm}$
10	*B*, *C*	*D*	*A*	$+\frac{vv}{3nm}-\frac{v^3}{4tnm}$
11	*B*, *D*	*C*	*A*	$+\frac{n}{2m}-\frac{nn}{3tm}-\frac{vv}{6nm}+\frac{v^3}{12tnm}$
12	*C*, *D*	*B*	*A*	$1-\frac{m}{2t}-\frac{n}{2m}+\frac{nn}{6tm}-\frac{vv}{6nm}+\frac{v^3}{12tnm}$

The ſum of all the above probabilities is unity.

QUESTION LXII.

The reſpective ages of four perſons, *A*, the youngeſt; *B*, the ſecond; *C*, the third; and *D*, the eldeſt; being given: it is required to find the probability of their dying, in any order, that ſhall be preſcribed.

SOLUTION.

The reſpective probabilities of all the poſſible varieties, in the order of ſurvivorſhip among four perſons, (whereof any two are ſuppoſed to die before either of the other two) were found in the laſt twelve queſtions, and their corollaries; and the reſpective probabilities that (of two perſons, both of whoſe lives are ſuppoſed to fail, in a given ſpace of time) either of them ſhall, within that time, die before the other, were found in queſtion 19: If, therefore, each of the firſt mentioned probabilities be, ſeverally, multiplied by each of thoſe laſt mentioned; the reſults will be the the probabilities now required. To render theſe proceſſes intelligible, and conciſe; this queſtion will be divided into twelve caſes, referring to the reſpective queſtions firſt above mentioned. And the reſults of each queſtion, and its corollaries, will be aſſumed therefrom.

If the ſolution of the 19th queſtion be applied, to every combination of two perſons, in the given four, viz. *A*, *B*, *C*, and *D*; then, the probabilities (during a limitted time) of the younger's dying before the elder, and of the elder's dying before the younger, in each of thoſe combinations, will be found in the following table; to which the reader may refer, if any difficulty ariſes in the following ſolutions.

Combinations of two perſons.	Probability, that the younger wil die, before the elder.	Probability, that the elder will die before the younger.
A, *B*	$\frac{m}{m+t}$	$\frac{t}{m+t}$
A, *C*	$\frac{n}{n+t}$	$\frac{t}{n+t}$
A, *D*	$\frac{v}{v+t}$	$\frac{t}{v+t}$
B, *C*	$\frac{n}{n+m}$	$\frac{m}{n+m}$
B, *D*	$\frac{v}{v+m}$	$\frac{m}{v+m}$
C, *D*	$\frac{v}{v+n}$	$\frac{n}{v+n}$

CASE

CASE I. referring to Queſt. 50, and its corollaries.

Order of dying.	Ages.	Solution.
A, B, C, D	unequal	$\left(\frac{m}{m+t} \times \frac{v^3}{12tmn} =\right) \frac{v^3}{\overline{m+t} \times 12tn}$;
B, A, C, D		$\left(\frac{t}{m+t} \times \frac{v^3}{12tmn} =\right) \frac{v^3}{\overline{m+t} \times 12mn}$;
A, B, C, D B, A, C, D	A=B	$\left(\frac{1}{2} \times \frac{v^3}{12mmn} =\right) \frac{v^3}{24\,mmn}$;
A, B, C, D	B=C	$\left(\frac{n}{n+t} \times \frac{v^3}{12tnn} =\right) \frac{v^3}{\overline{n+t} \times 12tn}$;
B, A, C, D		$\left(\frac{t}{n+t} \times \frac{v^3}{12tnn} =\right) \frac{v^3}{\overline{n+t} \times 12nn}$;

CASE

CASE I. continued.

Order of dying	Ages.	Solution.
A, B, C, D	$C=D$	$\left(\frac{m}{m+t} \times \frac{vv}{12tm} =\right) \frac{vv}{\overline{m+t} \times 12t}$;
B, A, C, D		$\left(\frac{t}{m+t} \times \frac{vv}{12tm} =\right) \frac{vv}{\overline{m+t} \times 12m}$:
A, B, C, D B, A, C, D	$A=B=C$	$\left(\frac{1}{2} \times \frac{v^3}{12n^3} =\right) \frac{v^3}{24n^3}$:
A, B, C, D	$B=C=D$	$\left(\frac{v}{v+t} \times \frac{v}{12t} =\right) \frac{vv}{\overline{v+t} \times 12t}$;
B, A, C, D		$\left(\frac{t}{v+t} \times \frac{v}{12t} =\right) \frac{v}{\overline{v+t} \times 12}$:
A, B, C, D B, A, C, D	all equal	$\left(\frac{1}{2} + \frac{1}{12} =\right)$ $\frac{7}{12}$.

CASE II. referring to queſt. 51, and its corollary.

Order of dying.	Ages.	Solution.
A, *C*, *B*, *D*	unequal.	$\left(\frac{n}{n+t} \times \frac{v^3}{12tmn} =\right) \frac{v^3}{\overline{n+t} \times 12tm}$;
C, *A*. *B*, *D*		$\left(\frac{t}{n+t} \times \frac{v^3}{12tmn} =\right) \frac{v^3}{\overline{n+t} \times 12mn}$:
A, *C*, *B*, *D*	*A*$=$*B*	$\left(\frac{n}{n+m} \times \frac{v^3}{12mmn} =\right) \frac{v^3}{\overline{n+m} \times 12mm}$;
C, *A*, *B*, *D*		$\left(\frac{m}{n+m} \times \frac{v^3}{12mmn} =\right) \frac{v^3}{\overline{n+m} \times 12mn}$:
A, *C*, *B*, *D*	*B*$=$*C*	$\left(\frac{n}{n+t} \times \frac{v^3}{12tnn} =\right) \frac{v^3}{\overline{n+t} \times 12tn}$;
C, *A*, *B*, *D*		$\left(\frac{t}{n+t} \times \frac{v^3}{12tnn} =\right) \frac{v^3}{\overline{n+t} \times 12nn}$:

CASE II. continued.

Order of dying.	Ages.		Solution.
A, C, *B*, *D*	C=D	$\left(\frac{v}{v+t}\times\frac{vv}{12tm}=\right)$	$\frac{v^3}{\overline{v+t}\times 12tm}$;
C, *A*, B, *D*		$\left(\frac{t}{v+t}\times\frac{vv}{12tm}=\right)$	$\frac{vv}{\overline{v+t}\times 12m}$:
A, C, B, D C, *A*, B, *D*	A=B=C	$\left(\frac{1}{2}\times\frac{v^3}{12n^3}=\right)$	$\frac{v^3}{24n^3}$:
A, C, *B*, D	B=C=D	$\left(\frac{v}{v+t}\times\frac{v}{12t}=\right)$	$\frac{vv}{\overline{v+t}\times 12t}$;
C, *A*, B, D		$\left(\frac{t}{v+t}\times\frac{v}{12t}=\right)$	$\frac{v}{\overline{v+t}\times 12}$;

CAS

CASE III. referring to queſt. 52, and its Corollary.

Order of dying.	Ages.	Solution.
B, *C*, *A*, *D*	unequal.	$\left(\frac{n}{n+m}\times\frac{v^3}{12tmn}=\right)\frac{v^3}{\overline{n+m}\times 12tm}$;
C, *B*, *A*, *D*	unequal.	$\left(\frac{m}{n+m}\times\frac{v^3}{12tmn}=\right)\frac{v^3}{\overline{n+m}\times 12tn}$:
B, *C*, *A*, *D*	*A*=*B*	$\left(\frac{n}{n+m}\times\frac{v^3}{12mmn}=\right)\frac{v^3}{\overline{n+m}\times 12mm}$;
C, *B*, *A*, *D*	*A*=*B*	$\left(\frac{m}{n+m}\times\frac{v^3}{12mmn}=\right)\frac{v^3}{\overline{n+m}\times 12mn}$:
B, *C*, *A*, *D* *C*, *B*, *A*, *D*	B=C;	$\left(\frac{1}{2}\times\frac{v^3}{12tnn}=\right)\frac{v^3}{24tnn}$:

CASE III. continued.

Order of dying.	Ages	Solution.
B, C, A, D	$C=D$	$\left(\frac{v}{v+m}\times\frac{vv}{12tm}=\right)\frac{v^3}{\overline{v+m}\times 12tm}$;
C, B, A, D		$\left(\frac{m}{v+m}\times\frac{vv}{12tm}=\right)\frac{vv}{\overline{v+m}\times 12t}$:
B, C, A, D C, B, A, D	$A=B=C$;	$\left(\frac{1}{2}\times\frac{v}{12n^3}=\right)\frac{v^3}{24n^3}$:
B, C, A, D C, B, A, D	$B=C=D$;	$\left(\frac{1}{2}\times\frac{v}{12t}=\right)\frac{v}{24t}$.

CASE IV. referring to queſt. 53, and its corollaries.

Order of dying.	Ages	Solution.
A, B, D, C	unequal.	$\left(\frac{m}{m+t}\times\overline{\frac{1}{3}-\frac{v}{4n}}\times\frac{vv}{mt}=\right)\overline{\frac{1}{3}-\frac{v}{4n}}\times\frac{vv}{\overline{m+t}\times t}$:
B, A, D, C		$\left(\frac{t}{m+t}\times\overline{\frac{1}{3}-\frac{v}{4n}}\times\frac{vv}{mt}=\right)\overline{\frac{1}{3}-\frac{v}{4n}}\times\frac{vv}{\overline{m+t}\times m}$:

Order of dying,	Ages.	Solution.
A, *B*, *D*, *C* *B*, *A*, *D*, *C*	$A=B$;	$\left(\frac{1}{2}\times\overline{\frac{1}{3}-\frac{v}{4n}}\times\frac{vv}{mm}=\right)\overline{\frac{1}{3}-\frac{v}{4n}}\times\frac{vv}{2mtn}$:
A, *B*, *D*, *C*	$B=C$	$\left(\frac{n}{n+t}\times\overline{\frac{1}{3}-\frac{v}{4n}}\times\frac{vv}{nt}=\right)\overline{\frac{1}{3}-\frac{v}{4n}}\times\frac{vv}{\overline{n+t}\times t}$:
B, *A*, *D*, *C*		$\left(\frac{t}{n+t}\times\overline{\frac{1}{3}-\frac{v}{4n}}\times\frac{vv}{nt}=\right)\overline{\frac{1}{3}-\frac{v}{4n}}\times\frac{vv}{\overline{n+t}\times n}$;
A, *B*, *D*, *C*	$C=D$	$\left(\frac{m}{m+t}\times\frac{vv}{12mt}=\right)\frac{vv}{\overline{n+t}\times 12t}$;
B, *A*, *D*, *C*		$\left(\frac{t}{m+t}\times\frac{vv}{12mt}=\right)\frac{vv}{\overline{m+t}\times 12m}$:
A, *B*, *D*, *C* *B*, *A*, *D*, *C*	$A=B=C$;	$\frac{1}{2}\times\overline{\frac{1}{3}-\frac{v}{4n}}\times\frac{vv}{nn}=\Big)\overline{\frac{1}{3}-\frac{v}{4n}}\times\frac{vv}{2nn}$.

CASE IV. continued.

Order of dying.	Ages.	Solution.
A, *B*, *D*, *C*	$B = C = D$	$\left(\frac{v}{v+t} \times \frac{v}{12t} =\right) \frac{vv}{\overline{v+t} \times 12t}$;
B, *A*, *D*, *C*		$\left(\frac{t}{v+t} \times \frac{v}{12t} =\right) \frac{v}{\overline{v+t} \times 12}$.

CASE V. referring to queſt. 54, and its corollaries.

A, *D*, *B*, *C*	unequal.	$\frac{v}{v+t} \times \overline{nn - vv + \frac{v^3}{2n}} \times \frac{1}{6tm}$;
D, *A*, *B*, *C*		$\frac{1}{v+t} \times \overline{nn - vv + \frac{v^3}{2n}} \times \frac{1}{6m}$:
A, *D*, *B*, *C*	$A = B$	$\frac{v}{v+m} \times \overline{nn - vv + \frac{v^3}{2n}} \times \frac{1}{6mm}$;
D, *A*, *B*, *C*		$\frac{1}{v+n} \times \overline{nn - vv + \frac{v^3}{2n}} \times \frac{1}{6m}$:

CASE V; continued.

Order of dying.	Ages.	Solution.
A, *D*, *B*, *C*	*B*=*C*	$\frac{v}{v+t} \times \overline{nn - vv + \frac{v^3}{2n}} \times \frac{1}{6tn}$;
D, *A*, *B*, *C*		$\frac{1}{v+t} \times \overline{nn - vv + \frac{v^3}{2n}} \times \frac{1}{6n}$:
A, *D*, *B*, *C*	*C*=*D*	$\frac{v}{v+t} \times \frac{vv}{12tm}$;
D, *A*, *B*, *C*		$\frac{1}{v+t} \times \frac{vv}{12m}$:
A, *D*, *B*, *C*	*A*=*B*=*C*	$\frac{v}{v+n} \times \overline{nn - vv + \frac{v^3}{2n}} \times \frac{1}{6nn}$;
D, *A*, *B*, *C*		$\frac{1}{v+n} \times \overline{nn - vv + \frac{v^3}{2n}} \times \frac{1}{6n}$:

CASE V. continued.

Order of dying.	Ages.	Solution.
A, D, B, C	$B=C=D$	$\frac{v}{v+t} \times \frac{v}{12t}$;
D, A, B, C		$\frac{1}{v+t} \times \frac{v}{12}$.

CASE VI. referring to queſt. 55, and its corollary.

Order of dying.	Ages.	Solution.
B, D, A, C	unequal	$\frac{v}{v+m} \times \overline{nn-vv+\frac{v^3}{2n}} \times \frac{1}{6tm}$;
D, B, A, C.		$\frac{1}{v+m} \times \overline{nn-vv+\frac{v^3}{2n}} \times \frac{1}{6t}$:
B, D, A, C	$A=B$	$\frac{v}{v+m} \times \overline{nn-vv+\frac{v^3}{2n}} \times \frac{1}{6mm}$;
D, B, A, C		$\frac{1}{v+m} \times \overline{nn-vv+\frac{v^3}{2n}} \times \frac{1}{6m}$:

CASE VI. continued.

Order of dying.	Ages.	Solution.
B, *D*, *A*, *C*	*B*=*C*	$\frac{v}{v+n} \times \overline{nn-vv+\frac{v^3}{2n}} \times \frac{1}{6tn}$;
D, *B*, *A*, *C*		$\frac{1}{v+n} \times \overline{nn-vv+\frac{v^3}{2n}} \times \frac{1}{6t}$:
B, *D*, *A*, *C*	*C*=*D*	$\frac{v}{v+m} \times \frac{vv}{12tm}$;
D, *B*, *A*, *C*		$\frac{1}{v+m} \times \frac{v}{12t}$:
B, *D*, *A*, *C*	*A*=*B*=*C*	$\frac{v}{v+n} \times \overline{nn-vv+\frac{v^3}{2n}} \times \frac{1}{6nn}$;
D, *B*, *A*, *C*		$\frac{1}{v+n} \times \overline{nn-vv+\frac{v^3}{2n}} \times \frac{1}{6n}$:
B, *D*, *A*, *C* *D*, *B*, *A*, *C*	*B*=*C*=*D*;	$\frac{1}{2} \times \frac{v}{12t}$.

CASE VII. referring to queſt. 56, and its corollaries.

Order of dying.	Ages.	Solution.
A, C, D, B	unequal	$\left(\frac{n}{n+t}\times\overline{\frac{1}{3}-\frac{v}{4m}}\times\frac{vv}{nt}=\right)\overline{\frac{1}{3}-\frac{v}{4m}}\times\frac{vv}{\overline{n+t}\times t}$;
C, A, D, B		$\left(\frac{t}{n+t}\times\overline{\frac{1}{3}-\frac{v}{4m}}\times\frac{vv}{nt}=\right)\overline{\frac{1}{3}-\frac{v}{4m}}\times\frac{vv}{\overline{n+t}\times n}$:
A, C, D, B	A=B	$\left(\frac{n}{n+m}\times\overline{\frac{1}{3}-\frac{v}{4m}}\times\frac{vv}{nm}=\right)\overline{\frac{1}{3}-\frac{v}{4m}}\times\frac{vv}{\overline{n+m}\times m}$;
C, A, D, B		$\left(\frac{m}{n+m}\times\overline{\frac{1}{3}-\frac{v}{4m}}\times\frac{vv}{nm}=\right)\overline{\frac{1}{3}-\frac{v}{4m}}\times\frac{vv}{\overline{n+m}\times n}$:
A, C, D, B	B=C	$\left(\frac{n}{n+t}\times\overline{\frac{1}{3}-\frac{v}{4n}}\times\frac{vv}{nt}=\right)\overline{\frac{1}{3}-\frac{v}{4n}}\times\frac{vv}{\overline{n+t}\times t}$;
C, A, D, B		$\left(\frac{t}{n+t}\times\overline{\frac{1}{3}-\frac{v}{4n}}\times\frac{vv}{nt}=\right)\overline{\frac{1}{3}-\frac{v}{4n}}\times\frac{vv}{\overline{n+t}\times n}$:

CASE VII. continued.

Order of dying.	Ages.	Solution.
A, *C*, *D*, *B*	$C=D$	$\left(\frac{v}{v+t}\times\overline{\frac{1}{3}-\frac{v}{4m}}\times\frac{v}{t}=\right)\overline{\frac{1}{3}-\frac{v}{4m}}\times\frac{vv}{\overline{v+t}\times t}$;
C, *A*, *D*, *B*		$\left(\frac{t}{v+t}\times\overline{\frac{1}{3}-\frac{v}{4m}}\times\frac{v}{t}=\right)\overline{\frac{1}{3}-\frac{v}{4m}}\times\frac{v}{v+t}$;
C, *A*, *D*, *B* *A*, *C*, *D*, *B*	$A=B=C$;	$\left(\frac{1}{2}\times\overline{\frac{1}{3}-\frac{v}{4n}}\times\frac{vv}{nn}=\right)\overline{\frac{1}{3}-\frac{v}{4n}}\times\frac{vv}{2nn}$:
A, *C*, *D*, *B*	$B=C=D$	$\left(\frac{v}{v+t}\times\frac{v}{12t}=\right)\frac{vv}{\overline{v+t}\times 12t}$;
C, *A*, *D*, *B*		$\left(\frac{t}{v+t}\times\frac{v}{12t}=\right)\frac{v}{\overline{v+t}\times 12}$.

CASE VIII. referring to queſt. 57, and its corollaries.

Order of dying.	Ages.	Solution.
A, D, C, B	unequal.	$\frac{v}{v+t}\times\overline{\frac{1}{2}-\frac{n}{3m}}\times\overline{n-1-\frac{v}{2m}}\times\frac{vv}{6n}\times\frac{1}{t}$;
D, A, C, B		$\frac{1}{v+t}\times\overline{\frac{1}{2}-\frac{n}{3m}}\times\overline{n-1-\frac{v}{2m}}\times\frac{vv}{6n}$:
A, D, C, B	A=B	$\frac{v}{v+m}\times\overline{\frac{1}{2}-\frac{n}{3m}}\times\overline{n-1-\frac{v}{2m}}\times\frac{vv}{6n}\times\frac{1}{m}$;
D, A, C, B		$\frac{1}{v+m}\times\overline{\frac{1}{2}-\frac{n}{3m}}\times\overline{n-1-\frac{v}{2m}}\times\frac{vv}{6n}$:
A, D, C, B	B=C	$\frac{v}{v+t}\times\overline{n-1-\frac{v}{2n}}\times\frac{vv}{n}\times\frac{1}{6t}$;
D, A, C, B		$\frac{1}{v+t}\times\overline{n-1-\frac{v}{2n}}\times\frac{vv}{n}\times\frac{1}{6}$:

CASE VIII. continued.

Order of dying.	Ages.	Solution.
A, D, C, B	$C=D$	$\frac{v}{v+t} \times \overline{\frac{1}{3} - \frac{v}{4m}} \times \frac{v}{t}$;
D, A, C, B		$\frac{1}{v+t} \times \overline{\frac{1}{3} - \frac{v}{4m}} \times v$:
A, D, C, B	$A=B=C$	$\frac{v}{v+n} \times \overline{\frac{1}{6} - \overline{1 - \frac{v}{2n}} \times \frac{vv}{6nn}}$;
D, A, C, B		$\frac{n}{v+n} \times \overline{\frac{1}{6} - \overline{1 - \frac{v}{2n}} \times \frac{v}{6nn}}$:
A, D, C, B	$B=C=D$	$\frac{v}{v+t} \times \frac{v}{12t}$;
D, A, C, B		$\frac{1}{v+t} \times \frac{v}{12}$.

CASE IX. referring to queſt. 58, and its corollaries.

Order of dying.	Ages.	Solution.
C, D, A, B	unequal.	$\frac{v}{v+n}\times\overline{\overline{m-1-\frac{n}{3m}}\times\overline{n-1-\frac{v}{2m}}\times\frac{vv}{3n}}\times\frac{1}{2t}$;
D, C, A, B		$\frac{n}{v+n}\times\overline{\overline{m-1-\frac{n}{3m}}\times\overline{n-1-\frac{v}{2m}}\times\frac{vv}{3n}}\times\frac{1}{2t}$:
C, D, A, B	*A*=*B*	$\frac{v}{v+n}\times\overline{\overline{m-1-\frac{n}{3m}}\times\overline{n-1-\frac{v}{2m}}\times\frac{vv}{3n}}\times\frac{1}{2m}$;
D, C, A, B		$\frac{n}{v+n}\times\overline{\overline{m-1-\frac{n}{3m}}\times\overline{n-1-\frac{v}{2m}}\times\frac{vv}{3n}}\times\frac{1}{2m}$:
C, D, A, B	*B*=*C*	$\frac{v}{v+n}\times\overline{\overline{n-1-\frac{v}{2n}}\times\frac{vv}{n}}\times\frac{1}{6t}$;
D, C, A, B		$\frac{n}{v+n}\times\overline{\overline{n-1-\frac{v}{2n}}\times\frac{vv}{n}}\times\frac{1}{6t}$:

CASE IX. continued.

Order of dying,	Ages.	Solution.
C, D, A, B *D, C, A, B*	$C=D$;	$\frac{1}{2}\times\overline{m-4-\frac{3v}{2m}\times\frac{v}{3}}\times\frac{1}{2t}$:
C, D, A, B	$A=B=C$	$\frac{v}{v+n}\times\overline{\frac{1}{6}-1-\frac{v}{2n}\times\frac{vv}{6nn}}$;
D, C, A, B		$\frac{n}{v+n}\times\overline{\frac{1}{6}-1-\frac{v}{2n}\times\frac{vv}{6nn}}$:
C, D, A, B *D, C, A, B*	$B=C=D$;	$\frac{1}{2}\times\frac{v}{12t}$.

CASE X. referring to queſt. 59, and its corollaries.

Order of dying.	Ages.	Solution.
B, *C*, *D*, *A*	unequal	$\left(\frac{n}{n+m}\times\overline{\frac{1}{3}-\frac{v}{4t}}\times\frac{vv}{nm}=\right)\overline{\frac{1}{3}-\frac{v}{4t}}\times\frac{vv}{\overline{n+m}\times m}$;
C, *B*, *D*, *A*		$\left(\frac{m}{n+m}\times\overline{\frac{1}{3}-\frac{v}{4t}}\times\frac{vv}{nm}=\right)\overline{\frac{1}{3}-\frac{v}{4t}}\times\frac{vv}{\overline{n+m}\times n}$:
B, *C*, *D*, *A*	$A=B$	$\left(\frac{n}{n+m}\times\overline{\frac{1}{3}-\frac{v}{4m}}\times\frac{vv}{nm}=\right)\overline{\frac{1}{3}-\frac{v}{4m}}\times\frac{vv}{\overline{n+m}\times m}$;
C, *B*, *D*, *A*		$\left(\frac{m}{n+m}\times\overline{\frac{1}{3}-\frac{v}{4m}}\times\frac{vv}{nm}=\right)\overline{\frac{1}{3}-\frac{v}{4m}}\times\frac{vv}{\overline{n+m}\times n}$:
B, *C*, *D*, *A* *C*, *B*, *D*, *A*	$B=C$;	$\left(\frac{1}{2}\times\overline{\frac{1}{3}-\frac{v}{4t}}\times\frac{vv}{nn}=\right)\overline{\frac{1}{3}-\frac{v}{4t}}\times\frac{vv}{2nn}$:

CASE X. continued.

Order of dying.	Ages.	Solution.
B, *C*, *D*, *A*	*C*=*D*	$\left(\frac{v}{v+m}\times\overline{\tfrac{1}{3}-\frac{v}{4t}}\times\frac{v}{m}=\right)\overline{\tfrac{1}{3}-\frac{v}{4t}}\times\frac{vv}{\overline{v+m}\times m}$;
C, *B*, *D*, *A*		$\left(\frac{m}{v+m}\times\overline{\tfrac{1}{3}-\frac{v}{4t}}\times\frac{v}{m}=\right)\overline{\tfrac{1}{3}-\frac{v}{4t}}\times\frac{v}{v+m}$:
B, *C*, *D*, *A* *C*, *B*, *D*, *A*	*A*=*B*=*C*;	$\left(\tfrac{1}{2}\times\overline{\tfrac{1}{3}-\frac{v}{4n}}\times\frac{vv}{nn}=\right)\overline{\tfrac{1}{3}-\frac{v}{4n}}\times\frac{vv}{2nn}$:
B, *C*, *D*, *A* *C*, *B*, *D*, *A*	*B*=*C*=*D*;	$\tfrac{1}{2}\times\overline{\tfrac{1}{3}-\frac{v}{4t}}$.

CASE XI. referring to queſt. 60, and its corollaries.

B, *D*, *C*, *A*	unequal	$\frac{v}{v+m}\times\overline{\tfrac{1}{2}-\frac{n}{3t}\times\overline{n-1}-\overline{\frac{v}{2t}}\times\frac{vv}{6n}}\times\frac{1}{m}$;
D, *B*, *C*, *A*		$\frac{1}{v+m}\times\overline{\tfrac{1}{2}-\frac{n}{3t}\times\overline{n-1}-\overline{\frac{v}{2t}}\times\frac{vv}{6n}}$;

CASE XI. continued.

Order of dying.	Ages.	Solution.
B, D, C, A	A=B	$\frac{v}{v+m} \times \overline{\tfrac{1}{2} - \frac{n}{3m} \times \overline{n-1-\frac{v}{2m}} \times \frac{vv}{6n}} \times \frac{1}{m};$
D, B, C, A		$\frac{1}{v+m} \times \overline{\tfrac{1}{2} - \frac{n}{3m} \times \overline{n-1-\frac{v}{2m}} \times \frac{vv}{6n}}:$
B, D, C, A	B=C	$\frac{v}{v+n} \times \overline{\tfrac{1}{2} - \frac{n}{3t} - \overline{1-\frac{v}{2t}} \times \frac{vv}{6nn}};$
D, B, C, A		$\frac{n}{v+n} \times \overline{\tfrac{1}{2} - \frac{n}{3t} - \overline{1-\frac{v}{2t}} \times \frac{vv}{6nn}}:$
B, D, C, A	C=D	$\frac{v}{v+m} \times \overline{2-\frac{3v}{2t}} \times \frac{v}{6m};$
D, B, C, A		$\frac{1}{v+m} \times \overline{2-\frac{3v}{2t}} \times \frac{v}{6}:$

CASE XI. continued.

Order of dying. Ages. Solution.

$$\left.\begin{array}{l} B,\ D,\ C,\ A \\ D,\ B,\ C,\ A \end{array}\right\} A=B=C \left\{\begin{array}{l} \frac{v}{v+n}\times\overline{\tfrac{1}{6}-\overline{1-\frac{v}{2n}}\times\frac{vv}{6nn}}\,; \\ \frac{n}{v+n}\times\overline{\tfrac{1}{6}-\overline{1-\frac{v}{2n}}\times\frac{vv}{6nn}}\,; \end{array}\right.$$

$$\left.\begin{array}{l} B,\ D,\ C,\ A \\ D,\ B,\ C,\ A \end{array}\right\} B=C=D;\ \tfrac{1}{2}\times\overline{\tfrac{1}{3}-\frac{v}{4t}}.$$

CASE XII, referring to queſt. 61, and its corollaries.

$$\left.\begin{array}{l} C,\ D,\ B,\ A \\ D,\ C,\ B,\ A \end{array}\right\} \text{unequal} \left\{\begin{array}{l} \frac{v}{v+n}\times\overline{1-\frac{m}{2t}-\overline{1-\frac{n}{3t}}\times\frac{n}{2m}-\overline{1-\frac{v}{2t}}\times\frac{vv}{6nm}}\,; \\ \frac{n}{v+n}\times\overline{1-\frac{m}{2t}-\overline{1-\frac{n}{3t}}\times\frac{n}{2m}-\overline{1-\frac{v}{2t}}\times\frac{vv}{6nm}}\,: \end{array}\right.$$

CASE XII. continued.

Order of dying.	Ages.	Solution.
C, D, B, A	*A*=*B*	$\frac{v}{v+n} \times \overline{\tfrac{1}{2} - \overline{1 - \frac{n}{3m}} \times \frac{n}{2m} - \overline{1 - \frac{v}{2m}} \times \frac{vv}{6nm}}$;
D, C, B, A		$\frac{n}{v+n} \times \overline{\tfrac{1}{2} - \overline{1 - \frac{n}{3m}} \times \frac{n}{2m} - \overline{1 - \frac{v}{2m}} \times \frac{vv}{6nm}}$:
C, D, B, A	*B*=*C*	$\frac{v}{v+n} \times \overline{\tfrac{1}{2} - \frac{n}{3t} - \overline{1 - \frac{v}{2t}} \times \frac{vv}{6nn}}$;
D, C, B, A		$\frac{n}{v+n} \times \overline{\tfrac{1}{2} - \frac{n}{3t} - \overline{1 - \frac{v}{2t}} \times \frac{vv}{6nn}}$:
C, D, B, A *D, C, B, A*	*C*=*D*;	$\tfrac{1}{2} \times \overline{1 - \frac{m}{2t} - \tfrac{2}{3} - \frac{v}{4t}} \times \frac{v}{m}$:

CASE XII. continued.

Order of dying. Ages. Solution.

$$\left.\begin{array}{l} C,\ D,\ B,\ A \\ D,\ C,\ B,\ A \end{array}\right\} A=B=C \left\{\begin{array}{l} \frac{v}{v+n} \times \frac{1}{6} - \overline{1 - \overline{\frac{v}{2n}} \times \frac{vv}{6nn}}; \\ \frac{n}{v+n} \times \frac{1}{6} - \overline{1 - \overline{\frac{v}{2n}} \times \frac{vv}{6nn}}: \end{array}\right.$$

$$\left.\begin{array}{l} C,\ D,\ B,\ A \\ D,\ C,\ B,\ A \end{array}\right\} B=C\mp D;\ \frac{1}{2} \times \overline{\frac{1}{3} - \frac{v}{4t}}.$$

QUESTION

QUESTION LXIII.

The reſpective ages of 4 perſons; *A*, the youngeſt; *B*, the ſecond; *C*, the third; and *D*, the eldeſt; being given: it is required, to find the preſent worth of an eſtate, or legacy, worth $\mathfrak{P}$ pounds, dependent on any order of ſurvivorſhip, that may be preſcribed.

SOLUTION.

If the ſeveral preſent worths, found in the 50th and the eleven following queſtions, and their corollaries, be multiplied by the ſame factors, as the probabilities, therein found, have been in the preceding queſtion; the products will be the ſeveral preſent worths required.

Note; when the preſent worth, reſulting from any of thoſe queſtions, or corollaries, conſiſts of a long expreſſion, and has not a diviſor, equal to the numerator of the fraction, into which it is to be multiplied; then the ſymbol $\mathfrak{A}$, with the addition of the queſtion, or corollary, and page where it may be found, will be wrote inſtead of that preſent worth; thus (in caſe 7) $\frac{n}{n+t}\times\mathfrak{A}$, is wrote for,

$$\frac{n}{n+t}\times\left\{\begin{array}{l} v\mathfrak{M}-\frac{vv}{6rm}- \\ \overline{v\mathfrak{N}-\frac{vv}{6rn}}\times\overline{2m-v}+ \\ \frac{vv}{6rn}\times\overline{n-v}\times 2 \end{array}\right\}\times\frac{\mathfrak{P}}{4mt}.$$

CASE I. referring to the preſent worths found in queſt 50, and its corollaries.

Order of dying.	Ages.	Solution.
A, B, C, D	unequal.	$\frac{1}{m+t} \times \overline{\mathfrak{P} - \frac{v}{6r}} \times \frac{vv\mathfrak{P}}{4tn}$;
B, A, C, D		$\frac{1}{m+t} \times \overline{\mathfrak{P} - \frac{v}{6r}} \times \frac{vv\mathfrak{P}}{4mn}$:
A, B, C, D B, A, C, D	A=B;	$\frac{1}{2} \times \overline{\mathfrak{P} - \frac{v}{6r}} \times \frac{vv\mathfrak{P}}{4mmn}$:
A, B, C, D	B=C	$\frac{1}{n+t} \times \overline{\mathfrak{P} - \frac{v}{6r}} \times \frac{vv\mathfrak{P}}{4tn}$;
B, A, C, D		$\frac{1}{n+t} \times \overline{\mathfrak{P} - \frac{v}{6r}} \times \frac{vv\mathfrak{P}}{4nn}$:

CASE

CASE I. continued.

Order of dying.	Ages.	Solution.
A, B, C, D	*C*=*D*	$\frac{1}{m+t}\times\overline{\mathfrak{D}-\frac{v}{6r}}\times\frac{v\mathfrak{P}}{4t}$;
B, A, C, D		$\frac{1}{m+t}\times\overline{\mathfrak{D}-\frac{v}{6r}}\times\frac{v\mathfrak{P}}{4m}$:
A, B, C, D *B, A, C, D*	*A*=*B*=*C*;	$\frac{1}{2}\times\overline{\mathfrak{D}-\frac{v}{6r}}\times\frac{vv\mathfrak{P}}{4n^3}$:
A, B, C, D	*B*=*C*=*D*	$\frac{v}{v+t}\times\overline{\mathfrak{D}-\frac{v}{6r}}\times\frac{\mathfrak{P}}{4t}$;
B, A, C, D		$\frac{1}{v+t}\times\overline{\mathfrak{D}-\frac{v}{6r}}\times\frac{\mathfrak{P}}{4}$:
A, B, C, D *B, A, C, D*	all equal;	$\frac{1}{2}\times\overline{\mathfrak{D}-\frac{v}{6r}}\times\frac{\mathfrak{P}}{4v}$.

CASE II. referring to the preſent worths, found in queſt. 51, and its corollary.

Order of dying.	Ages.	Solutioh.
A, C, B, D	unequal	$\frac{1}{n+t} \times \overline{\mathfrak{P} - \frac{v}{6r}} \times \frac{vv\mathfrak{P}}{4mt}$;
C, A, B, D		$\frac{1}{n+t} \times \overline{\mathfrak{P} - \frac{v}{6r}} \times \frac{vv\mathfrak{P}}{4mn}$:
A, C, B, D	*A=B*	$\frac{1}{n+m} \times \overline{\mathfrak{P} - \frac{v}{6r}} \times \frac{vv\mathfrak{P}}{4mm}$;
C, A, B, D		$\frac{1}{n+m} \times \overline{\mathfrak{P} - \frac{v}{6r}} \times \frac{vv\mathfrak{P}}{4mn}$:
A, C, B, D	*B=C*	$\frac{1}{n+t} \times \overline{\mathfrak{P} - \frac{v}{6r}} \times \frac{vv\mathfrak{P}}{4nt}$;
C, A, B, D		$\frac{1}{n+t} \times \overline{\mathfrak{P} - \frac{v}{6r}} \times \frac{vv\mathfrak{P}}{4nn}$:

CASE

CASE II. continued.

Order of dying.	Ages.	Solution.
A, C, B, D	C=D	$\frac{v}{v+t}\times\overline{\mathfrak{P}-\frac{v}{6r}}\times\frac{v\mathfrak{P}}{4mt}$;
C, A, B, D		$\frac{1}{v+t}\times\overline{\mathfrak{P}-\frac{v}{6r}}\times\frac{v\mathfrak{P}}{4m}$;
A, C, B, D C, A, B, D	A=B=C;	$\frac{1}{2}\times\overline{\mathfrak{P}-\frac{v}{6r}}\times\frac{vv\mathfrak{P}}{4n^3}$;
A, C, B, D	B=C=D	$\frac{v}{v+t}\times\overline{\mathfrak{P}-\frac{v}{6r}}\times\frac{\mathfrak{P}}{4t}$;
C, A, B, D		$\frac{1}{v+t}\times\overline{\mathfrak{P}-\frac{v}{6r}}\times\frac{\mathfrak{P}}{4}$.

C A S E III. referring to the preſent worths, found in queſt. 52, and its corollary.

Order of dying. Ages. Solution.

$$\begin{array}{l}
\left.\begin{array}{l} B,\ C,\ A,\ D \\ C,\ B,\ A,\ D \end{array}\right\}\text{unequal.}\left\{\begin{array}{l} \frac{1}{n+m} \times \overline{\mathfrak{P} - \frac{v}{6r}} \times \frac{vv\mathfrak{P}}{4mt}; \\ \frac{1}{n+m} \times \overline{\mathfrak{P} - \frac{v}{6r}} \times \frac{vv\mathfrak{P}}{4nt}: \end{array}\right. \\
\left.\begin{array}{l} B,\ C,\ A,\ D \\ C,\ B,\ A,\ D \end{array}\right\} A=B \left\{\begin{array}{l} \frac{1}{n+m} \times \overline{\mathfrak{A} - \frac{v}{6r}} \times \frac{vv\mathfrak{P}}{4mm}; \\ \frac{1}{n+m} \times \overline{\mathfrak{A} - \frac{v}{6r}} \times \frac{vv\mathfrak{P}}{4mn}: \end{array}\right. \\
\left.\begin{array}{l} B,\ C,\ A,\ D \\ C,\ B,\ A,\ D \end{array}\right\} B=C;\ \tfrac{1}{2} \times \overline{\mathfrak{A} - \frac{v}{6r}} \times \frac{vv\mathfrak{P}}{4tnn}:
\end{array}$$

CASE III. continued.

Order of dying.	Ages.	Solution.
B, *C*, *A*, *D*	$C=D$	$\frac{v}{v+m}\times\overline{\mathfrak{D}-\frac{v}{6r}}\times\frac{v\mathfrak{P}}{4mt}$;
C, *B*, *A*, *D*		$\frac{1}{v+m}\times\overline{\mathfrak{D}-\frac{v}{6r}}\times\frac{v\mathfrak{P}}{4t}$:
B, *C*, *A*, *D* *C*, *B*, *A*, *D*	$A=B=C$;	$\frac{1}{2}\times\overline{\mathfrak{D}-\frac{v}{6r}}\times\frac{vv\mathfrak{P}}{4n^3}$:
B, *C*, *A*, *D* *C*, *B*, *A*, *D*	$B=C=D$;	$\frac{1}{2}\times\overline{\mathfrak{D}-\frac{v}{6r}}\times\frac{\mathfrak{P}}{4t}$.

CASE IV. referring to the preſent worths, found in queſtion 53, and its corollaries.

A, *B*, *D*, *C*	unequal.	$\frac{1}{m+t}\times\overline{v\mathfrak{N}+\frac{2n-3v}{6rn}}\times v\times\frac{v\mathfrak{P}}{4t}$;
B, *A*, *D*, *C*		$\frac{1}{m+t}\times\overline{v\mathfrak{N}+\frac{2n-3v}{6rn}}\times v\times\frac{v\mathfrak{P}}{4m}$:

CASE

CASE IV. continued.

Order of dying.	Ages.	Solution.
A, B, D, C B, A, D, C	$A = B$;	$\frac{1}{2} \times \overline{v\mathfrak{R} + \frac{2n - 3v}{6rn}} \times v \times \frac{v\mathfrak{P}}{4mm}$:
A, B, D, C	$B = C$	$\frac{1}{n+t} \times \overline{v\mathfrak{R} + \frac{2n - 3v}{6rn}} \times v \times \frac{v\mathfrak{P}}{4^t}$;
B, A, D, C		$\frac{1}{n+t} \times \overline{v\mathfrak{R} + \frac{2n - 3v}{6rn}} \times v \times \frac{v\mathfrak{P}}{4^n}$:
A, B, D, C	$C = D$	$\frac{1}{m+t} \times \overline{\mathfrak{D} - \frac{v}{6r}} \times \frac{v\mathfrak{P}}{4^t}$;
B, A, D, C		$\frac{1}{m+t} \times \overline{\mathfrak{D} - \frac{v}{6r}} \times \frac{v\mathfrak{P}}{4^m}$:
A, B, D, C B, A, D, C	$A = B = C$;	$\frac{1}{2} \times \overline{v\mathfrak{R} + \frac{2n - 3v}{6rn}} \times v \times \frac{v\mathfrak{P}}{4^{nn}}$:

CASE IV. continued.

Order of dying. Ages. Solution.

$$\left.\begin{array}{l}A,\ B,\ D,\ C\\[2ex] B,\ A,\ D,\ C\end{array}\right\} B=C=D \left\{\begin{array}{l}\dfrac{v}{v+t}\times\overline{\mathfrak{A}-\dfrac{v}{6r}}\times\dfrac{\mathfrak{P}}{4t}:\\[2ex] \dfrac{1}{v+t}\times\overline{\mathfrak{A}-\dfrac{v}{6r}}\times\dfrac{\mathfrak{P}}{4}.\end{array}\right.$$

CASE V. referring to the preſent worths, found in queſt. 54, and its corollaries.

$$\left.\begin{array}{l}A,\ D,\ B,\ C\\[2ex] D,\ A,\ B,\ C\end{array}\right\} \text{unequal.} \left\{\begin{array}{l}\dfrac{v}{v+t}\times\overline{\overline{\mathfrak{N}-\dfrac{n}{6r}}\times\overline{n-\mathfrak{A}-\dfrac{v}{6r}}\times\overline{1-\dfrac{v}{2n}}}\times v\times\dfrac{\mathfrak{P}}{2mt};\\[2ex] \dfrac{1}{v+t}\times\overline{\overline{\mathfrak{N}-\dfrac{n}{6r}}\times\overline{n-\mathfrak{A}-\dfrac{v}{6r}}\times\overline{1-\dfrac{v}{2n}}}\times v\times\dfrac{\mathfrak{P}}{2m}:\end{array}\right.$$

CASE V. continued.

Order of dying.	Ages.	Solution.
A, D, B, C	*A=B*	$\frac{v}{v+m}\times\overline{\overline{\mathfrak{P}-\frac{n}{6r}}\times n-\overline{\mathfrak{D}-\frac{v}{6r}}\times\overline{1-\frac{v}{2n}}}\times v\times\frac{\mathfrak{P}}{2mm}$;
D, A, B, C		$\frac{1}{v+m}\times\overline{\overline{\mathfrak{P}-\frac{n}{6r}}\times n-\overline{\mathfrak{D}-\frac{v}{6r}}\times\overline{1-\frac{v}{2n}}}\times v\times\frac{\mathfrak{P}}{2m}$:
A, D, B, C	*B=C*	$\frac{v}{v+t}\times\overline{\overline{\mathfrak{P}-\frac{n}{6r}}\times n-\overline{\mathfrak{D}-\frac{v}{6r}}\times\overline{1-\frac{v}{2n}}}\times v\times\frac{\mathfrak{P}}{2nt}$;
D, A, B, C		$\frac{1}{v+t}\times\overline{\overline{\mathfrak{P}-\frac{n}{6r}}\times n-\overline{\mathfrak{D}-\frac{v}{6r}}\times\overline{1-\frac{v}{2n}}}\times v\times\frac{\mathfrak{P}}{2n}$:
A, D, B, C	*C=D*	$\frac{v}{v+t}\times\overline{\mathfrak{D}-\frac{v}{6r}}\times\frac{v\mathfrak{P}}{4mt}$;
D, A, B, C		$\frac{1}{v+t}\times\overline{\mathfrak{D}-\frac{v}{6r}}\times\frac{v\mathfrak{P}}{4m}$:

CASE V. continued.

Order of dying.	Ages.	Solution.
A, *D*, *B*, *C*	*A*=*B*=*C*	$\frac{v}{v+n}\times\overline{\overline{\mathfrak{N}-\frac{n}{6r}}\times n-\overline{\mathfrak{D}-\frac{v}{6r}}\times\overline{1-\frac{v}{2n}}}\times v\times\frac{\mathfrak{P}}{2nn}$;
D, *A*, *B*, *C*		$\frac{1}{v+n}\times\overline{\overline{\mathfrak{N}-\frac{n}{6r}}\times n-\overline{\mathfrak{D}-\frac{v}{6r}}\times\overline{1-\frac{v}{2n}}}\times v\times\frac{\mathfrak{P}}{2n}$;
A, *D*, *B*, *C*	*B*=*C*=*D*	$\frac{v}{v+t}\times\overline{\mathfrak{D}-\frac{v}{6r}}\times\frac{\mathfrak{P}}{4t}$;
D, *A*, *B*, *C*		$\frac{1}{v+t}\times\overline{\mathfrak{D}-\frac{v}{6r}}\times\frac{\mathfrak{P}}{4}$.

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CASE VI. referring to the preſent worths, found in Queſt. 55, and its corollary.

Order of dying.	Ages.	Solution.
B, *D*, *A*, *C*	unequal.	$\frac{v}{v+m}\times\overline{\overline{\mathfrak{P}-\frac{n}{6r}}\times n-\overline{\mathfrak{D}-\frac{v}{6r}}\times\overline{1-\frac{v}{2n}}}\times v\times\frac{\mathfrak{P}}{2mt}$;
D, *B*, *A*, *C*		$\frac{1}{v+m}\times\overline{\overline{\mathfrak{P}-\frac{n}{6r}}\times n-\overline{\mathfrak{D}-\frac{v}{6r}}\times\overline{1-\frac{v}{2n}}}\times v\times\frac{\mathfrak{P}}{2t}$:
B, *D*, *A*, *C*	*A*=*B*	$\frac{v}{v+m}\times\overline{\overline{\mathfrak{P}-\frac{n}{6r}}\times n-\overline{\mathfrak{D}-\frac{v}{6r}}\times\overline{1-\frac{v}{2n}}}\times v\times\frac{\mathfrak{P}}{2mm}$;
D, *B*, *A*, *C*		$\frac{1}{v+m}\times\overline{\overline{\mathfrak{P}-\frac{n}{6r}}\times n-\overline{\mathfrak{D}-\frac{v}{6r}}\times\overline{1-\frac{v}{2n}}}\times v\times\frac{\mathfrak{P}}{2m}$:
B, *D*, *A*, *C*	*B*=*C*	$\frac{v}{v+n}\times\overline{\overline{\mathfrak{P}-\frac{n}{6r}}\times n-\overline{\mathfrak{D}-\frac{v}{6r}}\times\overline{1-\frac{v}{2n}}}\times v\times\frac{\mathfrak{P}}{2tn}$;
D, *B*, *A*, *C*		$\frac{1}{v+n}\times\overline{\overline{\mathfrak{P}-\frac{n}{6r}}\times n-\overline{\mathfrak{D}-\frac{v}{6r}}\times\overline{1-\frac{v}{2n}}}\times v\times\frac{\mathfrak{P}}{2t}$:

CASE VI. continued.

Order of dying.	Ages.	Solution.
B, *Γ*, *A*, *C* *D*, *B*, *A*, *C*	*C*=*D*	$\frac{v}{v+m}\times\overline{\mathfrak{D}-\frac{v}{6r}}\times\frac{v\mathfrak{P}}{4mt}$; $\frac{1}{v+m}\times\overline{\mathfrak{D}-\frac{v}{6r}}\times\frac{v\mathfrak{P}}{4t}$:
B, *D*, *A*, *C* *D*, *B*, *A*, *C*	*A*=*B*=*C*	$\frac{v}{v+n}\times\overline{\overline{\mathfrak{N}-\frac{n}{6r}}\times n-\overline{\mathfrak{D}-\frac{v}{6r}}\times\overline{1-\frac{v}{2n}}}\times v\times\frac{\mathfrak{P}}{2nn}$; $\frac{1}{v+n}\times\overline{\overline{\mathfrak{N}-\frac{n}{6r}}\times n-\overline{\mathfrak{D}-\frac{v}{6r}}\times\overline{1-\frac{v}{2n}}}\times v\times\frac{\mathfrak{P}}{2n}$:
B, *D*, *A*, *C* *D*, *B*, *A*, *C*	*B*=*C*=*D*	; $\frac{1}{2}\times\overline{\mathfrak{A}-\frac{v}{6r}}\times\frac{\mathfrak{P}}{4t}$.

CASE VII. referring to the preſent worths, found in queſt. 56, and its corollaries.

Order of dying. Ages. Solution.

$$\left.\begin{matrix} A,\ C,\ D,\ B \\ C,\ A,\ D,\ B \end{matrix}\right\}\text{unequal}\left\{\begin{array}{l} \frac{n}{n+t}\times\mathfrak{A};\ \text{ſee queſt. 56, fol. 173.} \\ \frac{1}{n+t}\times\frac{\mathfrak{P}}{4m}\times\left\{\begin{array}{l} \overline{v\mathfrak{M}-\frac{vv}{6rm}}\times 2m-\overline{v\mathfrak{N}-\frac{vv}{6rn}}\times\overline{2m-v} \\ +\frac{vv}{6rn}\times\overline{n-v}\times 2: \end{array}\right. \end{array}\right.$$

$$\left.\begin{matrix} A,\ C,\ D,\ B \\ C,\ A,\ D,\ B \end{matrix}\right\}A=B\left\{\begin{array}{l} \frac{n}{n+m}\times\mathfrak{A};\ \text{ſee Corol. 1. queſt. 56. fol. 174.} \\ \frac{1}{n+m}\times\frac{\mathfrak{P}}{4m}\times\left\{\begin{array}{l} \overline{v\mathfrak{M}-\frac{vv}{6rm}}\times 2m-\overline{v\mathfrak{N}-\frac{vv}{6rn}}\times\overline{2m-v} \\ +\frac{vv}{6rn}\times\overline{n-v}\times 2: \end{array}\right. \end{array}\right.$$

CASE VII. continued.

Order of dying.	Ages.	Solution
A, *C*, *D*, *B*	*B*=*C*	$\frac{1}{n+t} \times \overline{v\mathfrak{N} + \frac{2n-3v}{6rn} \times v \times \frac{v\mathfrak{P}}{4t}}$;
C, *A*, *D*, *B*		$\frac{1}{n+t} \times \overline{v\mathfrak{N} + \frac{2n-3v}{6rn} \times v \times \frac{v\mathfrak{P}}{4n}}$:
A, *C*, *D*, *B*	*C*=*D*	$\frac{v}{v+t} \times \overline{v\mathfrak{M} - \frac{vv}{6rm}} - \overline{\mathfrak{D} - \frac{v}{6r}} \times \overline{\frac{2m-v}{2m}} \times \frac{\mathfrak{P}}{2t}$;
C, *A*, *D*, *B*		$\frac{1}{v+t} \times \overline{v\mathfrak{M} - \frac{vv}{6rm}} - \overline{\mathfrak{D} - \frac{v}{6r}} \times \overline{\frac{2m-v}{2m}} \times \frac{\mathfrak{P}}{2}$:
A, *C*, *D*, *B* *C*, *A*, *D*, *B*	*A*=*B*=*C*;	$\frac{1}{2} \times \overline{v\mathfrak{N} + \frac{2n-3v}{6rn} \times v \times \frac{v\mathfrak{P}}{4nn}}$:

CASE VII. continued.

Order of dying. Ages. Solution.

$$\left.\begin{array}{l} A,\ C,\ D,\ B \\ C,\ A,\ D,\ B \end{array}\right\} B=C=D \left\{\begin{array}{l} \frac{v}{v+t} \times \overline{\mathfrak{P} - \frac{v}{6r}} \times \frac{\mathfrak{P}}{4t}; \\ \frac{1}{v+t} \times \overline{\mathfrak{P} - \frac{v}{6r}} \times \frac{\mathfrak{P}}{4}. \end{array}\right.$$

CASE VIII. referring to the preſent worths, found in queſt. 57, and its corollaries.

$$\left.\begin{array}{l} A,\ D,\ C,\ B \\ D,\ A,\ C,\ B \end{array}\right\} \text{unequal.} \left\{\begin{array}{l} \frac{v}{v+t} \times \overline{{}^{n}\mathfrak{M} - \frac{nn}{6rm} - \overline{\mathfrak{P} - \frac{v}{6r}} \times \frac{2m-v}{2mn}} \times v \times \frac{\mathfrak{P}}{2t}; \\ \frac{1}{v+t} \times \overline{{}^{n}\mathfrak{M} - \frac{nn}{6rm} - \overline{\mathfrak{P} - \frac{v}{6r}} \times \frac{2m-v}{2mn}} \times v \times \frac{\mathfrak{P}}{2}: \end{array}\right.$$

CASE VIII. continued.

Order of dying.	Ages.	Solution.
A, D, C, B	$A = B$	$\frac{v}{v+m} \times \overline{n\mathfrak{M} - \frac{nn}{6rm} - \overline{\mathfrak{P} - \frac{v}{6r}} \times \frac{2m-v}{2mn}} \times v \times \frac{\mathfrak{P}}{2m}$;
D, A, C, B		$\frac{1}{v+m} \times \overline{n\mathfrak{M} - \frac{nn}{6rm} - \overline{\mathfrak{P} - \frac{v}{6r}} \times \frac{2m-v}{2mn}} \times v \times \frac{\mathfrak{P}}{2}$:
A, D, C, B	$B = C$	$\frac{v}{v+t} \times \overline{\mathfrak{N} - \frac{n}{6r} - \overline{\mathfrak{P} - \frac{v}{6r}} \times \frac{2n-v}{2nn}} \times v \times \frac{\mathfrak{P}}{2t}$;
D, A, C, B		$\frac{1}{v+t} \times \overline{\mathfrak{N} - \frac{n}{6r} - \overline{\mathfrak{P} - \frac{v}{6r}} \times \frac{2n-v}{2nn}} \times v \times \frac{\mathfrak{P}}{2}$:
A, D, C, B	$C = D$	$\frac{v}{v+t} \times \overline{v\mathfrak{M} - \frac{vv}{6rm} - \overline{\mathfrak{P} - \frac{v}{6r}} \times \frac{2m-v}{2m}} \times \frac{\mathfrak{P}}{2t}$;
D, A, C, B		$\frac{1}{v+t} \times \overline{v\mathfrak{M} - \frac{vv}{6rm} - \overline{\mathfrak{P} - \frac{v}{6r}} \times \frac{2m-v}{2m}} \times \frac{\mathfrak{P}}{2}$:

CASE VIII. continued.

Order of dying, Ages. Solution.

$$\left.\begin{array}{l}A,\ D,\ C,\ B\\ D,\ A,\ C,\ B\end{array}\right\}A=B=C\left\{\begin{array}{l}\frac{v}{v+n}\times\overline{\mathfrak{N}-\frac{n}{6r}-\mathfrak{D}-\frac{v}{6r}\times\frac{2n-v}{2nn}}\times v\times\frac{\mathfrak{P}}{2n};\\ \frac{1}{v+n}\times\overline{\mathfrak{N}-\frac{n}{6r}-\mathfrak{A}-\frac{v}{6r}\times\frac{2n-v}{2nn}}\times v\times\frac{\mathfrak{P}}{2}:\end{array}\right.$$

$$\left.\begin{array}{l}A,\ D,\ C,\ B\\ D,\ A,\ C,\ B\end{array}\right\}B=C=D\left\{\begin{array}{l}\frac{v}{v+t}\times\overline{\mathfrak{D}-\frac{v}{6r}}\times\frac{\mathfrak{P}}{4t};\\ \frac{1}{v+t}\times\overline{\mathfrak{D}-\frac{v}{6r}}\times\frac{\mathfrak{P}}{4}:\end{array}\right.$$

CASE IX. referring to the preſent worths, found in queſt. 58. and its corollaries.

Order of dying	Ages.	Solution.
C, D, A, B	unequal.	$\frac{v}{v+n}\times\mathfrak{A}$; ſee queſt. 58. fol. 183.
D, C, A, B		$\frac{n}{v+n}\times\mathfrak{A}$: ſee ditto.
C, D, A, B	$A=B$	$\frac{v}{v+n}\times\mathfrak{A}$; ſee corol. 1. queſt. 58. fol. 185.
D, C, A, B		$\frac{n}{v+n}\times\mathfrak{A}$: ſee ditto.
C, D, A, B	$B=C$	$\frac{v}{v+n}\times\overline{\mathfrak{N}-\frac{n}{6r}-\overline{\mathfrak{D}-\frac{v}{6r}}\times\frac{2n-v}{2nn}}\times v\times\frac{\mathfrak{P}}{2t}$;
D, C, A, B		$\frac{n}{v+n}\times\overline{\mathfrak{N}-\frac{n}{6r}-\overline{\mathfrak{D}-\frac{v}{6r}}\times\frac{2n-v}{2nn}}\times v\times\frac{\mathfrak{P}}{2t}$;

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CASE IX. continued.

Order of dying.	Ages	Solution.
C, D, A, B D, C, A, B	$C=D$;	$\frac{1}{2}\times\overline{\mathfrak{M}-\mathfrak{D}+\overline{\mathfrak{D}-\frac{v}{6r}}\times\frac{3v-2m}{4m}}\times\frac{\mathfrak{P}}{t}$:
C, D, A, B	$A=B=C$	$\frac{v}{v+n}\times\overline{\mathfrak{N}-\frac{n}{6r}-\overline{\mathfrak{D}-\frac{v}{6r}}\times\frac{2n-v}{2nn}}\times v\times\frac{\mathfrak{P}}{2n}$;
D, C, A, B		$\frac{1}{v+n}\times\overline{\mathfrak{N}-\frac{n}{6r}-\overline{\mathfrak{D}-\frac{v}{6r}}\times\frac{2n-v}{2nn}}\times v\times\frac{\mathfrak{P}}{2}$:
C, D, A, B D, C, A, B	$B=C=D$;	$\frac{1}{2}\times\overline{\mathfrak{D}-\frac{v}{6r}}\times\frac{\mathfrak{P}}{4t}$.

CASE X. referring to the preſent worths, found in queſt, 59, and its corollaries.

Order of dying. Ages. Solution.

$$\left.\begin{matrix} B,\ C,\ D,\ A \\ C,\ B,\ D,\ A \end{matrix}\right\}\text{unequal}\left\{\begin{matrix} \frac{n}{n+m}\times\mathfrak{A}; \\ \frac{m}{n+m}\times\mathfrak{A}: \end{matrix}\right\}\text{ ſee queſt. 59. fol. 190.}$$

$$\left.\begin{matrix} B,\ C,\ D,\ A \\ C,\ B,\ D,\ A \end{matrix}\right\}A=B\left\{\begin{matrix} \frac{n}{n+m}\times\mathfrak{A};\text{ ſee corol. 1. queſt. 59. fol. 191.} \\ \frac{1}{n+m}\times\frac{\mathfrak{P}}{4m}\times\left\{\begin{matrix} \overline{v\mathfrak{M}-\frac{vv}{6rm}}\times 2m-\overline{v\mathfrak{P}-\frac{vv}{6rn}}\times\overline{2m-v} \\ +\frac{vv}{6rn}\times\overline{n-v}\times 2: \end{matrix}\right. \end{matrix}\right.$$

$$\left.\begin{matrix} B,\ C,\ D,\ A \\ C,\ B,\ D,\ A \end{matrix}\right\}B=C;\ \tfrac{1}{2}\times\mathfrak{A}:\text{ ſee corol. 2. queſt. 59. fol. 192.}$$

CASE X. continued.

Order of dying.	Ages.	Solution.	
B, C, D, A	$C=D$	$\frac{v}{v+m}\times\mathfrak{A};$	see corol. 3. queſt. 59. fol. 192.
C, B, D, A		$\frac{m}{v+m}\times\mathfrak{A}:$	
B, C, D, A *C, B, D, A*	$A=B=C;$	$\frac{1}{2}\times\overline{v\mathfrak{N}+\frac{2n-3v}{6rn}\times v\times\frac{v\mathfrak{P}}{4nn}}:$	
B, C, D, A *C, B, D, A*	$B=C=D;$	$\frac{1}{2}\times\overline{v\mathfrak{F}-\mathfrak{D}+\mathfrak{C}-\frac{v}{6r}\times\frac{3v-2t}{4t}}\times\frac{\mathfrak{P}}{v}.$	

CASE XI. referring to the preſent worths, found in queſt. 60, and its corollaries.

Order of dying.	Ages.	Solution.	
B, D, C, A	unequal.	$\frac{v}{v+m}\times\mathfrak{A}$;	ſee queſt. 60. fol. 196.
D, B, C, A		$\frac{m}{v+m}\times\mathfrak{A}$:	
B, D, C, A	*A*=*B*	$\frac{v}{v+m}\times\mathfrak{A}$;	ſee corol. 1. queſt. 60. fol. 197.
D, B, C, A		$\frac{1}{v+m}\times\frac{\mathfrak{P}}{2}\times\overline{n\mathfrak{M}-\frac{nn}{6rm}-\mathfrak{C}-\frac{v}{6r}\times\frac{2m-v}{2mn}}\times v$:	
B, D, C, A	*B* = *C*	$\frac{v}{v+n}\times\mathfrak{A}$;	ſee corol. 2. queſt. 60. fol. 197.
D, B, C, A		$\frac{n}{v+n}\times\mathfrak{A}$:	

CASE XI. continued.

Order of dying.	Ages.	Solution.
B, D, C, A	$C=D$	$\frac{v}{v+m}\times\mathfrak{A}$; see corol. 3. queſt. 60. fol. 198.
D, B, C, A		$\frac{m}{v+m}\times\mathfrak{A}$;
B, D, C, A	$A=B=C$	$\frac{v}{v+n}\times\overline{\mathfrak{P}-\frac{n}{6r}-\overline{\mathfrak{P}-\frac{v}{6r}}\times\frac{2n-v}{2nn}}\times v\times\frac{\mathfrak{P}}{2n}$;
D, B, C, A		$\frac{1}{v+n}\times\overline{\mathfrak{P}-\frac{n}{6r}-\overline{\mathfrak{P}-\frac{v}{6r}}\times\frac{2n-v}{2nn}}\times v\times\frac{\mathfrak{P}}{2}$:
B, D, C, A D, B, C, A	$B=C=D$;	$\frac{1}{2}\times\overline{v\mathfrak{F}-\mathfrak{P}}+\overline{\mathfrak{P}-\frac{v}{6r}}\times\overline{\frac{3v-2t}{4t}}\times\frac{\mathfrak{P}}{v}$.

CASE XII. referring to the preſent worths, found in queſt. 61, and its corollaries.

Order of dying.	Ages.	Solution.	
C, D, B, A	unequal.	$\frac{v}{v+n} \times \mathfrak{A}$;	ſee queſt. 61. fol. 201.
D, C, B, A		$\frac{n}{v+n} \times \mathfrak{A}$:	
C, D, B, A	$A = B$	$\frac{v}{v+n} \times \mathfrak{A}$;	ſee corol. 1. queſt. 61. fol. 202.
D, C, B, A		$\frac{n}{v+n} \times \mathfrak{A}$:	
C, D, B, A	$B = C$	$\frac{v}{v+n} \times \mathfrak{A}$;	ſee corol. 2. ditto. 203.
D, C, B, A		$\frac{1}{v+n} \times \mathfrak{P}$; ×	$\left\{\begin{array}{l} n\mathfrak{F} - \mathfrak{N} + \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{n}{2t} \\ - \overline{\mathfrak{D} - \frac{v}{6r}} \times \overline{1 - \frac{v}{2t}} \times \frac{v}{2n}: \end{array}\right.$

CASE XII. continued.

Order of dying.	Ages.	Solution.
C, D, B, A *D, C, B, A*	*C*=*D*;	$\frac{1}{2} \times \overline{m\mathfrak{F} - \mathfrak{D} + \overline{\mathfrak{D} - \frac{v}{6r}} \times \frac{3v-2t}{4t}} \times \frac{\mathfrak{P}}{m}$:
C, D, B, A *D, C, B, A*	*A*=*B*=*C*	$\frac{v}{v+n} \times \mathfrak{A}$; ſee corol. 4. queſt. 61. fol. 204. $\frac{1}{v+n} \times \frac{\mathfrak{P}}{2} \times \overline{\mathfrak{P} - \frac{n}{6r}} - \overline{\mathfrak{D} - \frac{v}{6r}} \times \overline{\frac{2n-v}{2nn} \times v}$:
C, D, B, A *D, C, B, A*	*B*=*C*=*D*;	$\frac{1}{2} \times \overline{v\mathfrak{F} - \mathfrak{D} + \overline{\mathfrak{D} - \frac{v}{6r}} \times \frac{3v-2t}{4t}} \times \frac{\mathfrak{P}}{v}$.

QUESTION LXIV.

The reſpective ages of four perſons; *A*, the youngeſt; *B*, the ſecond; *C*, the third; and *D* the eldeſt; being given: it is required to find the probability, that C and *D*, the two elder, ſhall both ſurvive *A* and *B*, the two younger, and alſo to determine the preſent value of an eſtate, or legacy, worth 𝔓 pounds, dependent on that ſurvivorſhip.

SOLUTION.

If t, m, n, and v, ſeverally, repreſent the complements of the lives of *A*, *B*, *C*, and *D*, as in the preceding queſtions. Then firſt (by queſt. 50.) the probability that *A* and *B*, ſhall both die before either *C* or *D*; and that *C* and *D*, ſhall die, in the order in which they ſtand, is $\frac{v^3}{12tmn}$; and (by queſt. 53.) the probability that *A* and *B*, ſhall both die before either, *D* or *C*; and that *D* and *C* ſhall die, in the order in which they ſtand, is $\frac{vv}{3mt}-\frac{v^3}{4tmn}$; now, if theſe two probabilities be added together, their ſum $\left(\frac{vv}{3mt}-\frac{v^3}{6tmn}\text{, or}\right)$ $\overline{1-\frac{v}{2n}}\times\frac{vv}{3mt}$, will be the probability required.

Note, in the following ſolutions we ſhall (for brevity's ſake) denote the orders of ſurvivorſhip, treated off in the 50th, and eleven following queſtions, by writing, $\overline{A\ B}$, *C*, *D*, and ſimilar expreſſions, inſtead of ſaying that *A* and *B*, are both to die, before either *C* or *D*, and that *C* is alſo to die before *D*, or words ſimilar thereto, adapted to other orders of ſurvivorſhip.

Secondly; (by queſt. 50.) the preſent worth of an eſtate, dependent on the order, $\overline{A\ B}$, *C*, *D*, is 𝔓 −

$\overline{\mathbb{P} - \frac{v}{6r}} \times \frac{vv\mathbb{P}}{4mtn}$; and (by queſt. 53) that, of an eſtate, dependent on the order $\overline{A\,B}$, D, C, is $\overline{v\mathfrak{P} + \frac{2n-3v}{6rn}} \times v \times \frac{v\mathbb{P}}{4mt}$; therefore their ſum

$$\overline{\mathfrak{A} - \frac{v}{6r}} \times \frac{vv\mathbb{P}}{4mtn} + \overline{v\mathfrak{P} + \frac{2n-3v}{6rn}} \times v \times \frac{v\mathbb{P}}{4mt}, \text{ or)}$$

$$\overline{\overline{\mathfrak{A} - \frac{v}{6r}} \times \frac{v}{n} + v\mathfrak{P} + \frac{2n-3v}{6nr}} \times v \times \frac{v\mathbb{P}}{4mt}; \text{ that is}$$

$$\overline{\left(\frac{v}{n} + \mathfrak{A} + v\mathfrak{P} + \frac{2n-4v}{6r}\right.} \times \frac{v}{n} \times \frac{v\mathbb{P}}{4mt}, \text{ or)}$$

$$\overline{v\mathfrak{P} + \mathfrak{A} + \frac{2n-4v}{6r}} \times \frac{v}{n} \times \frac{v\mathbb{P}}{4mt},$$ will be the preſent worth required.

EXAMPLE.

What is the probability, that C and D (aged 66 and 76) ſhall both ſurvive, A and B (aged 43 and 54); and what is the preſent worth of a legacy of 25£. dependent on that ſurvivorſhip.?

Here $\frac{v}{2n} = \left(\frac{10}{40} =\right) \frac{1}{4}$; $1 - \frac{1}{4} = \frac{3}{4}$; $\frac{vv}{3mt} = \left(\frac{10 \cdot 10}{3 \cdot 3^2 \cdot 43} =\right) \frac{25}{24 \cdot 43}$;

Therefore $\left(\frac{3}{4} \times \frac{25}{24 \cdot 43} =\right) 0{,}01817$; will be the probability required.

Again $v\mathfrak{P} = 6{,}215$; $\mathfrak{A} = 4{,}318$; $\frac{v}{n} = \left(\frac{10}{20} =\right) \frac{1}{2}$; $2n - 4v = (2 \times 20 - 4 \times 10) = 0$; $\frac{v\mathbb{P}}{4mt} = \frac{10 \times 25}{4 \cdot 3^2 \cdot 43}$;

And $6{,}215 + \frac{4{,}318}{2} = 8{,}374$; Therefore $(8{,}374 \times \frac{10 \times 25}{4 \cdot 32 \cdot 43} =)$ 0,3804, will be the preſent worth required.

COROL. I.

If *A*, and *B*, are of equal ages; then $t = m$; and the anſwers will become $1 - \frac{v}{2n} \times \frac{vv}{3mm}$; and $\overline{v\mathfrak{N} + \overline{\mathfrak{D} + \frac{2n-4v}{6r}} \times \frac{v}{n}} \times \frac{v\mathfrak{P}}{4mm}$.

COROL. II.

If *B*, and *C*, are of equal ages; then $m = n$; and the anſwers will become $1 - \frac{v}{2n} \times \frac{vv}{3nt}$, and $\overline{v\mathfrak{N} + \overline{\mathfrak{D} + \frac{2n-4v}{6r}} \times \frac{v}{n}} \times \frac{v\mathfrak{P}}{4nt}$.

COROL. III.

If *C*, and *D*, are of equal ages; then $v\mathfrak{N} = \mathfrak{D}$, and $n = v$; whence the anſwers will become $(1 - \frac{1}{2} \times \frac{vv}{3mt}$ or) $\frac{vv}{6mt}$, and $(\overline{2\,\mathfrak{D} - \frac{2v}{6r}} \times \frac{v\mathfrak{P}}{4mt}$ or) $\overline{\mathfrak{D} - \frac{v}{6r}} \times \frac{v\mathfrak{P}}{2mt}$.

COROL. IV.

If *A*, *B*, and *C*, are of equal ages; then $t = m = n$; and the anſwers will become $1 - \frac{v}{2n} \times \frac{vv}{3nn}$, and $\overline{{}^{v}\mathfrak{P} + \overline{\mathfrak{D} + \frac{2n - 4v}{6r}} \times \frac{v}{n}} \times \frac{v\mathfrak{P}}{4nn}$.

COROL. V.

If *B*, *C*, and *D*, are of equal ages; then ${}^{v}\mathfrak{P} = \mathfrak{D}$, and $m = n = v$; whence the anſwers will become $\frac{v}{6t}$, and $\overline{\mathfrak{D} - \frac{v}{6r}} \times \frac{\mathfrak{P}}{2t}$.

COROL. VI.

If the four lives are of equal ages; then the anſwers will be $\frac{1}{6}$, and $\overline{\mathfrak{D} - \frac{v}{6r}} \times \frac{\mathfrak{P}}{2v}$.

QUESTION LXV.

The probability, that *B* and *D* ſhall both ſurvive *A* and *C*; and the preſent value of an eſtate, or legacy, worth $\mathfrak{P}$ pounds, dependent on that ſurvivorſhip, are required.

SOLUTION.

First. The probability of the order $\overline{A\,C}$, B, D, is (b queſt. 51) $\frac{v^3}{12tmn}$; and that of the order $\overline{A\,C}$, D, B, i (by queſt. 56) $\frac{vv}{3tn}-\frac{vv}{4tmn}$; therefore their ſum $\left(\frac{vv}{3tn}-\frac{v^3}{6tmn}\right.$, or) $\overline{1-\frac{v}{2m}}\times\frac{vv}{3tn}$, will be th probability required.

Secondly; The preſent worth depending on the orde $\overline{A\,C}$, B, D is (by queſt. 51) $\overline{\mathfrak{A}-\frac{v}{6r}}\times\frac{vv\mathfrak{P}}{4mtn}$, and that, on the order $\overline{A\,C}$, D, B is (by queſt. 56)

$$\left.\begin{array}{l}\overline{v\mathfrak{M}-\frac{vv}{6rm}}\times 2m-\overline{v\mathfrak{N}-\frac{vv}{6rn}}\times\overline{2m-v}\\ +\frac{vv}{6rn}\times\overline{n-v}\times 2\end{array}\right\}\times\frac{\mathfrak{P}}{4mt};$$

therefore their ſum will be the preſent worth required, viz.

$$\left(\begin{array}{l}\overline{v\mathfrak{M}-\frac{vv}{6rm}}\times 2m-\overline{v\mathfrak{N}-\frac{vv}{6rn}}\times\overline{2m-v}\\ +\frac{vv}{6rn}\times\overline{n-v}\times 2+\overline{\mathfrak{D}-\frac{v}{6r}}\times\frac{vv}{n}\end{array}\right\}\times\frac{\mathfrak{P}}{4mt}\text{, or)}$$

$$\left.\begin{array}{l}\overline{\overline{v\mathfrak{M}-\frac{vv}{6rm}}-\overline{v\mathfrak{N}-\frac{vv}{6rn}}}\times 2m+\mathfrak{D}\times\frac{vv}{n}\\ +\frac{2n-4v}{6r}\times\frac{vv}{n}+v\mathfrak{N}\times v\end{array}\right\}\times\frac{\mathfrak{P}}{4mt};$$

That is,

$$\overline{\left(v\mathfrak{M}-v\mathfrak{N}+\overline{\overline{\frac{1}{n}-\frac{1}{m}}\times\frac{vv}{6r}}\times 2m+v\mathfrak{N}\times v+\quad+\overline{\frac{2n-4v}{6r}}\times\frac{vv}{n}\times\frac{\mathfrak{P}}{4mt},\text{ or}\right)}$$

$$\overline{\overline{v\mathfrak{M}-v\mathfrak{N}}+\frac{\overline{m-n}\times v}{mn}\overline{\times\frac{v}{6r}}\times 2m+v\mathfrak{N}\times v+\mathfrak{D}+\overline{\frac{2n-4v}{6r}}\times\frac{vv}{n}\times\frac{\mathfrak{P}}{4mt}},$$

will be the preſent worth required.

EXAMPLE.

What is the probability, that *B* and *D* (aged 54 and 76) ſhall both ſurvive *A* and *C* (aged 43 and 66) and what is the preſent worth of a legacy of 25£. dependent on that ſurvivorſhip?

Here $\frac{v}{2m}\left(\frac{10}{64}=\right)\frac{5}{32}$; $1-\frac{5}{32}=\frac{27}{32}$; $\frac{vv}{3tn}=\left(\frac{10.10}{3.43.20}=\right)\frac{5}{129}$;

Whence $\left(\frac{27}{32}\times\frac{5}{129}=\right)$ 0,0327 will be the probability required.

Alſo $v\mathfrak{M}=6{,}925$; $v\mathfrak{N}=6{,}215$; $\frac{\overline{m-n}\times v}{mn}=\left(\frac{\overline{32-20}\times 10}{32\times 20}=\frac{12.10}{32.20}=\right)\frac{3}{16}$;

$\frac{v}{6r} = 1,603$; $m = 32$; $\mathfrak{C} = 4,318$; $2n - 4v = 0$; $\frac{vv}{n} = (\frac{100}{20} =)\ 5$; and $\frac{\mathfrak{P}}{4mt} = \frac{25}{4.32.43}$:

Then $\frac{3}{16} \times 1,603 = 0,301$; $6,925 - 6,215 = 0,710$; $0,710 + 0,301 = 1,011$; $1,011 \times 2 \times 32 = 64,704$; $4,318 \times 5 = 21,590$; $6,215 \times 10 = 62,150$; And $64,704 + 62,150 + 21,590 = 148,444$;

Whence $(148,444 \times \frac{25}{4.32.43} =)\ 0,674$, will be the preſent worth required.

COROL. I.

If *A*, and *B*, are of equal ages; then $t = m$; and the anſwers will become $\overline{1 - \frac{v}{2m}} \times \frac{vv}{3mn}$; and

$$\left.\begin{array}{l}\overline{v\mathfrak{M} - v\mathfrak{N} + \frac{\overline{m-n} \times v}{mn} \times \frac{v}{6r} \times 2m} \\ + v\mathfrak{N} \times v + \overline{\mathfrak{D} \times \frac{2n-4v}{6r}} \times \frac{vv}{n}\end{array}\right\} \times \frac{\mathfrak{P}}{4mm}.$$

COROL. II.

If *B*, and *C*, are of equal ages; then $v\mathfrak{M} = v\mathfrak{N}$, and $m = n$; whence the anſwers will become

$\overline{1 - \frac{v}{2n}} \times \frac{vv}{3tn}$, and

$$\left(\overline{v\mathfrak{N} \times v + \mathfrak{D} + \frac{2n-4v}{6r}} \times \frac{vv}{n} \times \frac{\mathfrak{P}}{4nt}\right)$$

$$\text{or } \overline{v\mathfrak{N} \times \frac{n}{v} + \mathfrak{D} + \frac{2n-4v}{6r}} \times \frac{\mathfrak{P}vv}{4nnt}.$$

COROL. III.

If *C*, and *D*, are of equal ages; then ${}^{v}\mathfrak{N}=\mathfrak{D}$, and $n=v$; whence the anſwers will become, $\overline{1-\frac{v}{2m}}\times\frac{v}{3t}$, and

$$\left.\begin{array}{l}(\overline{{}^{v}\mathfrak{M}-\mathfrak{A}+\overline{\frac{m-v}{m}}\times\frac{v}{6r}}\times 2m\\ +\mathfrak{A}\times v+\overline{\mathfrak{A}-\frac{2v}{6r}}\times v\end{array}\right\}\times\frac{\mathfrak{P}}{4mt},\text{ or)}$$

$$\overline{\overline{{}^{v}\mathfrak{M}-\mathfrak{A}}\times 2m+\frac{\overline{m-v}\times 2v}{6r}+\overline{\mathfrak{D}-\frac{v}{6r}}\times 2v}\times\frac{\mathfrak{P}}{4mt};$$

That is,

$$\overline{\overline{{}^{v}\mathfrak{M}-\mathfrak{A}}\times m+\frac{\overline{m-v}\times v}{6r}+\overline{\mathfrak{D}-\frac{v}{6r}}\times v}\times\frac{\mathfrak{P}}{2mt}.$$

COROL. IV.

If *A*, *B*, and *C*, are of equal ages; then ${}^{v}\mathfrak{M}={}^{v}\mathfrak{N}$, and $t=m=n$; whence the anſwers will become

$$\overline{1-\frac{v}{2n}}\times\frac{vv}{3nn},\text{ and}$$

$$\overline{{}^{v}\mathfrak{N}\times\frac{n}{v}+\mathfrak{D}+\frac{2n-4v}{6r}}\times\frac{vv\mathfrak{P}}{4nnn}.$$

COROL. V.

If *B* *C*, and *D*, are of equal ages; then ${}^{v}\mathfrak{M}={}^{v}\mathfrak{N}=\mathfrak{D}$, and $m=n=v$; whence the anſwers will become

$(\overline{1-\frac{1}{2}}\times\frac{v}{3t}$ or$)\ \frac{v}{6t}$. and $(\overline{\mathfrak{D}-\frac{v}{6r}}\times v\times\frac{\mathfrak{P}}{2vt}$ or$)$

$$\overline{\mathfrak{D}-\frac{v}{6r}}\times\frac{\mathfrak{P}}{2t}.$$

QUESTION LXVI.

The probability, that *A* and *D* ſhall both ſurvive *B* and *C*; and the preſent value of an eſtate, or legacy, worth $\mathfrak{P}$ pounds, depending on that ſurvivorſhip, are required.

SOLUTION.

Firſt, the probability of the order $\overline{BC}$, *A*, *D* is (by queſt. 52.) $\frac{v^3}{12tmn}$; and that of the order $\overline{BC}$, *D*, *A*, is (by queſt. 59.) $\frac{vv}{3mn} - \frac{v^3}{4tmn}$; therefore their ſum ($\frac{vv}{3mn} - \frac{v^3}{6tmn}$, or) $\overline{1 - \frac{v}{2t}} \times \frac{vv}{3mn}$, will be the probability required.

Secondly; the preſent worth depending on the order $\overline{BC}$, *A*, *D*, is (by queſt. 52) $\overline{\mathfrak{P} - \frac{v}{6r}} \times \frac{vv\mathfrak{P}}{4tmn}$; and that, on the order $\overline{BC}$, *D*, *A*, is (by queſt. 59.)

$$\mathfrak{P}\times\left\{\begin{array}{l}\dfrac{v\mathfrak{F}-v\mathfrak{M}}{v}\\ +\overline{\overline{v\mathfrak{M}-\dfrac{vv}{6rm}}\times 2m-\overline{v\mathfrak{N}-\dfrac{vv}{6rn}}\times\overline{2t-v}+\dfrac{vv}{6rn}\times\overline{n-v}\times 2}\times\dfrac{1}{4mt};\end{array}\right.$$

Therefore, their ſum

$$\mathfrak{P}\times\left\{\begin{array}{l}\dfrac{v\mathfrak{F}-v\mathfrak{M}}{v}\\ +\dfrac{1}{4mt}\times\left\{\begin{array}{l}\overline{v\mathfrak{M}-\dfrac{vv}{6rm}}\times 2m-\overline{v\mathfrak{N}-\dfrac{vv}{6rn}}\times\overline{2t-v}\\ +\dfrac{vv}{6rn}\times\overline{n-v}\times 2+\overline{\mathfrak{D}-\dfrac{v}{6r}}\times\dfrac{vv}{n}\end{array}\right\}\text{, or}\end{array}\right.$$

$$\mathfrak{P}\times\left\{\begin{array}{l}\dfrac{v\mathfrak{F}-v\mathfrak{M}}{v}\\ +\dfrac{1}{4mt}\times\overline{\overline{v\mathfrak{M}-\dfrac{vv}{6rm}}\times 2m-\overline{v\mathfrak{N}-\dfrac{vv}{6rn}}\times\overline{2t-v}+\overline{\mathfrak{D}+\dfrac{2n-3v}{6r}}\times\dfrac{vv}{n}},\end{array}\right.$$

will be the preſent worth required.

EXAMPLE.

What is the probability that *A* and *D* (aged 43 and 76) ſhall both ſurvive *B* and *C* (aged 54 and 66); and what is the preſent worth of a legacy of 25£. dependent on that ſurvivorſhop?

Here $\frac{v}{2t} = \left(\frac{10}{2.43} =\right) \frac{5}{43}$; $1 - \frac{5}{43} = \frac{38}{43}$; $\frac{vv}{3mn} = \left(\frac{10.10}{3.32.20} =\right) \frac{5}{96}$;

Therefore $\left(\frac{38}{43} \times \frac{5}{96} = \frac{19}{43} \times \frac{5}{48} =\right) 0{,}0461$, will be probability required.

Again $v\mathfrak{F} = 7{,}228$; $v\mathfrak{M} = 6{,}925$; $\overline{7{,}228 - 6{,}925} = 0{,}303$; and $\frac{0{,}303}{10} = 0{,}0303$; $\overline{v\mathfrak{M} - \frac{vv}{6rm}} \times 2m = 411{,}136$; and $\overline{v\mathfrak{N} - \frac{vv}{6rn}} \times 2t - v = 411{,}426$; by exam. queſt. 59. $\mathfrak{D} = 4{,}318$; $\frac{2n - 3v}{6r} = \left(\frac{10}{6r} =\right) 1{,}603$; $4{,}318 + 1{,}603 = 5{,}921$; $\frac{vv}{n} = 5$; $5{,}921 \times 5 = 29{,}605$; $411{,}136 - 411{,}426 + 29{,}605 = 29{,}315$; $\frac{29{,}315}{4.32.43} = 0{,}0053$; $0{,}0303 + 0{,}0053 = 0{,}0356$; and $(0{,}0356 \times 25 =)\ 0{,}890$, will be the preſent worth required.

COROL. I.

If *A*, and *B*, are of equal ages; then $v\mathfrak{F} = v\mathfrak{M}$, and $t = m$; whence the anſwers will become

$$1 - \frac{v}{2m} \times \frac{vv}{3mn}, \text{ and}$$

$\times\ \mathfrak{Q}$

$\frac{\mathfrak{P}}{4mm}$

$$\frac{\mathfrak{P}}{4mm} \times \overline{v\mathfrak{M} + \frac{vv}{6rm} \times 2m - v\mathfrak{N} - \frac{vv}{6rn} \times 2n - v + \mathfrak{A} + \frac{2n-3v}{6r} \times \frac{vv}{n}}.$$

COROL. II.

If *B*, and *C*. are of equal ages; then $v\mathfrak{M} = v\mathfrak{N}$, and $m = n$; whence the anſwers will become $\overline{1 - \frac{v}{2t}} \times \frac{vv}{3nn}$, and

$$\mathfrak{P} \times \left\{ \begin{array}{l} \dfrac{v\mathfrak{F} - v\mathfrak{N}}{v} \\ + \dfrac{1}{4nt} \times \overline{v\mathfrak{N} - \dfrac{vv}{6rn} \times 2n + v - 2t + \mathfrak{D} + \dfrac{2n-2v}{6r} \times \dfrac{vv}{n}}. \end{array} \right.$$

COROL. III.

If *C*, and *D*, are of equal ages; then $v\mathfrak{N} = \mathfrak{A}$, and $n = v$; whence the anſwers will become $\overline{1 - \frac{v}{2t}} \times \frac{v}{3m}$, and

$$\mathfrak{P} \times \left\{ \begin{array}{l} \frac{v\mathfrak{F} - v\mathfrak{M}}{v} \\ + \frac{1}{4mt} \times \overline{v\mathfrak{P} - \frac{vv}{6rm} \times \overline{2m - \mathfrak{D} - \frac{v}{6r} \times 2t - 3v}} \end{array}\right.$$

COROL. IV.

If *A*, *B*, and *C*, are of equal ages; then $v\mathfrak{F} = v\mathfrak{M} = v\mathfrak{N}$, and $t = m = n$; whence the anſwers will become $1 - \frac{v}{2n} \times \frac{vv}{3nn}$, and

$\left(\frac{\mathfrak{P}}{4nn} \times \overline{v\mathfrak{P} - \frac{vv}{6rn} \times v + \mathfrak{D} + \frac{2n-3v}{6r} \times \frac{vv}{n}}\right.$ or) $\frac{\mathfrak{P}}{4nn} \times \overline{v\mathfrak{P} \times v + \mathfrak{D} + \frac{2n-4v}{6r} \times \frac{vv}{n}}$; that is $\frac{vv\mathfrak{P}}{4n^3} \times \overline{v\mathfrak{N} \times \frac{n}{v} + \mathfrak{D} + \frac{2n-4v}{6r}}$.

COROL. V.

If *B*, *C*, and *D* are of equal ages; then $v\mathfrak{M} = v\mathfrak{N} = \mathfrak{D}$ and $m = n = v$; whence the anſwers will become $1 - \frac{v}{2t} \times \frac{1}{3}$ and

$\mathfrak{P} \times \overline{\frac{v\mathfrak{F} - \mathfrak{D}}{2v} + \frac{1}{4vt} \times \overline{\mathfrak{D} - \frac{v}{6r} \times 4v - 2t}}$ or)

$\frac{v\mathfrak{P}}{2t} \times \overline{v\mathfrak{F} - \mathfrak{D} + \mathfrak{D} - \frac{v}{6r} \times \frac{4v - 2t}{4t}}$.

QUES-

QUESTION LXVII.

The probability, that *B* and *C* ſhall both ſurvive *A* and *D*; and the preſent value of an eſtate, or legacy, worth $\mathfrak{P}$ pounds, depending on that ſurvivorſhip are required.

SOLUTION.

Firſt; The probability of the order $\overline{AD}$, *B*, *C* is (by queſt. 54) $\frac{nn}{6tm} - \frac{vv}{6tm} + \frac{v^3}{12ntm}$; and the probability of the order $\overline{AD}$, *C*, *B*, is (by queſt. 57) $\frac{n}{2t} - \frac{nn}{3\,m} - \frac{vv}{6tm} + \frac{v^3}{12ntm}$; therefore their ſum $\left(\frac{n}{2t} - \frac{nn}{6tm} - \frac{vv}{6tm} - \frac{vv}{6tn} + \frac{v^3}{6ntm}\right.$ or) $\overline{1 - \frac{n}{3m}} \times \frac{n}{2t} - \overline{\frac{1}{m} + \frac{1}{n} - \frac{v}{nm}} \times \frac{vv}{6t}$; that is ($\overline{1 - \frac{n}{3m}} \times \frac{n}{2t} - \frac{n+m-v}{nm} \times \frac{vv}{6t}$ or) $\overline{\overline{1 - \frac{n}{3m}} \times n - \frac{n+m-v}{nm} \times \frac{vv}{3}} \times \frac{1}{2t}$, will be the probability required.

Secondly; The preſent worth depending on the order $\overline{AD}$, *B*, *C*, is (by queſt. 54)

$$\overline{\mathfrak{P} - \frac{n}{6r}} \times \frac{n}{m} - \overline{\mathfrak{D} - \frac{v}{6r}} \times \overline{1 - \frac{v}{2n}} \times \frac{v}{m} \times \frac{\mathfrak{P}}{2t};$$

and that on the order $\overline{AD}$, *C*, *B*, is (by queſt. 57)

$$\overline{\mathfrak{M} - \frac{nn}{6rm} - \overline{\mathfrak{D} - \frac{v}{6r}} \times \frac{\overline{2m-v} \times v}{2mn}} \times \frac{\mathfrak{P}}{2t};$$

therefore their ſum

$$\frac{\mathfrak{P}}{2t}\times\left\{\begin{array}{l}\overline{\mathfrak{P}-\frac{n}{6r}}\times\frac{n}{m}-\overline{\mathfrak{D}-\frac{v}{6r}}\times\frac{\overline{2n-v}\times v}{2mn}\\ +\,{}^{n}\mathfrak{M}-\frac{nn}{6rm}-\overline{\mathfrak{D}-\frac{v}{6r}}\times\frac{\overline{2m-v}\times v}{2mn}\end{array}\right\}\text{ or}$$

$$\frac{\mathfrak{P}}{2t}\times\left\{\begin{array}{l}{}^{n}\mathfrak{M}-\frac{nn}{6rm}+\overline{\mathfrak{P}-\frac{n}{6r}}\times\frac{n}{m}-\\ \overline{\mathfrak{D}-\frac{v}{6r}}\times\frac{n+m-v}{mn},\end{array}\right\}\times v,$$ will be the preſent worth required.

EXAMPLE.

What is the probability that *B* and *C* (aged 54 and 66) ſhall both ſurvive *A* and *D* (aged 43 and 76); and what is the preſent worth of a legacy of 25£. depending on that ſurvivorſhip?

Here $\frac{n}{3m}=(\frac{20}{96}=)\,\frac{5}{24}$; $1-\frac{5}{24}=\frac{19}{24}$; $\frac{19}{24}\times 20=(\frac{19\times 5}{6}=)\,15,833$; $\frac{n+m-v}{nm}=(\frac{42}{20\cdot 32}=)\,\frac{21}{320}$; $\frac{vv}{3}=\frac{100}{3}$; $\frac{21}{320}\times\frac{100}{3}=(\frac{70}{32}=)\,2,188$; $15,833-2,188=13,645$; and $(\frac{13,645}{2\times 43}=)\,0,159$ will be the probability required.

Again ${}^{n}\mathfrak{M}-\frac{nn}{6rm}=7,888$; $\mathfrak{P}-\frac{n}{6r}=4,468$; $\frac{n}{m}=(\frac{20}{32}=)\,\frac{5}{8}$;

$4,468\times\frac{5}{8}=2,7925$; $\mathfrak{D}-\frac{v}{6r}=2,715$; $\frac{n+m-v}{mn}$

$\times v = \left(\frac{21 \times 10}{320} =\right) \frac{21}{32}$; $2,715 \times \frac{21}{32} = 1,782$; $7,888 + 2,793 - 1,782 = 8,899$; $\frac{\mathfrak{P}}{2t} = \frac{25}{86}$; therefore $(8,899 \times \frac{25}{86} =)\ 2,587$ will be the preſent worth required.

COROL. I.

If *A* and *B* are of equal ages; then $t = m$; and the anſwer will be

$$\overline{1 - \frac{n}{3m} \times n - \frac{n+m-v}{nm}} \times \frac{vv}{3} \times \frac{1}{2m}; \text{ and}$$

$$\frac{\mathfrak{P}}{2m} \times \left\{ \begin{array}{l} n\mathfrak{M} - \frac{nn}{6rm} \times \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{n}{m} \\ - \overline{\mathfrak{D} - \frac{v}{6r}} \times \frac{n+m-v}{mn} \times v. \end{array} \right.$$

COROL. II.

If *B* and *C* are of equal ages; then $\mathfrak{M} = \mathfrak{N}$, and $m = n$; whence the anſwers will become

$$\left(\overline{1 - \frac{1}{3}} \times \overline{n - \frac{2n-v}{nn} \times \frac{vv}{3}} \times \frac{1}{2t} \text{ or}\right) \overline{2n - \frac{2n-v}{nn}} \times vv \times \frac{1}{6t}, \text{ and}$$

$$\frac{\mathfrak{P}}{2t} \times \overline{\overline{\mathfrak{N} - \frac{n}{6r}} \times 2 - \overline{\mathfrak{D} - \frac{v}{6r}} \times \frac{2n-v}{nn}} \times v.$$

COROL. III.

If *C* and *D* are of equal ages; then $\mathfrak{N} = \mathfrak{D}$, and $n = v$; whence the anſwers will become

$\overline{(1-\frac{v}{3m}\times v-\frac{1}{v}}\times\frac{vv}{3}\times\frac{1}{2t}$ or)

$\frac{3m-v}{3m}\times\overline{v-\frac{vv}{3}}\times\frac{1}{2t}$; that is $\left(\frac{3m-v}{m}-1\times\frac{v}{6t}\right.$ or) $\frac{2m-v}{m}\times\frac{v}{6t}$; and

$\frac{\mathfrak{P}}{2t}\times\overline{v\mathfrak{M}-\frac{vv}{6rm}+\mathfrak{P}-\frac{v}{6r}\times\frac{v}{m}-1}$, or)

$\frac{\mathfrak{P}}{2t}\times\overline{v\mathfrak{M}-\frac{vv}{6rm}-\mathfrak{G}-\frac{v}{6r}}\times\frac{m-v}{m}$.

COROL. IV.

If *A*, *B*, and *C* are of equal ages; then ${}^{n}\mathfrak{M}=\mathfrak{P}$; and $t=m=n$; whence the anſwers will be

$\overline{(2n-\frac{2n-v}{nn}}\times vv\times\frac{1}{6n}$, or) $\frac{1}{3}-\overline{2n-v}\times\frac{vv}{6n^3}$;

and $\frac{\mathfrak{P}}{2n}\times\overline{\mathfrak{P}-\frac{n}{6r}\times 2-\mathfrak{P}-\frac{v}{6r}}\times\frac{2n-v}{nn}\times v$.

COROL. V.

If *B*, *C*, and *D* are of equal ages; then ${}^{n}\mathfrak{M}=\mathfrak{N}=\mathfrak{P}$, and $m=n=v$; whence the anſwers will be

$\left(\frac{2v-v}{v}\times\frac{v}{6t}\right.$ or) $\frac{v}{6t}$; and $\left(\frac{\mathfrak{P}}{2t}\times\overline{\mathfrak{P}-\frac{v}{6r}}\times\overline{1-\frac{\mathfrak{P}-v}{v}}\right.$

or) $\overline{\mathfrak{P}-\frac{v}{6r}}\times\frac{\mathfrak{P}}{2t}$.

QUES-

QUESTION LXVIII.

The probability that *A* and *C*, ſhall both ſurvive *B* and *D*; and the preſent value of an eſtate, or legacy, worth $\mathfrak{P}$ pounds, depending on that ſurvivorſhip are required.

SOLUTION.

Firſt, the probability of the order $\overline{B\,D}$, *A*, *C*, is (by queſt. 55.) $\frac{nn}{6tm}-\frac{vv}{6tm}+\frac{v^3}{12tnm}$; and that of the order $\overline{B\,D}$, *C*, *A*, is (by queſt. 60.) $\frac{n}{2m}-\frac{nn}{3tm}-\frac{vv}{6nm}+\frac{v^3}{12tnm}$; th. their ſum $\left(\frac{n}{2m}-\frac{nn}{6tm}-\frac{vv}{6tm}-\frac{vv}{6nm}+\frac{v^3}{6tnm}\right.$ or$\left.\right)$ $\overline{1-\frac{n}{3t}}\times\frac{n}{2m}-\overline{\frac{1}{t}+\frac{1}{n}-\frac{v}{tn}}\times\frac{vv}{6m}$; That is ($\overline{1-\frac{n}{3t}}\times\frac{n}{2m}-\frac{n+t-v}{tn}\times\frac{vv}{6m}$ or) $\overline{1-\frac{n}{3t}\times n-\frac{n+t-v}{tn}\times\frac{vv}{3}}\times\frac{1}{2m}$, will be the probability required.

Secondly, The preſent worth depending on the order $\overline{B\,D}$, *A*, *C*, is (by queſt. 55.)

$$\overline{\mathfrak{P}-\frac{n}{6r}}\times\frac{n}{2tm}-\overline{\mathfrak{P}-\frac{v}{6r}}\times\overline{1-\frac{v}{2n}}\times\frac{v}{2tm}\times\mathfrak{P};$$

and that on the order $\overline{B\,D}$, *C*, *A*, is (by queſt. 60.)

$$\left.\begin{array}{l}\frac{p\mathfrak{F}-n\mathfrak{M}}{n}+\overline{n\mathfrak{M}-\frac{nn}{6rm}}\times\frac{1}{2t}\\ -\overline{\mathfrak{P}-\frac{v}{6r}}\times\frac{\overline{2t-v}\times v}{4tnm}\end{array}\right\}\times\mathfrak{P};$$

therefore their sum

$$(\mathfrak{P}\times\left\{\frac{{}^n\mathfrak{F}-{}^n\mathfrak{M}}{n}+\overline{{}^n\mathfrak{M}-\frac{nn}{6rm}}\times\frac{1}{2t}-\overline{\mathfrak{D}-\frac{v}{6r}}\times\frac{\overline{2t-v}\times v}{4tnm}+\overline{\mathfrak{P}-\frac{n}{6r}}\times\frac{n}{2tm}-\overline{\mathfrak{D}-\frac{v}{6r}}\times\frac{\overline{2n-v}\times v}{4tnm}\right\}\text{, or)}$$

$$\mathfrak{P}\times\left\{\frac{{}^n\mathfrak{F}-{}^n\mathfrak{M}}{n}+\overline{{}^n\mathfrak{M}-\frac{nn}{6rm}}\times\frac{1}{2t}+\overline{\mathfrak{P}-\frac{n}{6r}}\times\frac{n}{2tm}-\overline{\mathfrak{D}-\frac{v}{6r}}\times\frac{t+n-v}{2tnm}\times v\right\}\text{; that is}$$

$$\mathfrak{P}\times\left\{\frac{{}^n\mathfrak{F}-{}^n\mathfrak{M}}{n}+\frac{1}{2t}\times\overline{{}^n\mathfrak{M}-\frac{nn}{6rm}+\overline{\mathfrak{P}-\frac{n}{6r}}\times\frac{n}{m}-\overline{\mathfrak{D}-\frac{v}{6r}}\times\frac{t+n-v}{nm}\times v}\right\}\text{, will be}$$

the present worth required.

EXAM.

EXAMPLE.

What is the probability that *A* and *C* (aged 43, and 66) ſhall ſurvive *B* and *D*, (aged 54, and 76) and what is the value of a legacy of 25 £. dependent on that ſurvivorſhip.

Here $\frac{n}{3t} = \left(\frac{20}{3 \cdot 43} =\right) \frac{20}{129}$; $1 - \frac{20}{129} = \frac{109}{129}$; $\frac{109}{129} \times 20 = 16{,}899$; $\frac{n+t-v}{tn} = \frac{53}{860}$; $\frac{vv}{3} = \frac{120}{3}$; $\frac{53}{860} \times \frac{120}{3} = \left(\frac{530}{258} =\right) 2{,}054$; $16{,}899 - 2{,}054 = 14{,}845$; therefore $\frac{14{,}845}{2 \cdot 32} =\Big)$ 0,232, will be the probability required.

Again $\frac{n\mathfrak{F} - n\mathfrak{M}}{n} = 0{,}04735$; $n\mathfrak{M} + \frac{nn}{6rm} = 7{,}888$; $\overline{\mathfrak{P} - \frac{n}{6r}} \times \frac{n}{m} = 2{,}792$; $\mathfrak{D} + \frac{v}{6r} = 2{,}715$; $\frac{t+n-v}{nm} = \frac{53}{640}$; $\frac{53}{640} \times 10 = \frac{53}{64}$; $2{,}715 \times \frac{53}{64} = 2{,}249$; $7{,}888 + 2{,}792 - 2{,}249 = 8{,}431$; $\frac{8{,}431}{2 \times 43} = 0{,}09803$; $0{,}09803 + 0{,}04735 = 0{,}14538$; and $(0{,}14538 \times 25 =)$ 3,635, will be the preſent worth required.

COROL. I.

If *A* and *B* are of equal ages; then $n\mathfrak{F} = n\mathfrak{M}$, and $t = m$; whence the anſwers will become

$$\overline{1 - \frac{n}{3m} \times n - \frac{n+m-v}{mn} \times \frac{vv}{3}} \times \frac{1}{2m}, \text{ and}$$

$\frac{\mathfrak{P}}{2m}$

$$\frac{\mathfrak{P}}{2m} \times \left\{ \begin{array}{l} n\mathfrak{M} - \frac{nn}{6rm} + \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{n}{m} \\ \overline{-\mathfrak{P} - \frac{v}{6r}} \times \frac{m+n-v}{mn} \times nv \end{array} \right.$$

COROL. II.

If *B* and *C* are of equal ages; then $n\mathfrak{M} = \mathfrak{N}$, and $m = n$; whence the anſwers will become

$$\overline{1 - \frac{n}{3t}} \times \overline{n - \frac{n+t-v}{tn} \times \frac{vv}{3}} \times \frac{1}{2n}, \text{ and}$$

$$\mathfrak{P} \times \left\{ \begin{array}{l} \frac{n\mathfrak{F} - \mathfrak{N}}{n} \\ + \frac{1}{2t} \times \overline{\overline{\mathfrak{N} - \frac{n}{6r}} \times 2 - \overline{\mathfrak{P} - \frac{v}{6r}} \times \frac{t+n-v}{nn} \times v}. \end{array} \right.$$

COROL. III.

If *C* and *D* are of equal ages; then $\mathfrak{N} = \mathfrak{V}$, and $n = v$; whence the anſwers will become

$$\overline{1 - \frac{v}{3t}} \times \overline{v - \frac{t}{tv} \times \frac{vv}{3}} \times \frac{1}{2m}, \text{ or}$$

$$\overline{1 - \frac{v}{3t}} \times \overline{v - \frac{v}{3}} \times \frac{1}{2m}; \text{ that is}$$

$$\left(\overline{1 - \frac{v}{3t}} - \frac{1}{3} \times \frac{v}{2m}, \text{ or}\right) \overline{\frac{2}{3} - \frac{v}{3t}} \times \frac{v}{2m} =$$

$$\overline{2 - \frac{v}{t}} \times \frac{v}{6m}; \text{ and}$$

$$\mathfrak{P} \times \left\{ \begin{array}{l} \frac{v\mathfrak{F} - v\mathfrak{M}}{v} \\ + \frac{1}{2t} \times \overline{v\mathfrak{M} - \frac{vv}{6rm} - \overline{\mathfrak{P} - \frac{v}{6r}} \times \frac{t-v}{m}}. \end{array} \right.$$

COROL. IV.

If *A*, *B*, and *C*, are of equal ages; then $s\mathfrak{T} = s\mathfrak{M} = \mathfrak{N}$, and $t = m = n$; whence the anſwers will become

$$\left(\overline{1 - \tfrac{1}{3} \times n - \frac{2n - v}{nn} \times \frac{vv}{3}} \times \frac{1}{2n}, \text{ or}\right)$$

$$\overline{\tfrac{1}{3}n - \frac{2n - v}{nn} \times \frac{vv}{3}} \times \frac{1}{2n}; \text{ that is}$$

$$\left(\overline{2n - \overline{2n - v}} \times \frac{vv}{nn} \times \frac{1}{6n} \text{ or}\right) \tfrac{1}{3} - \overline{2n - v} \times \frac{vv}{6n^3}; \text{ and}$$

$$\frac{\mathfrak{P}}{2n} \times \overline{\mathfrak{N} - \frac{n}{6r} \times \overline{2 - \mathfrak{V}} - \frac{v}{6r} \times \frac{2n - v}{nn} \times v}.$$

COROL. V.

If *B*, *C*, and *D* are of equal ages; then $s\mathfrak{M} = \mathfrak{N} = \mathfrak{V}$, and $m = n = v$; whence the anſwers will become

$$\tfrac{1}{3} - \frac{v}{6t}; \text{ and } \mathfrak{P} \times \overline{\frac{v\mathfrak{T} - \mathfrak{V}}{v} + \frac{1}{2t} \times \mathfrak{V} - \frac{v}{6r} \times \frac{2v - t}{v}}.$$

QUESTION LXIX.

The probability that *A* and *B*, the 2 youngeſt, ſhall ſurvive *C* and *D* the two eldeſt; and the preſent value of an eſtate, or legacy, worth $\mathfrak{P}$ pounds, depending on that ſurvivorſhip, are required.

SOLU-

SOLUTION.

Firſt, The probability of the order $\overline{C\,D}$, *A*, *B*, is (by queſt. 58) $\frac{m}{2t} - \frac{n}{2t} + \frac{nn}{6mt} - \frac{vv}{6nt} + \frac{v^3}{12nmt}$; and that, of the order $\overline{C\,D}$, *B*, *A* is (by queſt. 61) $1 - \frac{m}{2t} - \frac{n}{2m} + \frac{nn}{6tm} - \frac{vv}{6nm} + \frac{v^3}{12ntm}$; therefore their ſum ($1 - \frac{n}{2t} - \frac{n}{2m} + \frac{nn}{3tm} - \frac{vv}{6nt} - \frac{vv}{6nm} + \frac{v^3}{6ntm}$, or) $1 - \overline{\frac{3}{6t} + \frac{3}{6m} - \frac{2n}{6tm}} \times n - \overline{\frac{1}{t} + \frac{1}{m} - \frac{v}{tm}} \times \frac{vv}{6n}$; that is

($1 - \frac{3m+3t-2n}{6tm} \times n - \frac{m+t-v}{6tm} \times \frac{vv}{n}$, or)

$1 - \overline{\overline{\overline{m+t}\times 3 - 2n} \times n + \overline{m+t-v} \times \frac{vv}{n}} \times \frac{1}{6tm}$,

will be the probability required.

Secondly; the preſent worth, depending on the order $\overline{C\,D}$, *A*, *B*, (is by queſt. 58)

$\mathfrak{P} \times \left\{\begin{array}{l} \frac{\mathfrak{M}-\mathfrak{D}}{t} + \overline{\mathfrak{P} - \frac{n}{6r}} \times \frac{n}{2tm} \\ - \overline{\mathfrak{P} - \frac{v}{6r}} \times \frac{2m-v}{2n} \times \frac{v}{2tm} \end{array}\right.$; and that depending on the order $\overline{C\,D}$, *B*, *A*, is (by queſt. 61)

$\mathfrak{P} \times$

$$\overline{\mathfrak{P}\times\frac{m\mathfrak{F}-\mathfrak{N}}{m}+\overline{\mathfrak{N}-\frac{n}{6r}}\times\frac{n}{2tm}-\overline{\mathfrak{D}-\frac{v}{6r}}\times\overline{1+\frac{v}{2t}}\times\frac{v}{2nm}}\text{; therefore}$$

their ſum ($\mathfrak{P}\times\left\{\begin{array}{l}\frac{\mathfrak{M}-\mathfrak{N}}{t}+\overline{\mathfrak{N}-\frac{n}{6r}}+\frac{n}{2tm}-\overline{\mathfrak{D}-\frac{v}{6r}}\times\frac{2m-v}{4tmn}\times v\\+\frac{m\mathfrak{F}-\mathfrak{N}}{m}+\overline{\mathfrak{N}-\frac{n}{6r}}\times\frac{n}{2tm}-\overline{\mathfrak{D}-\frac{v}{6r}}\times\frac{2t-v}{4tmn}\times v\end{array}\right\}$, or)

$\frac{\mathfrak{P}}{tm}\times\left\{\begin{array}{l}\overline{\mathfrak{M}-\mathfrak{N}}\times m+\overline{m\mathfrak{F}-\mathfrak{N}}\times t\\+\overline{\mathfrak{N}-\frac{n}{6r}}\times n-\overline{\mathfrak{D}-\frac{v}{6r}}\times\frac{m+t-v}{2n}\times v\end{array}\right\}$ will be the preſent worth required.

EXAMPLE.

D (aged 76) has jointured his wife *C* (aged 66) in an eſtate of 1 £. per ann. which, after both their deaths, will come to *B* (aged 54) in right of his wife *A* (aged 43) the daughter of *D*, if both *B* and *A* be then alive: what is the probability of *B*'s becoming poſſeſſed of the eſtate; and what is the preſent worth of his intereſt therein?

Here $\overline{m+t}\times 3-2n=185$; $185\times 20=3700$; $\overline{m+t-v}=65$; $\frac{vv}{n}=5$; $65\times 5=325$; $3700+325=4025$; and $\frac{4025}{6.32.43}=0,488$: Then $(1-0,488=)$ $0,512$ will be the probability required.

Again $\mathfrak{M}=10,757$; $\mathfrak{N}=7,673$; $\mathfrak{M}-\mathfrak{N}=3,084$; $3,084\times 32=98,688$; $m\mathfrak{F}=12,578$; $12,578-7,673=4,905$; $4,905\times 43=210,915$; $\mathfrak{N}-\frac{n}{6r}=4,468$; $4,468\times 20=89,360$; $\mathfrak{P}-\frac{v}{6r}=2,715$; $m+t-v=65$; $\frac{65\times 10}{40}=\frac{65}{4}$; $2,715\times\frac{65}{4}=44,119$:

And $98,688+210,915+89,360-44,119=354,844$;

Therefore $(354,844\times\frac{25}{32\cdot 43}=)$ $6,447$ will be the preſent worth required.

COROL. I.

If *A* and *B* are of equal ages; then $m\mathfrak{F}=\mathfrak{M}$, and $t=m$; whence the anſwers will become

$$1-\overline{6m-2n}\times n+\overline{2m-v}\times\frac{vv}{n}\times\frac{1}{6mm}, \text{ and}$$

$$\frac{\mathfrak{P}}{mm}\times\left\{\begin{array}{l}\overline{\mathfrak{M}-\mathfrak{N}}\times 2m+\overline{\mathfrak{N}-\frac{n}{6r}}\times n\\ -\overline{\mathfrak{P}-\frac{v}{6r}}\times\frac{2m-v}{2n}\times v.\end{array}\right.$$

COROL. II.

If *B* and *C* are of equal ages; then $\mathfrak{M} = \mathfrak{N}$, and $= n$; whence the anſwers will become

$$\overline{1-n+t}\times\overline{3-2n}\times n+\overline{n+t-v}\times\frac{vv}{n}\times\frac{1}{6tn},$$

or) $1-\overline{3t+n}\times n+\overline{n+t-v}\times\frac{vv}{n}\times\frac{1}{6tn}$; and

$$\frac{\mathfrak{P}}{tn}\times\left\{\begin{array}{l}\overline{{}^{n}\mathfrak{F}-\mathfrak{N}}\times t+\overline{\mathfrak{N}-\frac{n}{6r}}\times n\\ -\overline{\mathfrak{V}-\frac{v}{6r}}+\frac{n+t-v}{2n}\times v.\end{array}\right.$$

COROL. III.

If *C* and *D* are of equal ages; then $\mathfrak{N} = \mathfrak{V}$, and $= v$; whence the anſwers will become

$$\overline{1-m+t}\times\overline{3-2v}\times v+\overline{m+t-v}\times v\times\frac{1}{6tm},$$

or) $1-\overline{m+t}\times\overline{4-3v}\times\frac{v}{6tm}$; and

$$\frac{\mathfrak{P}}{tm}\times\left\{\begin{array}{l}\overline{\mathfrak{M}-\mathfrak{V}}\times m+\overline{{}^{m}\mathfrak{F}-\mathfrak{V}}\times t\\ +\overline{\mathfrak{V}-\frac{v}{6r}}\times\frac{3v-m-t}{2}.\end{array}\right.$$

COROL. IV.

If *A*, *B*, and *C*, are of equal ages; then $\mathfrak{T}=\mathfrak{M}=\mathfrak{N}$, and $t=m=n$; whence the anſwers will become

$\overline{1-4nn+\overline{2n-v}}\times\frac{vv}{n}\times\frac{1}{6nn}$,

or $\frac{1}{3}-\overline{2n-v}\times\frac{vv}{6nnn}$; and

$\frac{\mathfrak{P}}{nn}\times\overline{\mathfrak{P}-\frac{n}{6r}\times n-\mathfrak{V}-\frac{v}{6r}\times\frac{2n-v}{2n}\times v}$.

COROL. V.

If *B*, *C*, and *D*, are of equal ages; then $\mathfrak{M}=\mathfrak{N}=\mathfrak{V}$, and $m=n=v$; whence the anſwers will become

$\overline{1-4t+v}\times\frac{1}{6t}$, and

$\frac{\mathfrak{P}}{tv}\times\overline{v\mathfrak{T}-\mathfrak{V}\times t+\mathfrak{V}-\frac{v}{6r}\times v-\frac{t}{2}}$.

QUESTION LXX.

The reſpective ages of four perſons, *A*, *B*, *C*, and *D*, being given; it is required to find the probability, that any one of them ſhall ſurvive the other three?

SOLUTION.

Let t, m, n, and v be the ſeveral complements of the lives of *A*, *B*, *C* and *D*; and let the orders of dying (whoſe probabilities were found in the 50th and eleven following queſtions) be denoted by $\overline{AB}$, C, D, or ſimilar expreſſions, as in the ſix preceding queſtions.

CASE I.

If the ſurvivor, D, be elder than the three perſons to be ſurvived.

This ſurvivorſhip will take place, when the four perſons die, in either of the following orders, viz. $\overline{AB}$, C, D; $\overline{AC}$, B, D; or $\overline{BC}$, A, D:

Now, the probability that the four perſons ſhall die in the order,

$\overline{AB}$, C, D is (by queſt. 50) $\frac{v^3}{12tnm}$,

$\overline{AC}$, B, D is (by queſt. 51) $\frac{v^3}{12tnm}$,

$\overline{BC}$, A, D is (by queſt 52) $\frac{v^3}{12tnm}$;

The ſum of which, viz. $\frac{v^3}{4tnm}$, will be the probability required.

EXAMPLE.

If the ages are 43, 54, 66, and 76; then their complements will be 43, 32, 20, and 10; and $\frac{10 \cdot 10 \cdot 10}{4 \cdot 43 \cdot 20 \cdot 32} = \left(\frac{25}{2.43.32} =\right)$ 0,009084 will be the probability required.

COROL. I.

If A and B are of equal ages; then $t = m$ and $\frac{v^3}{4nmm}$ will be the anſwer.

COROL.

COROL. II.

If *B* and *C* are of equal ages; then $m = n$; and $\frac{v^3}{4tnn}$ will be the anſwer.

COROL. III.

If *C* and *D* are of equal ages; then $n = v$; and $\frac{vv}{4tm}$ will be the anſwer.

COROL. IV.

If *A*, *B* and *C* are of equal ages; then $t = m = n$; an $\frac{v^3}{4n^3}$ will be the anſwer.

COROL. V.

If *B*, *C*, and *D* are of equal ages; then $m = n = v$; an $\frac{v}{4t}$ will be the anſwer.

COROL. VI.

If the four lives are of equal ages; then $\frac{1}{4}$ will be th probability required.

CASE II.

If *C*, the eldeſt but one, is to ſurvive the other three This ſurvivorſhip will take place, when they die in th orders $\overline{AB}$, *D*, *C*; $\overline{AD}$, *B*, *C*; and $\overline{BD}$, *A*, *C*:

Now the probability of their dying in the order

$$\left.\begin{array}{l}\overline{AB},\ D,\ C\\ \overline{AD},\ B,\ C\\ \overline{BD},\ A,\ C\end{array}\right\}\text{is (by queſt.}\left\{\begin{array}{ll}53) & \frac{vv}{3mt}-\frac{v^3}{4tnm},\\ 54) & \frac{nn}{6tm}-\frac{vv}{6mt}+\frac{v^3}{12tnm},\\ 55) & \frac{nn}{6tm}-\frac{vv}{6mt}+\frac{v^3}{12tnm};\end{array}\right.$$

The ſum, viz. $\left(\frac{nn}{3tm}-\frac{v^3}{12tnm}\text{ or}\right)\ \overline{nn-\frac{v^3}{4n}}\times\frac{1}{3tm}$, will be the probability required.

EXAMPLE.

The ages being as before, $nn = 400$; $\frac{v^3}{4n} = \frac{25}{2}$; $400 - \frac{25}{2} = 387{,}5$; Th. $\left(\frac{387.5}{3.32.43} =\right) 0{,}0939$ will be the probability required.

COROL. I.

If A and B are of equal ages; then $t = m$; and the anſwer will be $\overline{nn - \frac{v^3}{4n}} \times \frac{1}{3mm}$.

COROL. II.

If B and C are of equal ages; then $m = n$; and the anſwer will be $\overline{nn - \frac{v^3}{4n}} \times \frac{1}{3tn}$.

COROL. III

If *C* and *D* are of equal ages; then $n = v$; and the answer will be $(vv - \frac{vv}{4}) \times \frac{1}{3tm}$ or) $\frac{3vv}{4} \times \frac{1}{3tm}$; That is $\frac{vv}{4tm}$.

COROL. IV.

If *A*, *B*, and *C*, are of equal ages; then $t = m = n$; and the answer will become $(nn - \frac{v^3}{4n}) \times \frac{1}{3nn}$ or) $\frac{1}{3} - \frac{v^3}{12n^3}$.

COROL. V.

If *B*, *C*, and *D*, are of equal ages; them $m = n = v$; and the answer will be $\frac{v}{4t}$.

CASE III

If *B*, the youngeſt but one, is to ſurvive the other three.

This ſurvivorſhip will take place, when they die in the orders, $\overline{AC}$, *D*, *B*; $\overline{AD}$, *C*, *B*; and $\overline{CD}$, *A*, *B*:

Now the probability of their dying in the order

$$\left.\begin{array}{l} \overline{AC}, D, B \\ \overline{AD}, C, B \\ \overline{CD}, A, B \end{array}\right\} \text{is (by queſt.} \left\{\begin{array}{ll} 56) & \frac{vv}{3tn} - \frac{v^3}{4tnm}, \\ 57) & \frac{n}{2t} - \frac{nn}{3tm} - \frac{vv}{6tn} + \frac{v^3}{12tnm}, \\ 58) & \frac{m}{2t} - \frac{n}{2t} + \frac{nn}{6tm} - \frac{vv}{6tn} + \frac{v^3}{12tnm}; \end{array}\right.$$

The ſum, viz. $\left(\frac{m}{2t} - \frac{nn}{6tm} - \frac{v^3}{12tnm}\right.$ or) $\overline{3m - \frac{nn}{m} - \frac{v^3}{2nm}} \times \frac{1}{6t}$;

That is, $\overline{3m - nn + \frac{v^3}{2n}} \times \frac{1}{m} \times \frac{1}{6t}$, will be the probability required.

EXAMPLE.

The ages being as before, $3m = 96$; $nn = 400$; $\frac{v^3}{2n} = 25$; $400 + 25 = 425$; $\frac{425}{32} = 13,281$; $96 - 13,281 = 82,719$;

Therefore $\left(\frac{82,719}{0 \times 43} \doteq\right)$ 0,321 will be the probability required.

COROL. I.

If *A* and *B* are of equal ages; then $t = m$; and the anſwer will become $\left(\overline{3m - m + \frac{v^3}{2n}} \times \frac{1}{m} \times \frac{1}{6m}, \text{ or}\right)$ $\overline{\frac{1}{2} - nn + \frac{v^3}{2n}} \times \frac{1}{6mm}$.

COROL. II.

If *B* and *C* are of equal ages; then $m = n$; and the anſwer will become, $3n - nn + \frac{v^3}{2n} \times \frac{1}{n} \times \frac{1}{6t}$.

COROL. III.

If *C* and *D* are of equal ages; then $n = v$; and the anſwer will become

$\left(\overline{3m - vv + \frac{vv}{2}} \times \frac{1}{m} \times \frac{1}{6t} \text{ or}\right)$ $\overline{3m - \frac{3vv}{2m}} \times \frac{2}{6t}$.

COROL. IV.

If *A*, *B*, and *C*, are of equal ages; then $t = m = n$; and the anſwer will be $\left(\frac{1}{2} - nn + \frac{v^3}{2n} \times \frac{1}{6nn}, \text{ or}\right)$ $\frac{1}{2} - \frac{1}{6} - \frac{v^3}{12n^3}$; that is $\frac{1}{3} - \frac{v^3}{12n^3}$.

COROL. V.

If B, C, and D, are of equal ages; then $m=n=v$; and the anſwer will be $(3v - \frac{3v}{2}) \times \frac{1}{6t}$, or

$$\left(\frac{3v}{2} \times \frac{1}{6t} =\right) \frac{v}{4t}.$$

CASE IV.

If A, the youngeſt, is to ſurvive the other three.

This ſurvivorſhip will take place, when they die in the orders $\overline{BC}$, D, A; $\overline{BD}$, C, A; and $\overline{CD}$, B, A:

Now the probability of their dying in the order

$$\left.\begin{array}{l}\overline{BC}, D, A \\ \overline{BD}, C, A \\ \overline{CD}, B, A\end{array}\right\} \text{is (by queſt.} \left\{\begin{array}{ll}59) & \frac{vv}{3nm} - \frac{v^3}{4tmn}, \\ 60) & \frac{n}{2m} - \frac{nn}{3tm} - \frac{vv}{6nm} + \frac{v^3}{12tmn}, \\ 61) & 1 - \frac{m}{2t} - \frac{n}{2m} + \frac{nn}{6tm} - \frac{vv}{6nm} + \frac{v^3}{12tnm}.\end{array}\right.$$

The ſum, viz. $\left(1 - \frac{m}{2t} - \frac{nn}{6tm} - \frac{v^3}{12tmn}\right.$, or$\left.\right)$ $1 - \overline{3m - \frac{nn}{m} - \frac{v^3}{2mn}} \times \frac{1}{6t}$; that is, $1 - \overline{3m - nn + \frac{v^3}{2n}} \times \frac{1}{m} \times \frac{1}{6t}$, will be the probability required.

EXAMPLE.

The ages being as before, $3m = 96$; $nn = 400$; $\frac{v^3}{2n} = 25$; $\overline{400+25} \times \frac{1}{32} = 13,281$; $96 + 13,281 = 109,281$; and $\frac{109,281}{6 \times 43} = 0,424$;

Therefore $(1 - 0,424 =)$ $0,576$ will be the probability required.

COROL. I.

If *A* and *B* are of equal ages; then $t = m$; and the anſwer will be $(1 - 3m - nn + \frac{v^3}{2n} \times \frac{1}{m} \times \frac{1}{6m}$, or$)$ $\overline{\frac{1}{2} - nn + \frac{v^3}{2n}} \times \frac{1}{6mm}$.

COROL. II.

If *B* and *C* are of equal ages; then $m = n$; and the anſwer will be $1 - 3n - nn + \frac{v^3}{2n} \times \frac{1}{n} \times \frac{1}{6t}$.

COROL. III.

If *C* and *D* are of equal ages; then $n = v$; and the anſwer will be $(1 - 3m - vv + \frac{vv}{2} \times \frac{1}{m} \times \frac{1}{6t}$, or$)$ $\overline{1 - 3m + \frac{3vv}{2m}} \times \frac{1}{6t}$.

COROL. IV.

If *A*, *B*, and *C*, are of equal ages; then $t = m = n$; and the anſwer will be $(\frac{1}{2} - nn + \frac{v^3}{2n} \times \frac{1}{6nn}$ or) $\frac{1}{3} - \frac{v^3}{12n^3}$.

COROL. V.

If *B*, *C*, and *D*, are of equal ages; then $m = n = v$; and the anſwer will be $(1 - 3v + \frac{3v}{2} \times \frac{1}{6t}$, or) $1 - \frac{9v}{12t}$; that is $1 - \frac{3v}{4t}$.

The anſwers, to the above four caſes, when the lives are unequal, are as below.

Survived	Survivor.	Solution.
A, *B*, *C*.	*D*.	$\frac{v^3}{4tmn}$;
A, *B*, *D*.	*C*.	$\frac{nn}{3tm} - \frac{v^3}{12tmn}$;
A, *C*, *D*.	*B*	$\frac{m}{2t} - \frac{nn}{6tm} - \frac{v^3}{12tmn}$;
B, *C*, *D*.	*A*.	$1 - \frac{m}{2t} - \frac{nn}{6tm} - \frac{v^3}{12tmn}$;

The ſum of which is unity; and the ſolutions agree with thoſe given, in Mr. *De Moivre*'s treatiſe of annuities on lives.

QUESTION LXXI.

The reſpective ages of four perſons, *A*, *B*, *C*, and *D*, being given; it is required to find the preſent value of an eſtate, or legacy, worth $\mathfrak{P}$ pounds, which is to become the property of any one of them, if he ſurvive the other three.

SOLUTION.

Let the complements of life, the values of ſingle lives, and orders of ſurvivorſhip, be repreſented by the ſame ſymbols, as in the preceding queſtions.

CASE I.

If the ſurvivor, *D*, be elder than the three perſons to be ſurvived.

Then the preſent worth of an eſtate, or legacy, dependent on the order,

$\overline{AB}$, *C*, *D*, is (by queſt. 50) $\overline{\mathfrak{P} - \frac{v}{6r}} \times \frac{vv\mathfrak{P}}{4mtn}$,

$\overline{AC}$, *B*, *D*, is (by queſt. 51) $\overline{\mathfrak{P} - \frac{v}{6r}} \times \frac{vv\mathfrak{P}}{4mtn}$,

$\overline{BC}$, *A*, *D*, is (by queſt. 52) $\overline{\mathfrak{P} - \frac{v}{6r}} \times \frac{vv\mathfrak{P}}{4mtn}$;

Therefore their ſum, viz. $\overline{\mathfrak{P} - \frac{v}{6r}} \times \frac{3vv\mathfrak{P}}{4mtn}$, will be the preſent worth required.

EXAMPLE.

D (aged 76) will receive a legacy of 25£. if he ſurvives *A*, *B*, and *C* (aged 43, 54, and 66) what is the preſent value of his intereſt, in that legacy?

Here $\mathfrak{D} - \frac{v}{6r} = 2{,}715$; $\frac{3vv\mathfrak{P}}{4mtn} = \left(\frac{3\cdot10\cdot10\cdot25}{4\cdot3^2\cdot43\cdot20} = \right)\frac{3\cdot5\cdot25}{4\cdot3^2\cdot43}$;

Therefore $\left(2{,}715 \times \frac{3\cdot5\cdot25}{4\cdot3^2\cdot43} = \right) 0{,}185$ will be the preſent worth required.

COROL. I.

If *A* and *B* are of equal ages; then $t = m$; and the anſwer will be $\mathfrak{D} - \frac{v}{6r} \times \frac{3vv\mathfrak{P}}{4mmn}$.

COROL. II.

If *B* and *C* are of equal ages; then $m = n$; and the anſwer will be $\mathfrak{D} - \frac{v}{6r} \times \frac{3vv\mathfrak{P}}{4tnn}$.

COROL. III.

If *C* and *D* are of equal ages; then $n = v$; and the anſwer will be $\mathfrak{D} - \frac{v}{6r} \times \frac{3v\mathfrak{P}}{4mt}$.

COROL. IV.

If *A*, *B*, and *C*, are of equal ages; then $t = m = n$; and the anſwer will be $\mathfrak{D} - \frac{v}{6r} \times \frac{3vv\mathfrak{P}}{4nnn}$.

COROL. V.

If *B*, *C*, and *D*, are of equal ages; then $m = n = v$; and the anſwer will be $\mathfrak{A} - \frac{v}{6r} \times \frac{3\mathfrak{P}}{4t}$.

COROL. VI.

If the four perſons are of equal ages; the anſwer will be $\mathfrak{A} - \frac{v}{6r} \times \frac{3\mathfrak{P}}{4v}$.

CASE II.

If *C*, the eldeſt but one, is to ſurvive the other three.

Then the preſent worth of an eſtate, or legacy, dependent on the order,

$$\left.\begin{array}{l} \overline{A\,B},\ D,\ C \\ \overline{A\,D},\ B,\ C \\ \overline{B\,D},\ A,\ C \end{array}\right\} \text{is (by queſt.)} \left\{\begin{array}{ll} 53) & \overline{\frac{v\mathfrak{N}v}{2} + \frac{2n-3v}{6rn} \times \frac{vv}{2}} \times \frac{\mathfrak{P}}{2mt}, \\ 54) & \overline{\mathfrak{N} - \frac{n}{6r}} \times \overline{n - \mathfrak{D} - \frac{v}{6r}} \times \frac{2n-v}{2n} \times v \times \frac{\mathfrak{P}}{2mt}, \\ 55) & \overline{\mathfrak{N} - \frac{n}{6r}} \times \overline{n - \mathfrak{C} - \frac{v}{6r}} \times \frac{2n-v}{2n} \times v \times \frac{\mathfrak{P}}{2mt}; \end{array}\right.$$

The ſum $\frac{\mathfrak{P}}{2mt} \times \left\{\begin{array}{l} \frac{v\mathfrak{N}v}{2} + \frac{2n-3v}{6rn} \times \frac{vv}{2} \\ + \overline{\mathfrak{N} - \frac{n}{6r}} \times 2n - \mathfrak{D} \times \frac{2n-v}{n} \times v + \frac{2n-v}{2n} \times \frac{v}{6r} \times 2v \end{array}\right\}$

or, $\frac{\mathfrak{P}}{2mt} \left\{\begin{array}{l} \frac{v\mathfrak{N}v}{2} + \overline{\mathfrak{N} - \frac{n}{6r} \times 2n + \frac{2n-3v}{2n} \times \frac{v}{6r}} \times v \\ - \mathfrak{D} \times \frac{2n-v}{n} \times v + \frac{4n-2v}{2n} \times \frac{v}{6r} \times v \end{array}\right\}$; that is,

$$\frac{\mathfrak{P}}{2mt}\left\{\begin{array}{l}\frac{v\mathfrak{R}v}{2}+\overline{\mathfrak{R}-\frac{n}{6r}}\times 2n \\ -\ \mathfrak{D}\times\frac{2n-v}{n}\times v+\frac{6n-5v}{2n}\times\frac{vv}{6r},\end{array}\right\}$$

will be the preſent worth required.

EXAMPLE.

The ages being as before; $\frac{v\mathfrak{R}v}{2}=31,075$;

$\overline{\mathfrak{R}-\frac{n}{6r}}=4,468$; $4,468\times 2\times 20=178,720$; $\mathfrak{D}=4,318$; $\frac{2n-v}{n}\times v=\left(\frac{30\ 10}{20}=\right)15$; $4,318\times 15=64,770$; $\frac{6n-5v}{2n}\ v=\left(\frac{70.10}{2.20}=\right)17,5$; $\frac{v}{6r}=1,603$; $1,603\times 17,5=28,052$; $31,075+178,720-64,770+28,052=173,077$; and $\frac{\mathfrak{P}}{2.m.t}=\frac{25}{2.32.43}$;

Therefore $\left(173,077\times\frac{25}{2.32.43}=\right)$ 1,572 will be the preſent worth required.

COROL. I.

If *A*, and *B*, are of equal ages; then $t=m$; and the anſwer will be

$$\frac{\mathfrak{P}}{2mm}\times\left\{\begin{array}{l}\frac{v\mathfrak{R}v}{2}+\overline{\mathfrak{R}-\frac{n}{6r}}\times 2n \\ -\ \mathfrak{D}\,v\times\frac{2n-v}{n}+\frac{6n-5v}{2n}+\frac{vv}{6r}\end{array}\right\}$$

COROL. II.

If *B* and *C* are of equal ages; then $m = n$; and the anſwer will be

$$\frac{\mathfrak{P}}{2tn} \times \left\{ \begin{array}{l} \frac{{}^{v}\mathfrak{N}v}{2} + \overline{\mathfrak{N} - \frac{n}{6r}} \times 2n \\ - \mathfrak{D}v \times \frac{2n-v}{n} + \frac{6n-5v}{2n} \times \frac{vv}{6r}. \end{array} \right\}$$

COROL. III.

If *C* and *D* are of equal ages; then $\mathfrak{N} = {}^{v}\mathfrak{N} = \mathfrak{D}$, and $n = v$; whence the anſwer will become

$$\left(\frac{\mathfrak{P}}{2tm} \times \left\{ \begin{array}{l} \frac{\mathfrak{D}v}{2} + \overline{\mathfrak{D} - \frac{v}{6r}} \times 2v \\ - \mathfrak{C}v \times \frac{2v-v}{v} + \frac{6v-5v}{2v} \times \frac{vv}{6r}, \text{ or} \right) \end{array} \right\}$$

$$\frac{\mathfrak{P}}{2tm} \times \overline{\frac{\mathfrak{D}v}{2} + 2\mathfrak{C}v - \frac{2vv}{6r} - \mathfrak{P}v + \frac{1}{2} \times \frac{vv}{6r}}; \text{ that is,}$$

$$\left(\frac{\mathfrak{P}}{2tm} \times \overline{\frac{3\mathfrak{D}v}{2} - \frac{3vv}{2.6r}}, \text{ or} \right) \frac{3\mathfrak{P}v}{4tm} \times \overline{\mathfrak{D} - \frac{v}{6r}}.$$

COROL. IV.

If *A*, *B*, and *C*, are of equal ages; then $t = m = n$; and the anſwer will be

$$\frac{\mathfrak{D}}{2nn} \times \left\{ \begin{array}{l} \frac{{}^{v}\mathfrak{N}v}{2} + \overline{\mathfrak{N} - \frac{n}{6r}} \times 2n \\ - \mathfrak{D}v \times \frac{2n-v}{n} + \frac{6n-5v}{2n} \times \frac{vv}{6r}. \end{array} \right\}$$

COROL. V.

If *B*, *C*, and *D*, are of equal ages; then $\mathfrak{N} = v\mathfrak{N} = \mathfrak{A}$, and $m = n = v$; whence the anſwer will be $\frac{3\mathfrak{P}}{4t} \times \mathfrak{D} - \frac{v}{6r}$.

CASE III.

If *B*, the youngeſt but one, is to ſurvive the other three.
Then the preſent worth of an eſtate, or legacy, dependent on the order,

$$\left.\begin{array}{l} \overline{AC}, D, B \\ \overline{AD}, C, B \\ \overline{CD}, A, B \end{array}\right\} \text{is (by queſt.)} \left\{\begin{array}{ll} 56) & \frac{\mathfrak{P}}{t} \times \overline{\overline{v\mathfrak{M} - \frac{vv}{6rm}} \times \frac{1}{2} - \overline{v\mathfrak{N} - \frac{vv}{6rn}} \times \frac{2m-v}{4m} + \frac{vv}{6rn} \times \frac{2n-2v}{4m}}, \\ 57) & \frac{\mathfrak{P}}{t} \times \overline{\overline{n\mathfrak{M} - \frac{nn}{6rm}} \times \frac{1}{2} - \overline{\mathfrak{A} - \frac{v}{6r}} \times \frac{\overline{2n-v} \times v}{4mn}}, \\ 58) & \frac{\mathfrak{P}}{t} \times \overline{\overline{\mathfrak{M} - \mathfrak{N} + \mathfrak{N} - \frac{n}{6r}} \times \frac{n}{2m} - \overline{\mathfrak{D} - \frac{v}{6r}} \times \frac{\overline{2m-v} \times v}{4mn}}; \end{array}\right.$$

Which preſent worths are to be added together;

Now $-\overline{v\mathfrak{P} - \frac{vv}{6rn}} \times \frac{2m-v}{4m} = -v\mathfrak{P} \times \frac{2m-v}{4m} + \frac{vv}{6rn} \times \frac{2m-v}{4m}$,

$-\overline{\mathfrak{D} - \frac{v}{6r}} \times \frac{\overline{2m-v} \times v}{4mn} = -\mathfrak{C} \times \frac{\overline{2m-v} \times v}{4mn} + \frac{vv}{6rn} \times \frac{2m-v}{4m}$,

$-\overline{\mathfrak{D} - \frac{v}{6r}} \times \frac{\overline{2m-v} \times v}{4mn} = -\mathfrak{D} \times \frac{\overline{2m-v} \times v}{4mn} + \frac{vv}{6rn} \times \frac{2m-v}{4m}$,

And $+\frac{vv}{6rn} \times \frac{2n-2v}{4m} = +\frac{vv}{6rn} \times \frac{2n-2v}{4m}$;

Therefore the ſum of the four expreſſions, laſt mentioned, will be

$(-v\mathfrak{P} \times \frac{2m-v}{4m} - v\mathfrak{D} \times \frac{2m-v}{2mn} + \frac{vv}{6rn} \times \frac{6m+2n-5v}{4m}$, or)

$-v\mathfrak{P} \times \frac{2m-v}{4m} - v\mathfrak{D} \times \frac{2m-v}{2mn} + \frac{vv}{6rm} \times \frac{6m+2n-5v}{4n}$;

Alſo $\overline{{}^{v}\mathfrak{M} - \frac{vv}{6rm}} \times \frac{1}{2} = \frac{{}^{v}\mathfrak{M}}{2} - \frac{vv}{6rm} \times \frac{1}{2}$,

$\overline{{}^{n}\mathfrak{M} - \frac{nn}{6rm}} \times \frac{1}{2} = \frac{{}^{n}\mathfrak{M}}{2} - \frac{nn}{6rm} \times \frac{1}{2}$,

$\overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{n}{2m} = \frac{n\mathfrak{N}}{2m} - \frac{nn}{6rm} \times \frac{1}{2}$;

Therefore the ſum of the three expreſſions, laſt mentioned, will be

$$\left(\frac{{}^{v}\mathfrak{M} + {}^{n}\mathfrak{M} + \frac{n}{m}\mathfrak{N}}{2} - \frac{vv}{6rm} \times \frac{1}{2} - \frac{nn}{6rm}, \text{ or}\right)$$

$$\frac{{}^{v}\mathfrak{M} + {}^{n}\mathfrak{M}}{2} + \overline{\frac{\mathfrak{N}}{2} - \frac{n}{6r}} \times \frac{n}{m} - \frac{vv}{6rm} \times \frac{1}{2}:$$

But $\frac{vv}{6rm} \times \frac{6m+2n-5v}{4n} - \frac{vv}{6rm} \times \frac{1}{2} =$

$$\left(\frac{vv}{6rm} \times \frac{6m-5v}{4n}, \text{ or}\right) \frac{v}{6r} \times \frac{\overline{6m-5v} \times v}{4nm}.$$

Therefore the value required will be

$$\left(\frac{\mathfrak{P}}{t} \times \left\{ \begin{array}{l} \mathfrak{M} - \mathfrak{N} + \frac{1}{2} \times \overline{v\boldsymbol{\mathfrak{M}} + n\boldsymbol{\mathfrak{M}} + \frac{\mathfrak{N}}{2} - \frac{n}{6r}} \times \frac{n}{m} \\ - v\mathfrak{N} \times \frac{2m-v}{4m} - v\mathfrak{T} \times \frac{2m-v}{2mn} + \frac{v}{6r} \times \frac{\overline{6m-5v} \times v}{4nm} \end{array} \right\}, \text{ or}\right)$$

$$\frac{\mathfrak{P}}{t} \times \left\{ \begin{array}{l} \mathfrak{M} - \mathfrak{N} + \frac{1}{2} \times \overline{v\mathfrak{M} + n\mathfrak{M}} \\ + \frac{1}{m} \times \left\{ \begin{array}{l} \frac{1}{2}\mathfrak{N} - \frac{n}{6r} \times n - v\mathfrak{N} \times \frac{2m-v}{4} \\ - \frac{v}{2n} \times \mathfrak{D} \times \overline{2m - v - \frac{v}{6r}} \times \frac{6m-5v}{4}. \end{array} \right. \end{array} \right.$$

EXAMPLE.

If the ages be as before; then $\boldsymbol{\mathfrak{M}} - \boldsymbol{\mathfrak{N}} = (10{,}757 - 7{,}673 =)\ 3{,}084$; $\frac{1}{2} \times \overline{v\mathfrak{M} + n\boldsymbol{\mathfrak{M}}}$ $= (\frac{1}{2} \times 6{,}925 + 9{,}891 =)\ 8{,}408$; $\frac{1}{2}\boldsymbol{\mathfrak{N}} = 3{,}836$; $\frac{n}{6r} = 3{,}205$; $\overline{3{,}836 - 3{,}205} \times 20$

= 12,620;

$= 12{,}620$; $v\mathfrak{P} = 6{,}215$; $\frac{2m-v}{4} = \left(\frac{54}{4} =\right) \frac{27}{2}$; $6{,}215 \times \frac{27}{2} = 83{,}902$; $\mathfrak{D} = 4{,}318$; $2m - v = 54$; $4{,}318 \times 54 = 233{,}172$; $\frac{v}{6r} = 1{,}603$; $\frac{6m-5v}{2} = \left(\frac{192-50}{2} =\right) 71$; $1{,}603 \times 71 = 113{,}813$; $\frac{v}{2n} = \left(\frac{10}{40} =\right) \frac{1}{4}$; $233{,}172 - 113{,}813 = 119{,}359$; $119{,}359 \times \frac{1}{4} = 29{,}840$; $12{,}620 - 83{,}902 - 29{,}840 = -101{,}172$; $-101{,}172 \times \frac{1}{32} = -3{,}160$; $3{,}084 + 8{,}408 - 3{,}160 = 8{,}332$; $\frac{\mathfrak{P}}{t} = \frac{25}{43}$;

Therefore $\left(8{,}332 \times \frac{25}{43} =\right) 4{,}844$, will be the preſent worth required.

COROL. I.

If *A* and *B* are of equal ages; then $t = m$; and the anſwer will be,

$$\frac{\mathfrak{P}}{m} \times \left\{ \begin{array}{l} \mathfrak{M} - \mathfrak{P} + \frac{1}{2} \times \overline{v\mathfrak{M} + n\mathfrak{M}} \\ + \frac{1}{m} \times \left\{ \begin{array}{l} \frac{1}{2}\mathfrak{P} - \frac{n}{6r} \times n - v\mathfrak{P} \times \frac{2m-v}{4} \\ - \frac{v}{2n} \times \mathfrak{D} \times \overline{2m-v} - \frac{v}{6r} \times \frac{6m-5v}{2}. \end{array} \right. \end{array} \right.$$

COROL. II.

If *B* and *C* are of equal ages; then $\mathfrak{M} = {}^{n}\mathfrak{M} = \mathfrak{N}$, ${}^{v}\mathfrak{M} = {}^{v}\mathfrak{N}$, and $m = n$; whence the anſwer will be,

$$\frac{\mathfrak{P}}{t} \times \begin{cases} \frac{1}{2} \times \overline{{}^{v}\mathfrak{N} + \mathfrak{N}} \\ + \frac{1}{n} \times \begin{cases} \overline{\frac{1}{2}\mathfrak{N} - \frac{n}{6r}} \times n - {}^{v}\mathfrak{N} \times \frac{2n-v}{4} \\ - \frac{v}{2n} \times \mathfrak{P} \times \overline{2n - v - \frac{v}{6r} \times \frac{6n-5v}{2}}. \end{cases} \end{cases}$$

COROL. III.

If *C* and *D* are of equal ages; then $\mathfrak{N} = {}^{v}\mathfrak{P} = \mathfrak{D}$, and $n = v$; whence the anſwer will be,

$$\frac{\mathfrak{P}}{t} \times \begin{cases} \mathfrak{M} - \mathfrak{D} + \frac{1}{2} \times \overline{{}^{v}\mathfrak{M} + {}^{v}\mathfrak{M}} \\ + \frac{1}{m} \times \begin{cases} \overline{\frac{1}{2}\mathfrak{D} - \frac{v}{6r}} \times v - \mathfrak{D} \times \frac{2m-v}{4} \\ - \frac{1}{2} \times \mathfrak{A} \times \overline{2m - v - \frac{v}{6r} \times \frac{6m-5v}{2}}. \end{cases} \end{cases}$$

Now $\mathfrak{P} \times \frac{v}{2} - \mathfrak{P} \times \frac{2m-v}{4} - \mathfrak{A} \times \frac{2m-v}{2} = -\mathfrak{P} \times \frac{6m-5v}{4}$;

And $\frac{1}{2} \times \frac{v}{6r} \times \frac{6m-5v}{2} - \frac{v}{6r} \times v = \frac{v}{6r} \times \frac{6m-9v}{4}$:

Therefore the anſwer will become

$$\left(\frac{\mathfrak{P}}{t} \times \overline{\mathfrak{M} - \mathfrak{P} + {}^{v}\mathfrak{M}} - \frac{1}{m} \times \mathfrak{P} \times \frac{6m-5v}{4} - \frac{v}{6r} \times \frac{6m-9v}{4}, \text{ or}\right)$$

$$\frac{\mathfrak{P}}{t} \times \mathfrak{M} - \mathfrak{P} + {}^{v}\mathfrak{M} - \frac{1}{4m} \times \mathfrak{P} - \frac{v}{6r} \times \overline{6m-5v} + \frac{v}{6r} \times 4v.$$

COROL. IV.

If *A*, *B*, and *C* are of equal ages; then $\mathfrak{M} = {}^{n}\mathfrak{M} = \mathfrak{N}$, ${}^{v}\mathfrak{M} = {}^{v}\mathfrak{N}$, and $t = m = n$; whence the anſwer will be,

$\mathfrak{P}$

$$\frac{\mathfrak{P}}{n}\times\left\{\begin{array}{l}\frac{1}{2}\times\overline{v\mathfrak{P}+\mathfrak{M}}\\ +\frac{1}{n}\times\left\{\begin{array}{l}\frac{1}{2}\overline{\mathfrak{P}-\frac{n}{6r}}\times n-v\mathfrak{N}\times\frac{2n-v}{4}\\ -\frac{v}{2n}\times\mathfrak{D}\times\overline{2n-v-\frac{v}{6r}}\times\frac{6n-5v}{2}.\end{array}\right.\end{array}\right.$$

COROL. V.

If *B*, *C*, and *D* and are of equal ages: then $\mathfrak{M}=n\mathfrak{M}=v\mathfrak{M}=\mathfrak{N}=v\mathfrak{N}=\mathfrak{A}$, and $m=t=v$; whence the anſwer will be,

$$\left(\frac{\mathfrak{P}}{t}\times\overline{\mathfrak{D}-\frac{1}{4v}\times\overline{\mathfrak{D}-\frac{v}{6r}}\times v+\frac{v}{6r}\times 4v},\text{ or}\right)\frac{\mathfrak{P}}{t}\times\overline{\mathfrak{D}-\overline{\mathfrak{D}-\frac{v}{6r}}\times\frac{1}{4}-\frac{v}{6r}};$$

That is $\left(\frac{\mathfrak{P}}{t}\times\overline{\mathfrak{D}-\frac{v}{6r}-\overline{\mathfrak{D}-\frac{v}{6r}}\times\frac{1}{4}},\text{ or}\right)\frac{3\mathfrak{P}}{4t}\times\overline{\mathfrak{D}-\frac{v}{6r}}.$

CASE IV.

If *A*, the youngest, is to survive the other three.
Then the present worth of an estate, or legacy, dependent on the order,

$$\left.\begin{array}{l} \overline{BC}, D, A \\ \\ \overline{BD}, C, A \\ \\ \overline{CD}, B, A \end{array}\right\} \text{is (by question)} \left\{\begin{array}{l} 59)\ \mathfrak{P}\times\left\{\begin{array}{l} \frac{v\mathfrak{F}-v\mathfrak{M}}{v}+\overline{v\mathfrak{M}-\frac{vv}{6rm}}\times\frac{1}{2t} \\ -\overline{v\mathfrak{N}-\frac{vv}{6rn}}\times\frac{2t-v}{4mt}+\frac{vv}{6rn}\times\frac{2n-2v}{4mt}, \end{array}\right. \\ 60)\ \mathfrak{P}\times\left\{\begin{array}{l} \frac{m\mathfrak{F}-n\mathfrak{M}}{n}+\overline{n\mathfrak{M}-\frac{nn}{6rm}}\times\frac{1}{2t} \\ -\overline{\mathfrak{P}-\frac{v}{6r}}\times\frac{2t-v}{2mn}\times\frac{v}{2t}, \end{array}\right. \\ 61)\ \mathfrak{P}\times\left\{\begin{array}{l} \frac{m\mathfrak{F}-\mathfrak{N}}{m}+\overline{\mathfrak{N}-\frac{n}{6r}}\times\frac{n}{2tm} \\ -\overline{\mathfrak{P}-\frac{v}{6r}}\times\frac{2t-v}{2t}\times\frac{v}{2nm}; \end{array}\right. \end{array}\right.$$

which preſent worths are to be added together:

Now $\overline{v\mathfrak{M} - \frac{vv}{6rm}} \times \frac{1}{2t} = \frac{v\mathfrak{M}}{2t} - \frac{v}{6r} \times \frac{v}{2mt}$,

$$-\overline{v\mathfrak{P} - \frac{vv}{6rn}} \times \frac{2t-v}{4mt} = -v\mathfrak{P} \times \frac{2t-v}{4mt} + \frac{v}{6r} \times \frac{v}{2mt} \times \frac{2t-v}{2n},$$

$$+\frac{vv}{6rn} \times \frac{2n-2v}{4mt} = +\frac{v}{6r} \times \frac{v}{2mt} \times \frac{2n-2v}{2n},$$

$$-\overline{\mathfrak{D} - \frac{v}{6r}} \times \frac{2t-v}{2mn} \times \frac{v}{2t} = -\mathfrak{D} \times \frac{2t-v}{4mt} \times \frac{v}{n} + \frac{v}{6r} \times \frac{v}{2mt} \times \frac{2t-v}{2n},$$

$$-\overline{\mathfrak{A} - \frac{v}{6r}} \times \frac{2t-v}{2t} \times \frac{v}{2nm} = -\mathfrak{D} \times \frac{2t-v}{4mt} \times \frac{v}{n} + \frac{v}{6r} \times \frac{v}{2mt} \times \frac{2t-v}{2n};$$

Therefore, the ſum of the laſt mentioned five expreſſions will be

$$\left(\frac{v\mathfrak{M}}{2t} - \frac{2t-v}{4mt} \times \overline{v\mathfrak{P} + \mathfrak{D} \times \frac{2v}{n}} + \frac{v}{6r} \times \frac{v}{2mt} \times \overline{\frac{6t+2n-5v}{2n} - 1}, \text{ or}\right)$$

$$\frac{v\mathfrak{M}}{2t} - \frac{2t-v}{4mt} \times \overline{v\mathfrak{P} \times \frac{2v\mathfrak{D}}{n}} + \frac{v}{6r} \times \frac{6t-5v}{2n} \times \frac{v}{2mt}:$$

Again $\overline{n\mathfrak{M} - \frac{nn}{6rm}} \times \frac{1}{2t} = \frac{n\mathfrak{M}}{2t} - \frac{n}{6r} \times \frac{n}{2mt}$,

And $\overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{n}{2mt} = \frac{\mathfrak{N}}{2mt} - \frac{n}{6r} \times \frac{n}{2mt}$;

Therefore, the ſum of thoſe two expreſſions will be

$$\left(\frac{n\mathfrak{M}}{2t} + \frac{n\mathfrak{N}}{2mt} - \frac{n}{6r} \times \frac{n}{mt}, \text{ or}\right) \frac{n\mathfrak{M}}{2t} + \overline{\tfrac{1}{2}\mathfrak{N} - \frac{n}{6r}} \times \frac{n}{mt}:$$

Whence the ſum of all the expreſſions will be

$$\left(\mathfrak{P} \times \left\{\begin{array}{l} \frac{v\mathfrak{F} - v\mathfrak{M}}{v} + \frac{n\mathfrak{F} - n\mathfrak{M}}{n} + \frac{m\mathfrak{F} - \mathfrak{R}}{m} + \frac{v\mathfrak{M} + n\mathfrak{M}}{2t} \\ + \overline{\tfrac{1}{2}\mathfrak{N} - \frac{n}{6r}} \times \frac{n}{mt} - \overline{v\mathfrak{R} + \frac{2v\mathfrak{Q}}{n}} \times \frac{2t - v}{4mt} \\ + \frac{v}{6r} \times \frac{6t - 5v}{2n} \times \frac{v}{2mt}. \end{array}\right\}, \text{ or}\right)$$

$$\mathfrak{P} \times \left\{ \begin{array}{l} \frac{{}^{v}\mathfrak{F} - {}^{v}\mathfrak{H}}{v} + \frac{{}^{n}\mathfrak{F} - {}^{n}\mathfrak{M}}{n} \\ + \frac{1}{mt} \times \left\{ \begin{array}{l} \overline{{}^{m}\mathfrak{F} - \mathfrak{N}} \times t + \overline{{}^{n}\mathfrak{M} + {}^{v}\mathfrak{H}} \times \frac{m}{2} + \overline{\tfrac{1}{2}\mathfrak{N} - \frac{n}{6r}} \times n \\ \overline{- {}^{v}\mathfrak{N} + \frac{2v\mathfrak{H}}{n}} \times \frac{2t - v}{4} + \frac{v}{6r} \times \frac{\overline{6t - 5v} \times v}{4n}. \end{array} \right. \end{array} \right.$$

EXAMPLE.

D (aged 76) has jointured his ſecond wife *C* (aged 66) in an eſtate of 1£. per annum; which (after both their deceaſe) will deſcend to *B*, the daughter of *D*, aged 54: Now *D* has a nephew, *A* (aged 43) who is heir to the eſtate after the deceaſe of *D*, *C* and *B*, if he ſurvives them: what is the preſent worth of his intereſt therein?

Here $\frac{{}^{v}\mathfrak{F} - {}^{v}\mathfrak{M}}{v} = \left(\frac{7{,}228 - 6.025}{10} = \right) 0{,}0303$; $\frac{{}^{n}\mathfrak{F} - {}^{n}\mathfrak{H}}{n} = \left(\frac{10{,}838 - 9{,}891}{20}\right.$ $=)\ 0{,}0473$; $\overline{{}^{m}\mathfrak{F} - \mathfrak{N}} \times t = (\overline{12{,}578 - 7{,}673} \times 43 =)\ 210{,}915$; $\overline{{}^{n}\mathfrak{H} + {}^{v}\mathfrak{H}} \times \frac{m}{2} =$

9,891

$(\overline{9,841+6,925}\times 16 =)$ 269,056; $\frac{1}{2}\mathfrak{P} = 3,836$; $\frac{s}{6r} = 3,205$; $\overline{3,836-3,205}\times 20 = 12,620$; $v\mathfrak{D} = 6,215$; $\frac{2v}{n} = \left(\frac{2.10}{20}=\right)$ 1; $\frac{2v\mathfrak{C}}{n} = (\mathfrak{D}=)$ 4,318; $6,215 + 4,318 = 10,533$; $\frac{2t-v}{4} = \left(\frac{76}{4}=\right)$ 19; $10,533\times 19 = 200,127$; $\frac{v}{6r} = 1,603$; $\frac{\overline{6t-5v}\times v}{4n} = \left(\frac{208\times 10}{4\times 20}=\right)$ 26; $1,603\times 26 =$ 41,678:

Now $210,915 + 269,056 + 12,620 + 41,678 - 200,127 = 334,142$;

And $\frac{334,142}{32\times 43} = 0,2429$; then $0,0303 + 0,0473 + 0,2429 = 0,3205$;

Whence $(0,3205\times 25 =)$ 8,0125 will be the preſent worth required.

COROL. I.

If *A* and *B* are of equal ages; then $v\mathfrak{F} = v\mathfrak{M}$, $v\mathfrak{F} = v\mathfrak{H}$, $m\mathfrak{F} = \mathfrak{M}$, and $t = m$; whence the anſwer will become,

$$\frac{\mathfrak{P}}{mn} \times \left\{ \begin{array}{l} \overline{\mathfrak{M} - \mathfrak{N}} \times m + {}^{n}\mathfrak{M} + {}^{v}\mathfrak{M} \times \frac{n}{2} + \overline{\tfrac{1}{2}\mathfrak{N} - \frac{n}{6r}} \times n \\ - {}^{v}\mathfrak{N} + \frac{2v^{n}}{} \times \frac{2m-v}{4} + \frac{v}{6r} \times \frac{\overline{6m-5v} \times v}{4n}. \end{array} \right.$$

COROL. II.

If, *B* and *C* are of equal ages; then ${}^{v}\mathfrak{M} = {}^{v}\mathfrak{N}$, ${}^{n}\mathfrak{M} = \mathfrak{N}$, and $m = n$; whence the anſwer will become,

$$\mathfrak{P} \times \left\{ \begin{array}{l} \frac{{}^{v}\mathfrak{F} - {}^{v}\mathfrak{N}}{} + \frac{\mathfrak{F} - \mathfrak{N}}{n} + \\ \frac{1}{m} \times \left\{ \begin{array}{l} + \overline{\mathfrak{F} - \mathfrak{N}} \times t + \overline{\mathfrak{N} + {}^{v}\mathfrak{N}} \times \frac{n}{2} + \overline{\tfrac{1}{2}\mathfrak{N} - \frac{n}{6r}} \times n \\ - {}^{v}\mathfrak{N} + \frac{2v\mathfrak{D}}{n} \times \frac{2t-v}{4} + \frac{v}{6r} \times \frac{\overline{6r-5v} \times v}{4n}. \end{array} \right. \end{array} \right.$$

COROL. III.

If *C* and *D* are of equal ages; then $= \mathfrak{N} = {}^{v}\mathfrak{N} = \mathfrak{D}$, and $n = v$; whence the anſwer will become

$$\left(\mathfrak{P}\times\left\{\begin{array}{l}\dfrac{v\mathfrak{F}-v\mathfrak{M}}{v}+\dfrac{v\mathfrak{F}-v\mathfrak{M}}{v}\\ \mp\dfrac{1}{mt}\times\left\{\begin{array}{l}+m\mathfrak{F}-\mathfrak{A}\times t+v\mathfrak{M}+v\mathfrak{M}\times\dfrac{m}{2}+\frac{1}{2}\mathfrak{P}-\dfrac{v}{6r}\times v\\ -\mathfrak{P}+2\mathfrak{P}\times\dfrac{2t-v}{4}+\dfrac{v^{2}}{6r}\times\dfrac{6t-5v}{4}\end{array}\right.\end{array}\right\}\text{, or)}$$

$$\mathfrak{P}\times\left\{\begin{array}{l}\overline{v\mathfrak{F}-v\mathfrak{M}}\times\dfrac{2}{v}\\ +\dfrac{1}{mt}\times\overline{m\mathfrak{F}-\mathfrak{P}\times t+v\mathfrak{M}m-\mathfrak{P}-\dfrac{v}{6r}\times\dfrac{6t-5v}{4}-\dfrac{v}{6r}\times v}.\end{array}\right.$$

COROL. IV.

If *A*, *B* and *C*, are of equal ages; then $v\mathfrak{F}=v\mathfrak{M}=v\mathfrak{N}$, $m\mathfrak{F}=n\mathfrak{F}=n\mathfrak{M}=\mathfrak{N}$, and $t=m=n$; whence the answer will become

$$\left(\frac{\mathfrak{P}}{nn}\times\left\{\begin{array}{l}\mathfrak{N}+v\mathfrak{N}\times\dfrac{n}{2}\\ +\frac{1}{2}\mathfrak{N}-\dfrac{n}{6r}\times n-v\mathfrak{N}+\dfrac{2v\mathfrak{N}}{n}\times\dfrac{2n-v}{4}+\dfrac{vv}{6r}\times\dfrac{6n-5v\times v}{4n}\end{array}\right\}\text{, or)}$$

$$\frac{\mathfrak{P}}{nn}\times\left\{\overline{\frac{v\mathfrak{D}n}{2}+\mathfrak{P}-\frac{n}{6r}\times n}-v\mathfrak{P}+\frac{2v\mathfrak{D}}{n}\times\frac{2n-v}{4}+\frac{v}{6r}\times\frac{\overline{6n-5v}\times v}{4n}\right\};\text{ that is,}$$

$$\frac{\mathfrak{P}}{nn}\times\left\{\frac{v\mathfrak{P}v}{2}+\mathfrak{P}-\frac{n}{6r}\times n-\frac{2v\mathfrak{D}}{n}\times\frac{2n-v}{4}+\frac{v}{6r}\times\frac{\overline{6n-5v}\times v}{4n}\right\}.$$

COROL. V.

If B, C, and D are of equal ages; then ${}^{n}\mathfrak{M}={}^{v}\mathfrak{M}=\mathfrak{P}={}^{v}\mathfrak{P}=\mathfrak{D}$, and $m=n=v$; whence the anſwer will become

$$(\mathfrak{D}\times\left\{\begin{array}{l}\overline{v\mathfrak{F}-\mathfrak{D}}\times\frac{2}{v}\\+\frac{1}{vt}\times\overline{v\mathfrak{F}-\mathfrak{D}\times t}+\mathfrak{D}v-\mathfrak{D}-\frac{v}{6r}\times\frac{6t-5v}{4}-\frac{v}{6r}\times v\end{array}\right\},\text{ or)}$$

𝔓 ×

$$\mathfrak{P}\times\left\{\begin{array}{l}\overline{v\mathfrak{F}-\mathfrak{D}}\times\overline{\frac{2}{v}+\frac{1}{v}}\\ +\overline{\mathfrak{D}-\frac{v}{6r}}\times\overline{v-\frac{6t-5v}{4}}\times\frac{1}{vt};\end{array}\right. :$$

That is,

$$\frac{\mathfrak{P}}{v}\times\overline{\overline{v\mathfrak{F}-\mathfrak{D}}\times3+\overline{\mathfrak{D}-\frac{v}{6r}}\times\frac{9v-6t}{4t}}.$$

QUESTION LXXII.

Two perſons, *B*, and *C*, of given ages, agree to purchaſe a leaſe of lands of the clear value of 1 £. per ann. for their two lives; to be enjoyed equally during their joint lives, and then to belong wholly to the ſurvivor; what part ought each to pay of the purchaſe money?

SOLUTION.

Each of them is entituled to half the value of the annuity on their joint lives; more the value of the reverſion of the whole annuity, for his own life, after the deceaſe of the other.

Now if the value of the ſingle life of the elder be denoted by $\mathfrak{N}$, that of the younger by $\mathfrak{Y}$; and the value of their joint lives by $\overline{\mathfrak{N}\,\mathfrak{Y}}$,

Then half the value of the annuity, on their joint lives, will be $\frac{1}{2}\times\overline{\mathfrak{M}\,\mathfrak{N}}$;

The reverſion of the younger life, after the elder, will be $\mathfrak{M}-\overline{\mathfrak{M}\,\mathfrak{N}}$ (by queſt. 94. vol. 2.)

And the reverſion of the elder life, after the younger, will be $\mathfrak{N}-\overline{\mathfrak{Y}\,\mathfrak{N}}$;

Therefore the intereſt of the younger perſon will be $\mathfrak{M}-\frac{1}{2}\times\overline{\mathfrak{Y}\,\mathfrak{N}}$,

and that of the elder $\mathfrak{N}-\frac{1}{2}\times\overline{\mathfrak{M}\,\mathfrak{N}}$;

That is to ſay, the intereſt of each perſon may be found, by ſubtracting half the value of the joint lives of the two perſons, from the value of his own life.

Now if *m* denote the complement of the younger, and *n* the complement of the elder life;

Then $\mathfrak{P} - \mathfrak{P} - \frac{n}{6r} \times \frac{n}{2m}$ will be the value of their joint lives (by queſt. 4.) the half of which (viz. $\frac{1}{2}\mathfrak{P} - \mathfrak{P} - \frac{n}{6r} \times \frac{n}{4m}$) being taken ſeverally, from $\mathfrak{P}$ and $\mathfrak{P}$, the values of the ſingle lives, will leave

$\mathfrak{P} - \frac{1}{2}\mathfrak{P} + \mathfrak{P} - \frac{n}{6r} \times \frac{n}{4m}$ for the intereſt of the younger,

And $\frac{1}{2}\mathfrak{P} + \mathfrak{P} - \frac{n}{6r} \times \frac{n}{4m}$ for the intereſt of the elder.

EXAMPLE.

If *B* and *C* be, ſeverally, aged 54 and 66; then (by exam. 3. queſt. 4.) the value of their joint lives is 6,276;

Then 10,757 − 3,138 = 7,619 } will be the ſum { younger;
And 7,673 − 3,138 = 4,535 } to be paid by the { elder.

SCHOLIUM.

If the values of the intereſt of the above two perſons be added together, their ſum will be the value of an annuity on the longeſt of the two lives; and therefore, if the two lives be equal, the intereſt of each perſon will be $\frac{1}{2}$ the value of the longeſt of the two lives.

Again (if the lives be unequal) yet, if the value of the longeſt of the two lives, and the values of the two ſingle lives be given; the intereſt of each perſon may be found thus, *To, or from, half the value of the longeſt of the two lives, add, or ſubtract, half the difference of the values of the two ſingle lives; and the ſum, or remainder, will be the value of the intereſt of the younger, or elder perſon.*

For the difference of their intereſts will appear to be $\mathfrak{M} - \mathfrak{L}$ by ſubtracting the expreſſions above given.

QUESTION LXXIII.

Three perſons, *A*, *B*, and *C*, of given ages, *A* being younger than *B*, and *B* younger than *C*, have between them a leaſe of lands, in equal ſhares; which, on the deceaſe of either of them, is to devolve to the two ſurvivors, ſhare and ſhare alike; and upon the demiſe of either of them, it is to belong, wholly, to the laſt ſurvivor for his life; the preſent value of the intereſt of each perſon, in that leaſe is required?

SOLUTION.

A one of the perſons is intituled to $\frac{1}{3}$ of the annuity for the joint lives of the three perſons *A B* and *C*; to $\frac{1}{2}$ of the reverſion of the ſame annuity for the two joint lives of *A* and *B*, after the deceaſe of *C*; to $\frac{1}{2}$ of the reverſion of the ſame annuity for the two joint lives of *A* and *C*, after the deceaſe of *B*; and to the whole reverſion of the ſame for his own life, after the longeſt of the two lives *B* and *C*.

Now, if the values of the ſingle lives of *A*, *B*, and *C* (whoſe complements are *l*, *m*, and *n*) be called $\mathfrak{L}$, $\mathfrak{M}$ and $\mathfrak{N}$; if the values of the joint lives of any two of them, be denoted by $\overline{\mathfrak{L}\mathfrak{M}}$, $\overline{\mathfrak{L}\mathfrak{N}}$ and $\overline{\mathfrak{M}\mathfrak{N}}$; and the value of the three joint lives by $\overline{\mathfrak{L}\mathfrak{M}\mathfrak{N}}$:

Then one third of the value of an annuity, on three joint lives, will be $\frac{1}{3} \times \overline{\mathfrak{L}\mathfrak{M}\mathfrak{N}}$;

Half the reverſion of the joint lives of *A* and *B*, after the deceaſe of *C*, will be $\frac{1}{2}\times\overline{\mathfrak{F}\mathfrak{M}}-\frac{1}{2}\times\overline{\mathfrak{F}\mathfrak{M}\mathfrak{N}}$; and half the reverſion of the joint lives of *A* and *C*, after the deceaſe of *B*, will be $\frac{1}{2}\times\overline{\mathfrak{F}\mathfrak{N}}-\frac{1}{2}\times\overline{\mathfrak{F}\mathfrak{M}\mathfrak{N}}$, by queſt. 95. vol. 2.

Alſo the reverſion of the life of *A*, after the longeſt of *B* and *C* will be $\mathfrak{F}-\overline{\mathfrak{F}\mathfrak{M}}-\overline{\mathfrak{F}\mathfrak{N}}+\overline{\mathfrak{F}\mathfrak{M}\mathfrak{N}}$; by queſt. 101. vol. 2.

Whence the intereſt of *A*, in the annuity, will be $\mathfrak{F}-\frac{1}{2}\times\overline{\mathfrak{F}\mathfrak{M}}-\frac{1}{2}\times\overline{\mathfrak{F}\mathfrak{N}}+\frac{1}{3}\times\overline{\mathfrak{F}\mathfrak{M}\mathfrak{N}}$.

And, by arguing in the ſame manner, the intereſt of *B* will be $\mathfrak{M}-\frac{1}{2}\times\overline{\mathfrak{F}\mathfrak{M}}-\frac{1}{2}\times\overline{\mathfrak{M}\mathfrak{N}}+\frac{1}{3}\times\overline{\mathfrak{F}\mathfrak{M}\mathfrak{N}}$;

And, that of *C*, $\mathfrak{N}-\frac{1}{2}\times\overline{\mathfrak{F}\mathfrak{N}}-\frac{1}{2}\times\overline{\mathfrak{M}\mathfrak{N}}+\frac{1}{3}\times\overline{\mathfrak{F}\mathfrak{M}\mathfrak{N}}$.

Therefore, the intereſt of each perſon in the leaſe may be found, by adding one third part of the value of the three joint lives, to the value of his ſingle life; and ſubtracting therefrom, half the ſum of the joint lives of the perſon (whoſe intereſt is required) combined, ſeparately, with each of the other perſons concerned.

Hence *firſt*. The intereſt of *A*, the youngeſt, will by ſubtracting and adding the values above named, as found in queſtions 1, 4, and 9 appear to be $\mathfrak{F}-\frac{1}{2}\mathfrak{M}-\frac{1}{6}\mathfrak{N}$

$$+\frac{1}{4t}\times\overline{\mathfrak{M}-\frac{m}{6r}\times m}-\overline{\mathfrak{N}-\frac{n}{6r}\times\frac{2t-m-n}{3m}}\times n.$$

Secondly. The intereſt of *B*, the middle perſon will be $\frac{1}{2}\mathfrak{M}-\frac{1}{6}\mathfrak{N}$

$$+\frac{1}{4t}\times\overline{\mathfrak{M}-\frac{m}{6r}}\times m-\overline{\mathfrak{N}-\frac{n}{6r}\times\frac{2m-t-n}{3m}}\times n.$$

Thirdly, the intereſt of *C*, the eldeſt, will be

$$\frac{1}{6}\mathfrak{N}+\overline{\mathfrak{N}-\frac{n}{6r}}\times\frac{t+n+n}{12mt}\times n.$$

Now, if we add together the above given expreſſions of the ſeveral intereſts of *A*, *B*, and *C*, in the leaſe, viz.

$$\mathfrak{F}-\tfrac{1}{2}\mathscr{M}-\tfrac{1}{6}\mathfrak{N}+\overline{\mathfrak{M}-\frac{m}{6r}}\times\frac{m}{4t}-\overline{\mathfrak{N}-\frac{n}{6r}}\times\overline{2t-m-n}\times\frac{n}{12mt},$$

$$+\tfrac{1}{2}\mathfrak{M}-\tfrac{1}{6}\mathscr{N}+\overline{\mathscr{M}-\frac{m}{6r}}\times\frac{m}{4t}-\overline{\mathfrak{N}-\frac{n}{6r}}\times\overline{2m-t-n}\times\frac{n}{12mt},$$

$$+\tfrac{1}{3}\mathfrak{N}\qquad+\overline{\mathscr{N}-\frac{n}{6r}}\times\overline{t+m+n}\times\frac{n}{12mt};$$

The ſum, $\mathfrak{F}+\overline{\mathfrak{M}-\frac{m}{6r}}\times\frac{m}{2t}+\overline{\mathfrak{N}-\frac{n}{6r}}\times\frac{nn}{4mt}$, is the value of the longeſt of the three lives, ſee queſt. 14.

COROL. I.

If the two younger perſons, *A* and *B*, are of equal ages; then $\mathfrak{F}=\mathfrak{M}$, and $t=m$; therefore the values of the ſeveral intereſts will be,

$$\left.\begin{matrix}A\\ \text{or}\\ B\end{matrix}\right\}\tfrac{1}{2}\mathscr{M}-\tfrac{1}{6}\mathfrak{N}+\overline{\mathscr{M}-\frac{m}{6r}}\times\tfrac{1}{4}-\overline{\mathscr{N}-\frac{n}{6r}}\times\overline{m+n}\times\frac{n}{12mm},$$

$$\mathfrak{C}\quad\tfrac{1}{2}\mathfrak{N}+\overline{\mathscr{N}-\frac{n}{6r}}\times\overline{2m+n}\times\frac{n}{12mm}.$$

COROL. II.

If the two elder perſons, *B* and *C*, are of equal ages; then $\mathfrak{M}=\mathfrak{N}$, and $m=n$; therefore the value of their ſeveral intereſts will be

$$A \quad \mathfrak{r}-\frac{1}{3}\,\overline{\mathfrak{N}+\mathfrak{N}-\frac{n}{6r}}\times\frac{5n-2t}{12t},$$

$$\left.\begin{matrix}B\\ \text{or}\\ C\end{matrix}\right\} \quad \frac{1}{3}\,\overline{\mathfrak{N}+\mathfrak{N}-\frac{n}{6r}}\times\frac{2n+t}{12t}.$$

COROL. III.

If the three lives are of equal ages. then each of their ſeparate intereſts will be, $\frac{1}{3}\,\overline{\mathfrak{N}+\mathfrak{N}-\frac{n}{6r}}\times\frac{3}{4}$; which expreſſion is $\frac{1}{3}$ of the value of the longeſt of the three equal lives. See Corol. 3. Queſt. 14.

QUESTION LXXIV.

Three perſons *A*, *B*, and *C*, agree to purchaſe a leaſe of lands for their three lives; and they farther agree, that the income thereof ſhall be divided among them, in the following manner; viz. *A* and *B* are to enjoy it, in equal ſhares, during their joint lives; if *A* dies firſt, then *C* and *B* are to enjoy it equally for their joint lives, and the ſurvivor is to have the whole; but if *B* dies firſt, then *A* is to enjoy the whole during his life, and then it is to devolve to *C*, for the remainder of his life: what ought each perſon to contribute toward the purchaſe?

SOLUTION.

A, is entituled to $\frac{1}{2}$ the income, for the joint lives of *A* and *B*; and to the reverſion of the whole, after the death of *B*.

B, is entituled to ½ the income, for the joint lives of *A* and *B*; to ½ the reverſion, for the joint lives of *B* and *C*, after the death of *A*; and to the reverſion of the whole, after the longeſt of the two lives *A* and *C*.

C, is entituled to ½ the reverſion, for the joint lives *B* and *C*, after the deceaſe of *A*; and to the reverſion of the whole, after the longeſt of the two lives *A* and *B*.

Now putting *A*, *B*, *C*. for the value of the ſingle lives of thoſe perſons; $\overline{AB}$, $\overline{AC}$, $\overline{BC}$ ſeverally for the values of the two joint lives of *A*, *B*; *A*, *C*; *B*, *C*; and $\overline{ABC}$ for the value of the three joint lives of *A*, *B*, and *C*.

Then firſt the intereſt of *A* will be ($\frac{1}{2}\overline{AB}$, $+ A - \overline{AB}$; (queſt. 94. vol. 2.)

That is) $A - \frac{1}{2}\overline{AB}$.

Secondly. The intereſt of *B* will be ($\frac{1}{2}\overline{AB}$ $+ \frac{1}{2}\overline{BC} - \frac{1}{2}\overline{ABC}$ (By queſt. 95. vol. 2.)

$\left\{ + B - \overline{AB} + \overline{ABC} - \overline{BC} \right\}$ (By queſt. 101. vol. 2.)

That is) $B - \frac{1}{2}\overline{AB} - \frac{1}{2}\overline{BC} + \frac{1}{2}\overline{ABC}$

Thirdly. The intereſt of *C*, will be, $\frac{1}{2}\overline{BC} - \frac{1}{2}\overline{ABC}$.

$\left\{ + C - \overline{AC} + \overline{ABC} - \overline{BC} \right\}$ (By queſt. 101. vol. 2.)

That is) $C - \overline{AC} - \frac{1}{2}\overline{BC} + \frac{1}{2}\overline{ABC}$.

CASE I.

If *A*, be the youngeſt; *B* the ſecond; and *C*, the eldeſt; put *t*, *m*, and *n* for the complements of *A*, *B* and *C*:

Then firſt; $\mathfrak{T} - \frac{1}{2}\mathfrak{M} + \mathfrak{M} - \frac{m}{6r} \times \frac{m}{4t}$, will be the value of *A*'s intereſt in the leaſe.

Secondly, $\frac{1}{2}\mathfrak{M} + \frac{1}{4t} \times \overline{\overline{\mathfrak{M} - \frac{m}{6r}} \times m - \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{\overline{2m-n} \times n}{2m}}$, will be the value of *B*'s intereſt in the leaſe.

Thirdly, $\overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{\overline{2m+n} \times n}{8mt}$ will be the value of *C*'s intereſt.

If the above found values of the intereſt of *A*, *B*, and *C*, be added together, viz.

A, $\mathfrak{F} - \frac{1}{2}\mathfrak{M} + \overline{\mathfrak{M} - \frac{m}{6r}} \times \frac{m}{4t}$

B, $\frac{1}{2}\mathfrak{M} + \overline{\mathfrak{M} - \frac{m}{6r}} \times \frac{m}{4t} - \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{\overline{2m-n} \times n}{8mt}$,

C, $+ \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{\overline{2m+n} \times n}{8mt}$;

The ſum, $\mathfrak{F} + \overline{\mathfrak{M} - \frac{m}{6r}} \times \frac{m}{2t} + \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{nn}{4mt}$, will be the value of the longeſt of the three three lives. See queſt. 14.

COROL. I.

If the two younger lives *A* and *B*, are of equal ages; then $\mathfrak{F} = \mathfrak{M}$, and $t = m$;

Therefore the value of the intereſts of

A $= \frac{1}{2}\mathfrak{M} + \overline{\mathfrak{M} - \frac{m}{6r}} \times \frac{1}{4}$,

B $= \frac{1}{2}\mathfrak{M} + \overline{\mathfrak{M} - \frac{m}{6r}} \times \frac{1}{4} - \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{\overline{2m-n} \times n}{8mm}$,

C $= + \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{\overline{2m+n} \times n}{8mm}$.

COROL. II.

If the two elder perſons, *B* and *C*, are of equal ages; then $\mathfrak{M} = \mathfrak{N}$, and $m = n$; therefore the value of the intereſts of,

A $= \mathfrak{S} - \frac{1}{2}\mathfrak{N} + \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{n}{4t}$,

B $= \frac{1}{2}\mathfrak{N} + \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{n}{8t}$,

C $= + \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{3n}{8t}$.

COROL.

COROL. III.

If the three perſons are of equal ages; then the value of the intereſts of,

$$A=\tfrac{1}{2}\,\mathfrak{N}+\overline{\mathfrak{N}-\frac{n}{6r}}\times\tfrac{1}{4},$$

$$B=\tfrac{1}{2}\,\mathfrak{N}+\overline{\mathfrak{N}-\frac{n}{6r}}\times\tfrac{3}{8},$$

$$C=+\,\overline{\mathfrak{N}-\frac{n}{6r}}\times\tfrac{3}{8}.$$

CASE II.

If A, be the youngeſt; B, the eldeſt; and C, the middle aged perſon; put t, n, and m, for the complements of A, B, and C; then, firſt; $\mathfrak{T}-\frac{1}{2}\mathfrak{N}+\overline{\mathfrak{N}-\frac{n}{6r}}\times\frac{n}{4t}$, will be the value of A's intereſt.

Secondly, $\frac{1}{2}\mathfrak{N}+\overline{\mathfrak{N}-\frac{n}{6r}}\times\frac{nn}{8mt}$, will be the value of B's intereſt.

Thirdly, $\overline{\mathfrak{M}-\frac{m}{6r}}\times\frac{m}{2t}-\overline{\mathfrak{N}-\frac{n}{6r}}\times\frac{\overline{2m-n}\times n}{8mt}$ will be the value of C's intereſt.

Now the the ſum of theſe three intereſts, viz.

$$A=\mathfrak{T}-\tfrac{1}{2}\mathfrak{N}\quad+\overline{\mathfrak{N}-\frac{n}{6r}}\times\frac{n}{4t},$$

$$B=\quad+\tfrac{1}{2}\mathfrak{N}\quad+\overline{\mathfrak{N}-\frac{n}{6r}}\times\frac{nn}{8mt},$$

$$C=\quad+\overline{\mathfrak{M}-\frac{m}{6r}}\times\frac{m}{2t}-\overline{\mathfrak{N}-\frac{n}{6r}}\times\frac{\overline{2m-n}\times n}{8mt};$$

That is, $\mathfrak{T}+\overline{\mathfrak{M}-\frac{m}{6r}}\times\frac{m}{2t}+\overline{\mathfrak{N}-\frac{n}{6r}}\times\frac{nn}{4mt}$, will be the value of the longeſt of the three lives as before.

COROL. I.

If the two younger perſons, *A* and *C*, are of equal ages; then $\mathfrak{F}=\mathfrak{M}$, and $t=m$; therefore the intereſt of

$$A = \mathfrak{M} - \tfrac{1}{2}\mathfrak{N} \quad + \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{n}{4tn},$$

$$B = \quad + \tfrac{1}{2}\mathfrak{N} \quad + \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{nn}{8mm}$$

$$C = \quad + \overline{\mathfrak{N} - \frac{m}{6r}} \times \tfrac{1}{2} - \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{\overline{2m-n} \times n}{8mm}.$$

COROL. II.

If the two elder perſons, *C* and *B*, are of equal ages; then $\mathfrak{M}=\mathfrak{N}$, and $m=n$; therefore the intereſt of

$$\left.\begin{aligned} A &= \mathfrak{F} - \tfrac{1}{2}\mathfrak{N} + \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{n}{4t}, \\ B &= \quad + \tfrac{1}{2}\mathfrak{N} + \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{n}{8t}, \\ C &= \quad + \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{3n}{8t}; \end{aligned}\right\} \begin{array}{l}\text{The ſame as in} \\ \text{the ſecond} \\ \text{Corol. of} \\ \text{Caſe 1.}\end{array}$$

CASE III.

If *A*, be the middle aged perſon; *B*, the youngeſt; and *C*, the eldeſt of the three.

Put *m*, *t*, and *n* for the complements of *A*, *B*, and *C*, then, firſt; $\tfrac{1}{2}\mathfrak{M} + \overline{\mathfrak{M} - \frac{m}{6r}} \times \frac{m}{4t}$ will be the value of *A*'s intereſt.

Secondly, $\mathfrak{F}-\frac{1}{2}\mathfrak{M}+\frac{1}{4t}\times\overline{\mathfrak{m}-\frac{m}{6r}\times m-\mathfrak{N}-\frac{n}{6r}\times\frac{\overline{2t-n}\times n}{2m}}$, will be the value of *B*'s intereſt.

Thirdly, $\mathfrak{N}-\frac{n}{6r}\times\frac{\overline{2t+n}\times n}{8mt}$; will be the value of *C*'s intereſt.

Now the ſum of theſe three intereſts, viz.

$$A\text{'s}=\frac{1}{2}\mathfrak{M}+\overline{\mathfrak{m}-\frac{m}{6r}}\times\frac{m}{4t},$$

$$B\text{'s}=\mathfrak{F}-\frac{1}{2}\mathfrak{M}+\overline{\mathfrak{M}-\frac{m}{6r}}\times\frac{m}{4t}-\overline{\mathfrak{N}-\frac{n}{6r}}\times\frac{\overline{2t-n}\times n}{8mt},$$

$$C\text{'s}=\overline{\mathfrak{N}-\frac{n}{6r}}\times\frac{\overline{2t+n}\times n}{8mt};$$

That is, $\mathfrak{F}+\overline{\mathfrak{M}-\frac{m}{6r}}\times\frac{m}{2t}+\overline{\mathfrak{N}-\frac{n}{6r}}\times\frac{nn}{4mt}$, will be the value of the longeſt of the three lives, as before.

COROL.

$$A = \tfrac{1}{3}\mathfrak{T} + \overline{\mathfrak{N} - \frac{n}{6r}} \times \text{[illegible]}$$

$$B = \mathfrak{T} - \tfrac{1}{3}\mathfrak{T} + \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{3n - 2t}{8t},$$

$$C = \quad + \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{2t + n}{8t}.$$

COROL. I.

If the two younger perſons, *B* and *A*, are of equal ages; then $\mathfrak{T} = \mathfrak{M}$, and $t = m$; therefore the intereſt of

$$A = \tfrac{1}{3}\mathfrak{M} + \overline{\mathfrak{M} - \frac{m}{6r}} \times \tfrac{1}{4},$$

$$B = \tfrac{1}{2}\mathfrak{M} + \overline{\mathfrak{M} - \frac{m}{6r}} \times \tfrac{1}{4} - \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{\overline{2m - n} \times n}{8mm},$$

$$C = \quad + \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{\overline{2m + n} \times n}{8mm}.$$

Which values are the ſame with thoſe, in corol. 1. caſe

COROL. II.

If the two elder perſons, *A* and *C*, are of equal ages; then $\mathfrak{M} = \mathfrak{N}$, and $m = n$; therefore the intereſt of

CASE IV.

If *A*, be the middle aged perſon; *B*, the eldeſt; and *C*, the youngeſt of the three.

Put *m*, *n*, and *t*, for the complements of *A*, *B*, and *C*: then, firſt,

$\mathfrak{M} - \frac{1}{2}\mathfrak{N} + \mathfrak{P} - \frac{n}{6r} \times \frac{n}{m}$, will be the value of *A*'s intereſt.

Secondly, $\frac{1}{2}\mathfrak{N} + \overline{\mathfrak{P} - \frac{n}{6r} \times \frac{nn}{8mt}}$, will be the value of *B*'s intereſt.

Thirdly,

$\mathfrak{F} - \mathfrak{M} + \frac{1}{2t} \times \overline{\mathfrak{M} - \frac{m}{6r} \times m - \mathfrak{P} - \frac{n}{6r} \times \frac{\overline{2t-n} \times n}{4m}}$, will be the value of *C*'s intereſt.

Now the ſum of theſe three intereſts, viz.

$$A\text{'s} = \mathfrak{M} - \tfrac{1}{2}\mathfrak{N} + \overline{\mathfrak{P} - \frac{n}{6r} \times \frac{n}{4m}}$$

$$B\text{'s} = + \tfrac{1}{2}\mathfrak{N} + \overline{\mathfrak{P} - \frac{n}{6r} \times \frac{nn}{8mt}},$$

$$C\text{'s} = \mathfrak{F} - \mathfrak{M} + \overline{\mathfrak{M} - \frac{m}{6r}} \times \frac{m}{2t} - \overline{\mathfrak{P} - \frac{n}{6r}} \times \frac{\overline{2t-n} \times n}{8mt};$$

That is, $\mathfrak{F} + \mathfrak{M} - \frac{m}{6r} \times \frac{m}{2t} + \mathfrak{P} - \frac{n}{6r} \times \frac{nn}{4mt}$, will be the value of the longeſt of the three lives as before.

COROL. I.

If the two younger perſons, *A* and *C*, are of equal ages; then $\mathfrak{F} = \mathfrak{M}$, and $t = m$; therefore the intereſt of [illegible]

$$A = \mathfrak{M} - \tfrac{1}{2}\mathfrak{N} + \overline{\mathfrak{P} - \frac{n}{6r}} \times \frac{n}{4m},$$

$$B = + \tfrac{1}{2}\mathfrak{N} + \overline{\mathfrak{P} - \frac{n}{6r}} \times \frac{nn}{8mm},$$

$$C = + \overline{\mathfrak{M} - \frac{m}{6r}} \times \tfrac{1}{2} - \overline{\mathfrak{P} - \frac{n}{6r}} \times \frac{\overline{2m-n} \times n}{8mm},$$

COROL. II.

If the two elder perſons, *A* and *B*, are of equal ages; then $\mathfrak{M} = \mathfrak{N}$, and $m = n$; therefore the intereſt of

$$A = \tfrac{1}{2}\mathfrak{N} + \overline{\mathfrak{P} - \frac{n}{6r}} \times \tfrac{1}{2},$$

$$B = \tfrac{1}{2}\mathfrak{P} + \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{n}{8t},$$

$$C = n - \mathfrak{P} + \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{5n-2t}{8t}.$$

CASE V.

If *A* be the eldeſt perſon; *B*, the youngeſt; and *C*, the middle aged perſon.

Put m, t, and m, for the complements of *A*, *B*, and *C*; then firſt, $\tfrac{1}{2}\mathfrak{P} + \overline{\mathfrak{P} - \frac{n}{6r}} \times \frac{n}{4t}$, will be the value of *A*'s intereſt. Secondly, $t - \tfrac{1}{2}\mathfrak{M} + \frac{1}{4t} \times \overline{\mathfrak{M} - \frac{m}{6r}} \times m - \overline{\mathfrak{P} - \frac{n}{6r}} \times \frac{\overline{2t-n} \times n}{2m}$, will be the value of *B*'s intereſt.

Thirdly, $\frac{1}{2}\mathfrak{MN} - \frac{1}{2}\mathfrak{N} + \frac{1}{4t} \times \overline{\mathfrak{M} - \frac{m}{6r}} \times m + \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{\overline{2t-2m+n} \times n}{2m}$,

will be the value of *C*'s intereſt.

Now the ſum of theſe three intereſts, viz.

$$A\text{'s} = \frac{1}{2}\mathfrak{N} + \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{n}{4t},$$

$$B\text{'s} = \mathfrak{F} - \frac{1}{2}\mathfrak{MN} + \overline{\mathfrak{MN} - \frac{m}{6r}} \times \frac{m}{4t} + \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{\overline{2t-n} \times n}{8mt},$$

$$C\text{'s} = +\frac{1}{2}\mathfrak{MN} - \frac{1}{2}\mathfrak{N} + \overline{\mathfrak{M} - \frac{m}{6r}} \times \frac{m}{4t} + \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{\overline{2t-2m+n} \times n}{8mt};$$

That is, $\mathfrak{F} + \overline{\mathfrak{MN} - \frac{m}{6r}} \times \frac{m}{2t} + \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{nn}{4mt}$, will be the value of the longeſt of the three lives, as before.

COROL. I.

If the two younger perſons, *B* and *C*, are of equal ages; then $\mathfrak{F} = \mathfrak{M}$, m; therefore the intereſt of

$$A = \tfrac{1}{2}\mathfrak{P} + \overline{\mathfrak{M} - \frac{n}{6r}} \times \frac{n}{4m},$$

$$B = \tfrac{1}{2}\mathfrak{N} + \overline{\mathfrak{M} - \frac{m}{6r}} \times \tfrac{1}{4} - \overline{\mathfrak{P} - \frac{n}{6r}} \times \frac{\overline{2m - n} \times n}{8mm},$$

$$C = \tfrac{1}{2}\mathfrak{M} - \tfrac{1}{2}\mathfrak{P} + \overline{\mathfrak{M} - \frac{m}{6r}} \times \tfrac{1}{4} + \overline{\mathfrak{P} - \frac{n}{6r}} \times \frac{nn}{8mm}.$$

COROL. II.

If the two elder perſons, *A* and *C*, are of equal ages; then $\mathfrak{M} = \mathfrak{N}$, and $m = n$; therefore the intereſt of

$$A = \tfrac{1}{2}\mathfrak{N} + \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{n}{4t},$$

$$B = \mathfrak{K} - \tfrac{1}{2}\mathfrak{N} + \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{3n - 2t}{8t},$$

$$C = + \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{2t + n}{8t}$$

CASE VI.

If *A*, be the eldeſt perſon; *B*, the ſecond; and *C*, the youngeſt. Put *n*, *m*, and *t*, for the complements of *A*, *B*, and *C*.

Then firſt; $\tfrac{1}{2}\mathfrak{P} + \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{n}{4m}$, will be the value of *A*'s intereſt.

Secondly $\frac{1}{2}\mathfrak{M}+\frac{1}{4t}\times\overline{\mathfrak{M}-\frac{m}{6r}}\times m-\overline{\mathfrak{N}-\frac{n}{6r}}\times\frac{\overline{2m-n}\times n}{2m}$, will be the value of *B*'s intereſt.

Thirdly, $\mathfrak{P}-\frac{1}{2}\mathfrak{M}-\frac{1}{2}\mathfrak{N}+\frac{1}{4t}\times\overline{\mathfrak{Q}-\frac{m}{6r}}\times m+\overline{\mathfrak{N}-\frac{n}{6r}}\times\frac{\overline{2m+n-2t}\times n}{2m}$, will be the value of *C*'s intereſt.

Now the ſum of theſe three intereſts, viz.

$$A\text{'s}=\quad+\frac{1}{2}\mathfrak{N}\quad+\overline{\mathfrak{N}-\frac{n}{6r}}\times\frac{n}{4m},$$

$$B\text{'s}=\quad+\frac{1}{2}\mathfrak{M}\quad+\overline{\mathfrak{M}-\frac{m}{6r}}\times\frac{m}{4t}-\overline{\mathfrak{N}-\frac{n}{6r}}\times\frac{\overline{2m-n}\times n}{8mt},$$

$$C\text{'s}=\mathfrak{P}-\frac{1}{2}\mathfrak{Q}-\frac{1}{2}\mathfrak{N}+\overline{\mathfrak{Q}-\frac{m}{6r}}\times\frac{m}{4t}+\overline{\mathfrak{N}-\frac{n}{6r}}\times\frac{\overline{2m+n-2t}\times n}{8mt};$$

That is, $\mathfrak{P}+\overline{\mathfrak{M}-\frac{m}{6r}}\times\frac{m}{2t}+\overline{\mathfrak{N}-\frac{n}{6r}}\times\frac{nn}{4mt}$, will be the value of the longeſt of the three lives, as before.

EXAMPLE.

A (aged 66) has a brother *B* (aged 54) and a daughter *C* (aged 43); they agree to purchaſe a leaſe (for the longeſt of their three lives) of lands, of the clear value of 1 £. per annum; and it is farther agreed, that the two brothers ſhall divide the profits, equally, during their joint lives; and, if the younger brother dies firſt, that the elder ſhall enjoy the whole, during his life; after which, it is to deſcend to his daughter; but, if the elder brother dies firſt, then his daughter and her uncle are to divide the profits, equally, for their joint lives; and the ſurvivor is to poſſeſs the whole: What ought each perſon to contribute toward the purchaſe?

Anſwer. *A*'s intereſt = 4,534.
B's = 6,068.
C's = 4,737.

The ſum = 15,339, which is the value of the longeſt of the three lives, ſee exam. to queſt. 14.

COROL. I.

If, *B* and *C*, the two younger, are of equal ages; then $\mathfrak{R}=\mathfrak{M}$, and $r=m$; whence the intereſt of

A =

$$A = \tfrac{1}{2}\mathfrak{N} + \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{n}{4m},$$

$$B = \tfrac{1}{2}\mathfrak{M} + \overline{\mathfrak{M} - \frac{m}{6r}} \times \tfrac{1}{2} - \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{\overline{2m-n} \times n}{8mm}$$

$$C = \tfrac{1}{2}\mathfrak{M} - \tfrac{1}{2}\mathfrak{N} + \overline{\mathfrak{M} - \frac{m}{6r}} \times \tfrac{1}{4} + \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{nn}{8mm}$$

COROL. II.

If, *A* and *B*, the two elder be of equal ages; then $\mathfrak{M} = \mathfrak{N}$, and $m = n$; whence the interest of

$$A = \tfrac{1}{2}\mathfrak{N} + \overline{\mathfrak{N} - \frac{n}{6r}} \times \tfrac{1}{4},$$

$$B = \tfrac{1}{2}\mathfrak{N} + \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{n}{8t},$$

$$C = \mathfrak{F} - \mathfrak{N} + \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{5n-2t}{8t}.$$

QUESTION LXXV.

Three perſons, *C*, *B*, and *A*, agree to purchaſe a leaſe of lands, for the longeſt of their three lives; the profits of which are to be, equally, divided between the two perſons, *C* and *B*, during their joint lives; upon the death of either, to be equally divided between *C* and *A*, or *B* and *A*, the two remaining perſons; and laſtly, to be enjoyed wholly by the ſurvivor: what ought each perſon to contribute toward the purchaſe?

SOLUTION.

By proceeding in the methods above perſued, it will appear, that

CASE I.

If, *C* and *B*, the two perſons who come firſt into poſſeſſion, be younger than *A*, the perſon in expectation.

(Putting *t*, *m*, and *n*, for the complements of *C*, *B*, and *A*) firſt,

$$\mathfrak{T} - \tfrac{1}{2}\mathfrak{M} + \frac{1}{4t} \times \overline{\overline{\mathfrak{M} - \frac{m}{6r}} \times m - \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{\overline{2t - n} \times n}{2m}},$$

will be the value of *C*'s intereſt.

Secondly,

$$\tfrac{1}{2}\mathfrak{M} + \frac{1}{4t} \times \overline{\overline{\mathfrak{M} - \frac{m}{6r}} \times m - \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{\overline{2m - n} \times n}{2m}},$$

will be the value of *B's* intereſt.

Thirdly, $\overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{\overline{t + m} \times n}{4mt}$, will be the value of *A*'s intereſt.

CASE II.

If, *B* and *C*, the two poſſeſſors, be one elder, and the other younger, than the expectant, *A*.

(Putting *n*, *t*, and *m*, for the complements of *B*, *C*, and *A*) then firſt,

$$\mathfrak{T}-\tfrac{1}{2}\mathfrak{M}+\frac{1}{4t}\times\overline{\overline{\mathfrak{M}-\frac{m}{6r}}\times m-\overline{\mathfrak{N}-\frac{n}{6r}}\times\frac{\overline{2t-n}\times n}{2m}},$$

will be the value of *C*'s intereſt.

Secondly, $\tfrac{1}{2}\mathfrak{MN}+\overline{\mathfrak{N}-\frac{n}{6r}}\times\frac{nn}{8mt}$, will be the value of *B*'s intereſt.

Thirdly,

$$\tfrac{1}{2}\mathfrak{M}-\tfrac{1}{2}\mathfrak{N}+\overline{\mathfrak{MN}-\frac{m}{6r}}\times\frac{m}{4t}+\overline{\mathfrak{N}-\frac{n}{6r}}\times\frac{n}{4m},$$

will be the value of *A*'s intereſt.

CASE III.

If, *C* and *B*, the two poſſeſſors are elder than *A*, the expectant. (Putting *n*, *m*, *t*, for the complements of *C*, *B*, *A*) then, firſt, *C*'s intereſt will be the ſame, as *B*'s in caſe 2; becauſe *B* was then the elder of the poſſeſſors; and his intereſt depended, on the ſame combinations, as *C*'s does now.

Secondly, *B*'s intereſt will be the ſame, as in caſe 1. for he was then the younger of the poſſeſſors, and his intereſt depended on the ſame combinations as now.

Thirdly,

$$\mathfrak{T}-\tfrac{1}{2}\times\overline{\mathfrak{MN}-\mathfrak{N}}+\frac{1}{4t}\times\overline{\overline{\mathfrak{M}-\frac{m}{6r}}\times m+\overline{\mathfrak{N}-\frac{n}{6r}}\times n},$$

will be the value of *A*'s intereſt.

QUESTION LXXVI.

Three perſons, *C*, *B*, and *A*, have a leaſe of lands for the longeſt of their three lives; the income of which, *C* is to enjoy during his life; but after his deceaſe it is to be divided equally between *B* and *A* during their joint lives; and the ſurvivor of them is to have the whole: what are the values of their ſeveral intereſts therein?

SOLUTION. CASE I.

If the poſſeſſor, *C*, be younger than either of the expectants, *B* or *A*; (putting *t*, *m*, and *n*, for the complements of *C*, *B*, and *A*) then, firſt, the intereſt of *C* will be, $\mathfrak{F}$, the value of his life.

2dly, $\overline{\mathfrak{M}-\frac{m}{6r}}\times\frac{m}{2t}-\overline{\mathfrak{N}-\frac{n}{6r}}\times\frac{\overline{2m-n}\times n}{8mt}$, will be the value of *B*'s intereſt.

Thirdly, $\overline{\mathfrak{N}-\frac{n}{6r}}\times\frac{\overline{2m+n}\times n}{8mt}$, will be the value of *A*'s intereſt.

CASE II.

If the expectants, *A* and *B*, be one younger, and the other elder, than the poſſeſſor *C*. (Putting *m*, *n*, and *t*, for the complements of *C*, *B*, and *A*) then, the intereſt of the three perſons will be,

$$C = \mathfrak{M}$$

$$B = +\overline{\mathfrak{N}-\frac{n}{6r}}\times\frac{\overline{2t+n}\times n}{8mt},$$

$$A = \mathfrak{F}-\mathfrak{M}+\overline{\mathfrak{M}-\frac{m}{6r}}\times\frac{m}{2t}-\overline{\mathfrak{N}-\frac{n}{6r}}\times\frac{\overline{2t-n}\times n}{8mt};$$

CASE III.

If the poſſeſſor C, be elder than the two expectants, A and B. Put n, m, and t, for the complements of C, B, and A; then the intereſt of the three perſons will be,

$$C = \mathfrak{N}.$$

$$B = \tfrac{1}{2}\mathfrak{M} - \tfrac{1}{2} = + \overline{\mathfrak{M} - \frac{m}{6r}} \times \frac{m}{4t} + \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{\overline{2t-2m+n} \times n}{8mt},$$

$$A = \mathfrak{F} - \tfrac{1}{2}\mathfrak{M} - \tfrac{1}{2}\mathfrak{N} + \overline{\mathfrak{M} - \frac{m}{6r}} \times \frac{m}{4t} + \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{\overline{2m-2t+n} \times n}{8mt}.$$

QUESTION LXXVII.

An eſtate of 1 £. per annum is leaſed on two lives (whoſe complements are n and m) the income of which is to be received by A, ſo long as both lives continue in being; but is (upon the firſt demiſe) to belong to B during the continuance of the remaining life; what is the preſent value of the intereſt of B in that leaſe?

SOLU-

SOLUTION.

From the value of the longeſt of the two lives $\mathfrak{M} + \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{n}{2m}$,

Take the value of the joint lives $\mathfrak{N} - \overline{\mathfrak{P} - \frac{n}{6r}} \times \frac{n}{2m}$;

The remainder will be $\mathfrak{M} - \mathfrak{P} + \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{n}{m}$, the value required.

And, if the two lives are of the ages of 10 and 39; Then 7,197 will expreſs that value.

QUESTION LXXVIII.

An eſtate of 1 £. per annum is leaſed for three lives (whoſe complements are *t*, *m*, and *n*) the income of which is to belong to *A*, ſo long as all the three lives are in being; but is (upon the firſt demiſe) to become the property of *B*, during the continuance of the longeſt of the remaining two lives: what is the preſent value of *B*'s intereſt in that leaſe?

SOLUTION.

$\mathfrak{T} - \mathfrak{P} + \overline{\mathfrak{M} - \frac{m}{6r}} \times \frac{m}{2t} + \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{t+m}{2mt} n$, will be the value required; and, if the three lives are of the ages, 10, 39, and 48; then 10,775 will expreſs that value.

QUESTION LXXIX.

An eſtate of 1 £. per annum is leaſed for three lives (whoſe complements are *t*, *m*, and *n*) the income of which is to be received by *A*, ſo long as any two of the three lives are in being; but, upon the demiſe of the ſecond, is to belong to *B*, during the remainder of the third life; what is the intereſt of *B* in that leaſe?

SOLUTION.

$$\mathfrak{T} - \mathfrak{M} + \overline{\mathfrak{M} - \frac{m}{6r}} \times \frac{m}{t} - \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{2t+2m-3n}{4mt}\, n,$$

will be the value required; and, if the 3 lives are of the ages 10, 39, and 48; then 5,122 will expreſs that value.

QUESTION LXXX.

An eſtate of 1£. per annum is leaſed on three lives (whoſe complements are *t*, *m*, and *n*) the income of which is to belong to *A*, for the three joint lives; to *B*, during the remainder of the joint lives of the two firſt ſurvivors; and to *C*, for the remainder of the life of the laſt ſurvivor: what is the preſent value of the intereſt of *B* in that leaſe?

SOLUTION.

$$\mathfrak{M} - \mathfrak{N} - \overline{\mathfrak{M} - \frac{m}{6r}} \times \frac{m}{2t} + \overline{\mathfrak{N} - \frac{n}{6r}} \times \frac{4m+4t-3n}{4mt} \times n,$$

will be the value required; and if the three lives are of the ages 10, 39, and 48; then 5,653 will expreſs that value.

QUESTION LXXXI.

There is a leaſe of an eſtate, granted at a pepper corn per annum, for *m* years; with this condition, that the tenant (at the expiration thereof) may renew the ſame paying a fine of *H* pounds; what is the value of the landlord's intereſt therein, at the time of any ſuch renewa (excluſive of the fine then received) allowing compound intereſt at a given rate?

SOLUTION.

The preſent worth of *H* pounds, due at the end of *m* years, is (per queſt. 144. part 2. vol. 1, if *r* be the amount of 1£. in 1 year) $\frac{H}{r^m}$; the preſent worth of *H* pounds, due at the end of 2 *m* years, is $\frac{H}{r^{2m}}$; at the end of 3 *m* years, $\frac{H}{r^{3m}}$; conſequently, the landlord's intereſt in the above eſtate will be $\frac{H}{r^m} + \frac{H}{r^{2m}} + \frac{H}{r^{3m}}$ &c. ad infinitum. Therefore $\frac{H}{r^m - 1}$, is the value required; whence, the preſent value of 620£. payable every 21 years, for ever; at 5 per cent. compound intereſt, will be 347,153£.

QUESTION LXXXII.

Suppoſing, that a copyhold tenant pays to the lord of the manor (in which his copyhold is ſituated) a certain ſum (*H*) on his admiſſion, and that every ſucceſſor does the like; it is required to find how much the lord's intereſt in that copyhold, for ever, is worth in preſent money; and by what ſum, the rentroll of his eſtate ought to be encreaſed, on that account?

SOLUTION.

In order to anſwer this queſtion, generally, it will be neceſſary to conceive a mean age, at which the tenants (one with another) may be ſuppoſed to be admitted; let the value of a life of that age be *P*; then (by queſt. 88, vol. II.) The reverſion of a perpetuity of 1£. after a life whoſe value is $\mathfrak{P}$, will be $P - \mathfrak{P}$: but in this caſe, upon the death of the firſt tenant, the lord is entituled to the perpetuity of the intereſt of the fine H; therefore the preſent value of the fine, to be received at the admiſſion of the next tenant, will be $\overline{P - H} \times dH$; that is $\overline{1 - d\mathfrak{P}} \times H$.

Now, if $\mathfrak{P}$ be conceived to be the preſent value of an annuity, for m years certain; then (by the proceſs uſed in the ſcholium to queſtion 89. vol. 2.) it will appear, that

$$r^m - 1 = \frac{d\mathfrak{P}}{1 - d\mathfrak{P}}:$$

But (by the laſt queſtion) the perpetual recurrency of the fine H, to be received at the expiration of every m years, will be $\frac{H}{r^m - 1}$; that is H divided by $\frac{d\mathfrak{P}}{1 - d\mathfrak{P}}$, which quotient will be $\frac{1 - d\mathfrak{P}}{d\mathfrak{P}} \times H$, the worth of the lord's intereſt in the copyhold for ever, immediately after he has received the admiſſion fine.

And ſince this preſent worth, $\frac{1 - d\mathfrak{P}}{d\mathfrak{P}} \times H$, will purchaſe a perpetuity, equal to its intereſt, for one year (which may be found by multiplying it by, d, the intereſt of 1£. for 1 year) therefore the rentroll of the lord's eſtate ſhould be encreaſed by $\frac{1 - d\mathfrak{P}}{\mathfrak{P}} \times H$, on account of this copyhold.

EXAMPLE.

Suppoſe that for a certain copyhold 100£. fine is paid on every admiſſion; and that the tenants admitted thereto are (one with another) of the age of 25 years; what is the perpetual recurrency of thoſe fines worth in preſent money, and how much ſhould the rent roll of the lord's eſtate be encreaſed, on account of thoſe fines; allowing compound intereſt at 4 per cent?

The value of a life of 25 years, ſecured by land, is 15,504; which being multiplied by 0,04, produces 0,62016.

If (1—0,62016 =) 0,37984, be multiplied by 100, the product will be 37,984.

Now 37,984, divided by 0,62016, quotes 61,25, for the preſent value, required; and 37,984, divided by 15,504, quotes 2,45, for the increaſe of the rent-roll.

But there are two caſes, which, though not ſo generally uſeful, yet are to be conſidered, viz.

1ſt. What the preſent worth of the lord's intereſt is, immediately before the receipt of a fine, on admiſſion; and this is evidently $\left(\frac{1-d\mathfrak{P}}{d\mathfrak{P}}H+H=\right)\frac{H}{d\mathfrak{P}}$.

Secondly. What that preſent worth will be, at any time after ſuch an admiſſion, when the preſent tenant's life is worth leſs than $\mathfrak{P}$.

Put the value of the preſent tenant's life $=\mathfrak{W}$; then will the fine, to be received at the next admiſſion, be worth $\overline{1-d\mathfrak{W}}\times H$; which is to be added, to the value of perpetual the recurrency of all the uſual fines, except the firſt; whence

$\frac{1-d\mathfrak{P}^{2}}{d\mathfrak{P}}+1-d\mathfrak{W}\times H$, will be the anſwer.

QUESTION LXXXIII.

Suppoſing that the cuſtom of a manor is ſuch, that a tenant at his admiſſion is to pay, to the lord, the ſum H, as a fine; what will the preſent value of the tenant's intereſt be, in a copyhold eſtate of 1£. per annum, clear of all quitrents, &c?

SOLUTION.

From P the preſent value of the perpetuity of 1£. per, ann. take the lord's intereſt in the copyhold, found by the laſt queſtion, and the remainder will be the intereſt of the tenant therein; that is, immediately after admiſſion, and having paid the fine, $P - \frac{1-d\mathfrak{P}}{d\mathfrak{P}}H$;

Before the ſaid fine is paid, $P \times \frac{\mathfrak{P}-H}{\mathfrak{P}}$;

At any other time, $P - \frac{\overline{1-d\mathfrak{P}}^2}{d\mathfrak{P}} + \overline{1-d\mathfrak{D}} \times H.$

And, if the ſum H, to be paid for a fine at every admiſſion be 1 year's clear rent; then $H = 1$; and the purchaſer of a copyhold, is to pay, $P \times \frac{\mathfrak{P}-1}{\mathfrak{P}}$, years purchaſe for the ſame; viz. 1 year's purchaſe to the lord, as a fine, and the reſt to the former tenant.

QUESTION LXXXIV.

Suppoſing that the cuſtom of a manner is ſuch, that a tenant (who holds 1£. per ann. improved rent) ſhall, thereout, pay to the lord $\frac{b}{a}$ yearly quit-rent; and ſhall moreover pay c years quit-rent, as a fine, at every admiſſion: what is the preſent value of the tenant's intereſt therein?

SOLUTION.

Here the tenant enjoys only $(1 - \frac{b}{a} =) \frac{a-b}{a}$ yearly clear rent, the perpetuity of which is $\frac{a-b}{a}P$; alſo $\frac{cb}{a}$ will be equal to H: whence ſuch a copyhold will coſt $\left(\frac{a-b}{a}P - \frac{P\,cb}{a\mathfrak{P}} =\right) \frac{\overline{a-b} \times \mathfrak{P} - cb}{a\mathfrak{P}}P$ year's purchaſe, the fine payable at admiſſion being included.

EX-

EXAMPLE.

Whence the purchaſe of a copyhold eſtate, of 1£. per ann. paying (1 s, or) $\frac{1}{20}$£. yearly quitrent, and (5 years quitrent or) $\frac{5}{20}$£. as a fine on renewal, will be 23,347£.

SCHOLIUM.

What has been ſaid in the foregoing queſtions, concerning a copyhold; may be applied to a perpetual advowſon; if we ſuppoſe (with Mr. *De Moivre*) that the patron would receive a ſum of money, *H*, at the death of each incumbent; for the alienating his right of the next preſentation, if the law did not forbid the alienation, in that circumſtance of time.

This being ſuppoſed, the preſent value of the next preſentation will be $\overline{1-d\mathfrak{N}}\times H$, the value of the perpetual advowſon, immediately after a preſentation will be $\frac{1-d\mathfrak{N}}{d\mathfrak{N}}H$; or (when the incumbent's life is worth but $\mathfrak{W}$ years purchaſe) $\overline{\frac{\overline{1-d\mathfrak{N}}^{2}}{d\mathfrak{N}}+\overline{1-d\mathfrak{W}}}\times H$; and laſtly, the perpetual advowſon, with an immmediate reſignation, will be worth $\frac{H}{d\mathfrak{N}}$; in which $\mathfrak{N}$ repreſents the value of a life of that age, at which the clergymen may be ſuppoſed (one with another) to be preſented to the living; and *d* the intereſt of 1£. for 1 year.

In like manner, when eſtates are leaſed for lives, with condition of renewing a ſingle life, when the ſame ſhall fail, for a certain ſum *H*; that caſe exactly co-incides with the above inſtance of a copyhold: and if $\mathfrak{M}$ repreſent the value of a life of that age, at which the leſſees (one with another) become intereſted in the eſtate; $\mathfrak{N}$, the value of the life, in poſſeſſion, after ſome years of the leaſe are expired; *P* the value of the perpetuity, and *d* the intereſt of 1£. for 1 year. Then, immediately after the leaſe granted and fine paid, the leſſor's intereſt will be $\frac{1-d\mathfrak{M}}{d\mathfrak{M}}H$; and the leſſees $P-\frac{1-d\mathfrak{M}}{d\mathfrak{MP}}H$;

Upon the death of the leſſee, before the renewal, and payment of the fine,

The leſſor's intereſt will be $\frac{H}{d\mathfrak{MP}}$; and the leſſees heirs,

$$P\times\frac{\mathfrak{M}-H}{\mathfrak{MP}}:$$

And at any other time,

The Leſſor's, $\frac{\overline{1-d\mathfrak{MP}}^2}{d\mathfrak{MP}}+\overline{1-d\mathfrak{P}}\times H$; and the leſſees $P-\frac{\overline{1-d\mathfrak{M}}^2}{d\mathfrak{MP}}+\overline{1-d\mathfrak{P}}\times H$.

But, notwithſtanding the ſame expreſſion anſwers the above 3 caſes, yet the age, whoſe life is repreſented by $\mathfrak{P}$ or $\mathfrak{M}$, will in different caſes, be very different; for in copyholds, it commonly happens that the tenant to be admitted ſucceeds his father, grandſather, uncle, brother, &c. as in freehoold eſtates; and conſequently a perſon of any age whatever may be admitted: but none can be preſented to a living, till they are above twenty, and conſequently (in the caſe of a perpetual advowſon) the mean age will be elder than in the caſe of the copyhold: and in a leaſehold, for lives, it may be preſumed, that the beſt age poſſible will always be put in; at leaſt, the leſſor's fine ſhould be proportioned, in the ſame manner, as if it were ſo; and therefore $\mathfrak{M}$ may, in that caſe, be generally eſteemed to be the value of a life of 10 years: but in the other two caſes, the age will be beſt fixed from obſervations.

N. B. An ingenious friend of the author's thinks that the ages of the incumbents, when they are inducted, may be partly fixed by the value of the livings; thus, for livings of a 100, 200, 300, and 400£. per ann. at 25, 34, 42, and 50 years reſpectively.

QUESTION LXXXV.

There is an eſtate of 1£. per ann. which is leaſed on two lives; with condition, that when both lives are extinct, the ſame may be renewed for two other ſuch lives, on paying the fine K; the intereſts of the leſſor and leſſee, in that eſtate, are required.

SOLUTION.

If $L\overline{\mathfrak{M}\mathfrak{N}}$ denote the value of an annuity on the longeſt of the two lives, then by arguing, as in queſt. 82; the intereſt of the leſſor immediately after ſigning the leaſe, and receiving the fine, will be $\frac{1-dL\overline{\mathfrak{M}\mathfrak{N}}}{dL\overline{\mathfrak{M}\mathfrak{N}}}K$; the rent-roll of the leſſor's eſtate ought to be encreaſed by $\frac{1-dL\overline{\mathfrak{M}\mathfrak{N}}}{L\overline{\mathfrak{M}\mathfrak{N}}}K$, on account of the perpetual recurrency of theſe fines: the intereſt of the leſſor at the extinction of the ſecond life, and before receiving the fine on renewal will be $\frac{K}{dL\overline{\mathfrak{M}\mathfrak{N}}}$; and at any other time, when the lives or life in poſſeſſion are worth but $\mathfrak{D}$ year's purchaſe, then the intereſt of the leſſor will be $\frac{1-dL\overline{\mathfrak{M}\mathfrak{N}}^2}{dL\overline{\mathfrak{M}\mathfrak{N}}} + 1-d\mathfrak{D} \times K$: the intereſt of the leſſee being in each caſe, the difference between the perpetuity and the value ſo found.

COROL.

Hence, the values of the reſpective intereſts of the leſſor and leſſee, of any leaſe for lives, renewable only at its expiration, may be found, by uſing the value of the lives, after the manner above expreſſed.

QUESTION LXXXVI.

It is propoſed to let an eſtate, by a leaſe for two lives, with a condition that the leſſee (or his heirs) may, on every failure of one life, put in another, on paying a fine certain. then, as it is to be preſumed that the life, ſo to be put in on renewal will be the beſt poſſible (viz. a life aged 10 years) it is required to find the age of that life, which ſhould be joined with a life of ten years, in the leaſe firſt granted, ſo as to render it probable, that the ſurviving life, after the firſt failure, may then be of the ſame age, with the elder in the original leaſe; and conſequently, that every renewed leaſe may be juſt of the ſame value, as that firſt granted.

SOLUTION.

Let the complements of the lives, for which the leaſe is to be granted, be t, and m; then

From the expectation of the longeſt of the two lives, $\Big\}\ \frac{t}{2}+\frac{mm}{6t}$,

Take the expectation of the two joint lives, $\Big\}\ \frac{m}{2}-\frac{mm}{6t}$;

The remainder, $\frac{t}{2}-\frac{m}{2}+\frac{2mm}{6t}$,

will be the expectation of the ſurvivor.

Now ſince the ſurvivor is (by queſtion) to be of the ſame age, with the elder life, in the leaſe firſt granted, the complement of his life will be m, and the expectation $\frac{m}{2}$;

Therefore $\frac{t}{2}-\frac{m}{2}+\frac{2mm}{6t}=\frac{m}{2}$,

And (by reduction) $\frac{3-\sqrt{3}\times t}{2}=m$.

Now ſince $\sqrt{3} = 1,732$; and $t = (86-10=)\ 76$; therefore $\left(\frac{3-1,732}{2} \times 76 =\right) 48,18 = m$.

That is 48 will be, nearly, equal to the complement of the age required; conſequently $(86-48=)$ 38 will be that age; and, if the original leaſe be granted, for two lives of the ages of 10, and 38 years, then every ſucceeding leaſe will probably be of the ſame value, with the original.

QUESTION LXXXVII.

It is propoſed to grant a leaſe of an eſtate of 1£. per annum, for two lives of the reſpective ages of 10 and 38 years, with a condition, that the leſſee (or his heirs) may, on every firſt failure, put in a new life, aged 10 years, on paying a fine certain: what ſum ought that fine to be fixed at, in the original leaſe?

SOLUTION.

The preſent value of the reverſion of a younger life, after an elder, is given in caſe 2. queſt. 20, whence it will appear, by calculation, that 5,208, will be the fine required.

QUESTION LXXXVIII.

It being propoſed to grant a leaſe of an eſtate of 1£. per annum, for two lives of 10 and 38 years, with conditions that the leſſee (or his heirs) may perpetually renew the ſame; either at the firſt death, for a fine *H* of 5,208£. or at the ſecond, for $(K=)$ 18,948 (the value of the longeſt of the two lives): what fine ought to be paid on granting the original leaſe?

SOLU-

SOLUTION.

It is evident, that the leſſor (by granting the above leaſe) alienates the whole perpetuity, except the value of thoſe fines; now it may be obtained by arguing as in queſtions 82 and 85; that the preſent worth of all the fines denoted by *H*, will be $\left(\frac{1 - d^{\overline{[illegible]}}}{d^{\overline{[illegible]}}} H =\right)$ 5.798; and the preſent worth of all the fines denoted by *K* will be $\left(\frac{1 - d^{l}\,\overline{[illegible]}}{d^{L}.\,\overline{[illegible]}} K =\right)$ 6,052.

Now, ſince the leſſee (or his heirs) are at liberty, either to renew at the firſt, or ſecond death; and, in caſe the one is conſtantly performed, the leſſor will not be, at all, entituled to the other; therefore the mean of the values of the two fines, viz. $\left(\frac{5{,}798 + 6{,}052}{2} =\right)$ 5,925, may be conſidered as the value which he has reſerved: and conſequently (25—5,925 =) 19,075 will be the fine, which ought to be paid on granting the firſt leaſe.

SCHOLIUM.

If the fines *H* and *K*, could have been proportioned with the deſired accuracy, their values would have been equal: but the inequality has two cauſes; the firſt is, that the age 38 is (in queſt. 86.) found without any regard to the intereſt of money; the ſecond is, that the ſaid age, 38, is not exact, tho' the neareſt whole number thereto; the true age (upon thoſe principles) being (86—48,18 =) 37,82.

QUESTION LXXXIX.

There is a leaſe for two lives; which may, on the firſt demiſe, be filled up with a life, whoſe complement is *t*, on paying a fine certain, *H*; and when both lives are extinct, it may be renewed for two lives, whoſe complements are *t*, and *m*, on paying the fine *K*: Now, one of the lives having failed, and the complement of the ſurvivor being *c*, it is required to determine, whether it is the leſſee's intereſt to renew the leaſe (by putting in a life) now; or to ſtay till that life is alſo extinct, and then to put in two lives?

SOLUTION.

If the leſſee were to renew at the time that each ſingle life fails, he muſt then pay the fine, *H*, again, when the joint lives, whoſe complements are *t*, and *c*, ſhall fail; but if he conſtantly renews at the expiration of the ſecond life, he then pays the fine, *K*, again, when the longeſt of the two lives, whoſe complements are *t* and *m* ſhall fail; whence (by queſt. 82) the intereſt of the leſſor, immediately before thoſe renewals, will (in the firſt caſe) be $\frac{H}{d\,\overline{\mathfrak{T}\mathfrak{C}}}$; and (in the ſecond) $\frac{K}{dL\,.\,\overline{\mathfrak{T}\mathfrak{M}}}$: but, if we value thoſe two intereſts at the the failure of the firſt life, the latter (not being due till the death of the life in being, whoſe value is $\mathfrak{C}$) will be then worth, but

$$\overline{1-d\mathfrak{C}}\times\frac{K}{dL\,.\,\overline{\mathfrak{T}\mathfrak{M}}}\text{, by queſt. 89. vol. 2.}$$

Now, if the fines, *H* and *K*, are ſo proportioned, as to make it indifferent to the leſſee, whether he renews at the firſt or ſecond demiſe; then $\frac{H}{d\,\overline{\mathfrak{T}\mathfrak{C}}}=\frac{\overline{1-d\mathfrak{C}}\times K}{dL\,.\,\overline{\mathfrak{T}\mathfrak{M}}}$;

$$\text{or }\frac{H}{\overline{\mathfrak{T}\mathfrak{C}}}=\frac{\overline{1-d\mathfrak{C}}\times K}{L\,.\,\overline{\mathfrak{T}\mathfrak{M}}}.$$

But, when the fines are proportioned as in the laſt queſtion, $L\,.\,\overline{\mathfrak{T}\mathfrak{M}}=K$; whence $\frac{H}{\overline{\mathfrak{T}\mathfrak{C}}}=1-d\mathfrak{C}$.

From whence it will be evident, that, if $\frac{\overline{1-d\mathfrak{C}}\times K}{L.\overline{\mathfrak{F}\mathfrak{M}}}$ or (in the caſe laſt mentioned) $1-d\mathfrak{C}$ exceeds $\frac{H}{\overline{\mathfrak{F}\mathfrak{C}}}$, then it will be the leſſor's intereſt, that the leaſe ſhould not be renewed till the life in being is extinct; and conſequently, the leſſee's, to renew now: on the contrary, when $\frac{H}{\overline{\mathfrak{F}\mathfrak{C}}}$ exceeds $\frac{\overline{1-d\mathfrak{C}}\times K}{L.\overline{\mathfrak{F}\mathfrak{M}}}$ or $1-d\mathfrak{C}$, then it will be the leſſor's intereſt to have the leaſe renewed at the preſent death, and the leſſee's not to renew till the other life has failed.

COROL.

Hence, when the ages, and fines, are properly proportioned; as in queſtions 86, 87, and 88; if (on the failure of the firſt life) the ſurvivor is of an age, leſs than 38, it will be the intereſt of the leſſee, not to renew, till the remaining life fails; but if the ſurvivor is elder than 38, then to renew directly.

QUESTION XC.

It is propoſed to let an eſtate, by a leaſe for three lives (the youngeſt of which, may be aged 10 years) with condition, that the leſſee (or his heirs) may, on every failure of a ſingle life, put in another, aged 10 years, on paying a fine certain of *H* pounds; and that (on the failure of two, out of the three lives) he may put in two others, ſuch as may make the renewed leaſe of the ſame value with that firſt granted, for a fine certain of *K* pounds: it is required, to find the ages of thoſe two lives, which ſhould be joined (in the original leaſe) with the life, aged 10 years, ſo as to render it probable, that the leaſe, to be granted in caſe of a renewal on the firſt death, may be of the ſame value, with that firſt granted.

SOLU-

SOLUTION.

Let the complements of the lives, for which the leaſe is firſt granted, be t, m, and n; then,

Firſt, from the expectation of the longeſt of the three lives, viz. $\frac{t}{2} + \frac{mm}{6t} + \frac{n^3}{12mt}$,

Take the expectation of the three joint lives, viz.

$$\frac{n}{2} - \frac{nn}{6m} - \frac{nn}{6t} + \frac{n^3}{12mt};$$

Then will the remainder, viz.

$$\frac{t}{2} - \frac{n}{2} + \frac{mm}{6t} + \frac{nn}{6m} + \frac{nn}{6t},$$

be the expectation of the two ſurvivors; but ſince the leſſee (or his heirs) may, in this caſe, put in a life, whoſe complement is t, therefore the complements of theſe ſurvivors lives ought to be m, and n; and, conſequently, their expectation $\frac{m}{2} + \frac{nn}{6m}$:

Hence, $\frac{t}{2} - \frac{n}{2} + \frac{mm}{6t} + \frac{nn}{6m} + \frac{nn}{6t} = \frac{m}{2} + \frac{nn}{6m}$;

Or, $\frac{nn}{6t} = \frac{m}{2} + \frac{n}{2} - \frac{t}{2} - \frac{mm}{6t}$.

Secondly, from the expectation of the longeſt of the three lives, viz. $\frac{t}{2} + \frac{mm}{6t} + \frac{n^3}{12mt}$,

take the expectation of the longeſt of any two lives of thoſe three, viz.

$$\frac{m}{2} - \frac{mm}{6t} + \frac{nn}{6m} + \frac{nn}{6t} - \frac{2nnn}{12mt};$$

And the remainder, viz.

$$\frac{t}{2} - \frac{m}{2} + \frac{2mm}{6t} - \frac{nn}{6m} - \frac{nn}{6t} + \frac{2n^3}{12mt},$$

wil

will be the expectation of the life ſurviving thoſe two; which, becauſe the leaſe, then made, is to be of equal value with the original, will be a life whoſe complement is n, and expectation $\frac{n}{2}$; therefore

$$\frac{t}{2} - \frac{m}{2} + \frac{2mm}{6t} - \frac{nn}{6m} - \frac{nn}{6t} + \frac{3n^3}{12mt} = \frac{n}{2},$$

Or

$$\frac{mm}{6t} - \frac{nn}{6m} - \frac{nn}{6t} + \frac{3n^3}{12mt} = \frac{m}{2} + \frac{n}{2} - \frac{t}{2} - \frac{mm}{6t}.$$

Now by comparing the two equations together, it will appear, that,

$$\frac{mm}{6t} - \frac{nn}{6m} - \frac{nn}{6t} + \frac{3n^3}{12mt} = \frac{nn}{6t},$$

That is $2m^3 - 2tnn - 4mnn + 3n^3 = 0$;

This equation (if we write $(86-10=)$ 76 for t) becomes $2m^3 - 152nn - 4mnn + 3n^3 = 0$;

Where the whole numbers 58, and 38, are (nearly) equal to the values of m, and n:

Therefore, the two ages required will be $(86-58=)$ 28, and $(86-38=)$ 48; and (when a renewal is to be made, after two of the three lives have failed) no 2 lives ſhould be put in, the expectation on the longeſt of which exceeds $\frac{76}{2} + \frac{58 \times 58}{6 \times 76} =)$ 45,38.

QUESTION XCI.

It is propoſed to grant a leaſe of an eſtate of 1£. per annum, for three lives, of the reſpective ages of 10, 28, and 48; with a condition, that the leſſee (or his heirs) may on every firſt failure, put in a new life, aged 10 years, on paying a fine certain: what ſum ought that fine to be fixed at, in the original leaſe?

SOLU-

SOLUTION.

The preſent value of the reverſion of a younger life after two elder lives is inveſtigated in caſe 3. queſt. 29. and by applying the reſult to the preſent caſe it will appear, that 2,676 £. will be the fine required.

QUESTION XCII.

It is propoſed to grant a leaſe of an eſtate of 1 £. per annum, for three lives, of the reſpective ages of 10, 28, and 48; with a condition, that the leſſee (or his heirs) may, on every failure of two lives, put in two new lives, aged 10 and 28 years, on paying a fine certain: that fine is required?

SOLUTION.

The preſent value of the reverſion of the longeſt of 2 younger lives after an elder life, is found in caſe 3. queſt. 28. which being applied to the preſent caſe will give 7,508 for the fine required.

QUESTION XCIII.

It is propoſed to grant a leaſe of an eſtate of 1 £. per annum, for three lives, aged 10, 28, and 48; with conditions that the leſſee (or his heirs) may, perpetually, renew the ſame; either, at the firſt death, for a fine of 2,676; or, when two of the lives have failed, for 7,508; or, when all three are extinct, for 19,510 (the value of the longeſt of the three lives): what fine ought to be paid, on granting the original leaſe?

SOLUTION.

Let $\overline{\mathfrak{L}\mathfrak{M}\mathfrak{N}}$ repreſent the value of the three joint lives; $\overline{\mathfrak{L}\mathfrak{M}\mathfrak{N}}^{\overset{1\,1}{1\,1\,1}}$, the value of the longeſt of any two, out of the three; $L.\overline{\mathfrak{L}\mathfrak{M}\mathfrak{N}}$, the longeſt of the three lives; *H*, the fine 2,676; *K*, the fine 7,508; and *G*, the fine 19,510.

Then the preſent worth of all the fines, denoted by *H*, will be $\frac{1-d\,\overline{\mathfrak{L}\mathfrak{M}\mathfrak{N}}}{d\,\overline{\mathfrak{L}\mathfrak{M}\mathfrak{N}}}$; of all the fines, denoted by *K*,

$\frac{1-d\,\overline{\mathfrak{L}\mathfrak{M}\mathfrak{N}}^{\overset{1\,1}{1\,1\,1}}}{d\,\overline{\mathfrak{L}\mathfrak{M}\mathfrak{N}}^{\overset{1\,1}{1\,1\,1}}}$; and of all the fines, denoted by *G*,

$\frac{1-d\,L.\overline{\mathfrak{L}\mathfrak{M}\mathfrak{N}}}{d\,L.\overline{\mathfrak{L}\mathfrak{M}\mathfrak{N}}}$,

whence, by calculation, firſt, 4,700 will be the value of all the fines, denoted by *H*.

Secondly, 4,732 will be the value of all the fines, denoted by *K*.

And thirdly 5,490 will be the value of all the fines, denoted by *G*.

Then (by arguing as in queſtion 88, $\frac{4{,}700+4{,}732+5{,}490}{3}=$) 4,974 will be the value of the leſſor's intereſt in all the fines; and (25—4,974 =) 20,026, the fine to be paid, on granting the original leaſe.

QUESTION XCIV.

There is an eſtate of 1£. per annum, which is leaſed, on three lives; with conditions, that (when a ſingle life drops) another, whoſe complement is *t*, may be put in, on paying the fine *H*; if two lives are extinct, then two other lives, whoſe complements are t and m, may be added, on paying the fine K; and, if all the three lives are expired, then a new leaſe for three lives, whoſe complements are t, m, and n, ſhall be granted, on paying the fine G. Let it now be ſuppoſed, that one of the three lives is extinct, and that the complements of the two remaining lives are c and b; it is required to determine, whether it will be the leſſee's intereſt to put in a life at this firſt demiſe, or to ſtay till the ſecond, and then to put in two?

SOLUTION.

If the leſſee were to renew, at the time that each ſingle life fails, he then muſt pay the fine, H, again, when one of the three lives whoſe complements are t, c, and b, ſhall fail; but, if he conſtantly renews at the ſecond death, he is then to pay the fine, K, a ſecond time, when two lives out of the three, whoſe complements are t, m, and c, ſhall fail; and then e the complement of the ſurvivor of the lives whoſe complements are c and b, will (by the proceſs in queſt. 86, probably) be worth

$$\left(\overline{2\times\frac{c}{2}-\frac{b}{2}+\frac{2bb}{6c}}=\right)c-b+\frac{2bb}{3c}.$$

Now, in the firſt caſe, the intereſt of the leſſor, immediately before thoſe renewals, will be $\frac{H}{d\,\overline{\mathfrak{F}\mathfrak{C}\mathfrak{B}}}$; and, in the ſecond caſe, it will be $\frac{K}{d\,\overline{\overline{\mathfrak{F}\mathfrak{M}}\mathfrak{C}1\frac{1}{1}\frac{1}{1}}}$; the latter of which (becauſe the fine K is not payable, till the failure of one of the lives, whoſe complements are c and b) will be, at the time of the firſt demiſe, worth, only,

$$\overline{1-d\,\overline{\mathfrak{C}\mathfrak{B}}}\times\frac{K}{d\,\overline{\mathfrak{F}\mathfrak{M}}\mathfrak{C}1\frac{1}{1}1}.$$

Now, if the fines, H and K, are ſo proportioned, as to make it indifferent to the leſſee, whether he renews at the firſt, or ſecond demiſe; then,

$$\frac{H}{d\mathfrak{F}\overline{\mathfrak{C}\mathfrak{B}}} = \frac{\overline{1-d\overline{\mathfrak{C}\mathfrak{B}}}\times K}{d\times\overline{\mathfrak{M}\mathfrak{C}}_{1\,1}^{1\,1}}.$$

Whence, if $\frac{\overline{1-d\overline{\mathfrak{C}\mathfrak{B}}}\times K}{\mathfrak{F}2b\overline{\mathfrak{C}}_{1\,1}^{1\,1}}$ exceeds $\frac{H}{\mathfrak{F}\overline{\mathfrak{C}\mathfrak{B}}}$, then it will be the leſſor's intereſt, that the leaſe ſhould not be renewed till the next life fails; and conſequently the leſſee's to renew now; but, if $\frac{H}{\mathfrak{F}\overline{\mathfrak{C}\mathfrak{B}}}$ exceeds $\frac{\overline{1-d\overline{\mathfrak{C}\mathfrak{B}}}\times K}{\mathfrak{F}ab\overline{\mathfrak{C}}_{1\,1}^{1\,1}}$, then it will be the leſſor's intereſt to have the leaſe renewed now; and the leſſee's, to poſtpone it.

Note. When the ages and fines are properly proportioned, as in queſtions 90, 91, 92, and 93; if, on the failure of the firſt life, the joint lives of the two ſurvivors is worth more, than the joint lives of two perſons, aged 28 and 48, it will be the leſſee's intereſt to poſtpone the renewal, till one of them fails; but if the joint lives of 28 and 48, are worth more, than the joint lives of the ſurvivors, it will be his intereſt to renew directly; which is evident, from the ſolutions of thoſe queſtions.

QUESTION XCV.

In ſuch a leaſe as that mentioned in the laſt queſtion; let us ſuppoſe, that two, of the three lives, have failed, the complement of the ſurvivor being c; Then, will it be the intereſt of the leſſee to renew directly, or to ſtay till the remaining life is alſo extinct.

SOLUTION.

If the leſſee were to renew conſtantly on the failure of the ſecond life, he is to pay the fine, *K*, a ſecond time, when two lives, out of the three whoſe complements are t, m, and c, ſhall fail; but if he conſtantly renews when all the three lives are extinct, then he will be to pay the fine, G, a ſecond time, upon the failure of all the three lives, whoſe complements are t, m, and n.

Then, in the firſt caſe, the intereſt of the leſſor, immediately before the renewals will be $\frac{K}{d\,\overline{\mathfrak{T}\mathfrak{M}\mathfrak{C}}^{\,1\,1\,1}}$, and $\frac{G}{dL\,.\,\overline{\mathfrak{T}\mathfrak{M}\mathfrak{N}}}$; the latter of which (becauſe the fine G is not payable till the failure of the life whoſe complement is c) will, at the time of the ſecond demiſe, be worth, only, $\frac{\overline{1-d\mathfrak{C}}\times G}{dL\,.\,\overline{\mathfrak{T}\mathfrak{M}\mathfrak{N}}}$.

Now, if the fines, K and G, are ſo proportioned, as to make it indifferent to the leſſee, whether he renews at the ſecond or third death; then,

$$\frac{K}{d\,\overline{\mathfrak{T}\mathfrak{M}\mathfrak{C}}^{\,1\,1\,1}}=\frac{\overline{1-d\mathfrak{C}}\times G}{dL\,.\,\overline{\mathfrak{T}\mathfrak{M}\mathfrak{N}}}.$$

Whence, if $\frac{\overline{1-d\mathfrak{C}}\times G}{L\,.\,\overline{\mathfrak{T}\mathfrak{M}\mathfrak{N}}}$ exceeds $\frac{K}{\overline{\mathfrak{T}\mathfrak{M}\mathfrak{C}}^{\,1\,1\,1}}$, then it will be the leſſor's intereſt, that the leaſe ſhould not be renewed till the third life fails; and, conſequently, the leſſee's to renew directly: But, if $\frac{K}{\overline{\mathfrak{T}\mathfrak{M}\mathfrak{C}}^{\,1\,1\,1}}$ exceeds $\frac{\overline{1-d\mathfrak{C}}\times G}{L\,.\,\overline{\mathfrak{T}\mathfrak{M}\mathfrak{N}}}$. then it will be the leſſor's intereſt to have the leaſe renewed now, and the leſſee's to poſtpone it.

Note. When the ages and fines are properly proportioned, as in queſtions 90, 91, 92, and 93; then $L\,.\,\overline{\mathfrak{T}\mathfrak{M}\mathfrak{N}}=G$; and, if the remaining life (whoſe complement is c) be younger than 48; it will be the intereſt of the leſſee not to renew, till the failure thereof; but if the ſurvivor is elder than 48, then to renew directly.

QUESTION XCVI.

What ſum ought to be paid for inſuring, for one year, a given ſum, s, on a life, whoſe complement is n?

SOLUTION,

Since the probability of the life's failing in that year is $\frac{1}{n}$, and (if that happens) the inſurer is to pay to the inſured, the ſum, s; therefore, the premium, for that inſurance, ſhould be $\frac{s}{n}$.

EXAMPLE.

If the ſum of 100 £. is inſured on a life, aged ſixty ſix years; then $n = (86-66=)\ 20$; and $(\frac{100}{20} =)\ 5$ £. will be the required premium.

Note. The premium of inſurance is uſually paid down, at the time of inſuring; but the ſum inſured does (in caſe of the death of the perſon whoſe life is inſured) generally remain unpaid, till the end of the year, and ſometimes longer; now (if the computation be made, as above) it is evident, that the premium is but juſt, adequate to the hazard: and conſequently, if the inſurer is to expect any profit, on the tranſaction; the intereſt of the premium (which by the above cuſtom he enjoys) for one year, will be a very moderate one; being (in the above example) but 5s. if he could make 5 £. per cent. of his money.

QUESTION XCVII.

What conſtant yearly ſum, of which the firſt payment is to be made immediately, ought to be paid, for inſuring the ſum, s, on a life whoſe complement is n?

SOLU-

SOLUTION.

By the laſt queſtion, the premium for inſuring, the firſt year, will be $\frac{s}{n}$; and, when that is elapſed, then, for the ſecond year, $\frac{s}{n-1}$; for the third, $\frac{s}{n-2}$ &c.

But, foraſmuch as the premium, for the ſecond, third, &c. year, will not be payable, unleſs the inſured ſurvives the firſt, ſecond, &c. year; the probabilities of which are $\frac{n-1}{n}$, $\frac{n-2}{n}$, &c. therefore, the premiums for the ſecond, third, &c. year, being eſtimated at the beginning of the firſt year, will be $\frac{n-1}{n}\times\frac{s}{n-1}$. $\frac{n-2}{n}\times\frac{s}{n-2}$, &c. or $\frac{s}{n}$, $\frac{s}{n}$, &c.

Again, becauſe the ſaid premiums for the 2d. 3d. &c. year, do not become payable, till the expiration of the 1ſt. 2d. &c. year; the preſent worth of them, computed at the beginning of the firſt year, will be $\frac{s}{nr}$, $\frac{s}{nr^2}$, &c. and, ſince one payment is to be made immediately, therefore

$$\left(\frac{s}{n}+\frac{s}{nr}+\frac{s}{nr^2}\ (n)=\right)$$

$$\frac{s}{n}\times\overline{\frac{1}{1}+\frac{1}{r}+\frac{1}{r^2}+\frac{1}{r^3}\ (n)},$$

will be the preſent worth of all the yearly premiums, that can be received by the inſurer; for, by the hypotheſis of equal decrements, the life will be neceſſarily extinct, at the end of the nth year; and conſequently, $\frac{s}{nr^{n-1}}$ will be the preſent worth of the laſt premium, that he can poſſibly receive.

Now $\frac{1}{1} + \frac{1}{r} + \frac{1}{r^2} (n) = (r \times \overline{\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} (n) =)}$ $\frac{r}{r-1} \times \overline{1 - \frac{1}{r^n}}$ (by queſt. 15. vol. 2.) therefore (putting $\frac{r}{r-1} = P$, and $\frac{1}{r^n} = p$) $\frac{s}{n} + \frac{s}{nr} + \frac{s}{nr^2} (n)$ the preſent worth of all the unequal premiums, which can be received by the inſurer, will be $\frac{srP}{n} \times \overline{1-p}$.

But the preſent worth of all the conſtant annual ſums, which the inſurer is to receive, for inſuring the life in queſtion, will (if N repreſents the value of a life, not ſecured by land, whoſe complement is n; and a, the yearly ſum, ſo to be received) be ($a + Na$ or) $\overline{1 + N} \times a$; for the ſum, a, is to be immediately received, and the receipt thereof is to be annually continued, during the life, whoſe value is N; alſo, if the life fails, in any part of a year, no ſum is to be received at the end of that year; for then, the ſum inſured is to be paid to the inſured, or his aſſigns.

It follows, therefore, that $\frac{srP}{n} \times \overline{1-p}$, the preſent worth of all the yearly unequal premiums, which the inſurer would receive, if the life was to be annually inſured; will be equal to, $\overline{1+N} \times a$, the preſent worth of all the equal premiums, which he is to receive, if he engages to inſure the life, on the receipt of a conſtant yearly ſum:

That is $\overline{1 + N} \times a = \frac{srP}{n} \times \overline{1-p}$;

Therefore $a = \frac{srP}{n} \times \frac{1-p}{1+N}$.

Example. What conſtant annual premium ſhould be paid, for inſuring 100£. on a life, aged 66, allowing 4 per cent. compound intereſt?

Here $s = 100$; $r = 1{,}04$; $P = 25$; $n = (86 - 66 =)\ 20$; $p = 0{,}4564$; $1 - p = 0{,}5436$; $N = 7{,}333$; and $1 + N = 8{,}333$:

Therefore $\left(\frac{100\times 1{,}04\times 25\times 0{,}5436}{20\times 8{,}333}=\right)$ 8,481 will be the annual premium required.

Scholium. The above queſtion may be, otherwiſe, ſolved, in the following manner: the inſured purchaſes the ſum *s*, to be paid to him, or his aſſigns, on the deceaſe of a life, whoſe value is *N*; by the payment of the ſum, *a*, in hand, and by the annual payment of the like ſum, *a*, during the continuance of the ſaid life.

Now (by queſt 89. vol, 2.) the preſent worth of 1£. payable on the failure of the propoſed life, is $\frac{\overline{1-p}\times r}{n\times\overline{r-1}}$, or $1-\overline{r-1}\times N$; and, conſequently, the preſent value of *s* will be equal to $\frac{\overline{1-p}\times rPs}{n}$ (becauſe $\frac{1}{r-1}=P$), or $\overline{1-\overline{r-1}\times N}\times s$:

But $\overline{1+N}\times a=\left(\frac{\overline{1-p}\times rPs}{n}\text{ or}\right)\ \overline{1-\overline{r-1}\times N}\times s$;

Therefore $a=\left(\frac{\overline{1-p}\times rPs}{\overline{1+N}\times n}\text{ or}\right)\ \frac{\overline{1-\overline{r-1}\times N}\times s}{1+N}$.

In the above example; $r-1=0{,}04$; $7{,}333\times 0{,}04=0{,}29332$; $1-0{,}29332=0{,}70668$; and $0{,}70668\times 100=70{,}668$:

Therefore $\left(\frac{70{,}668}{8{,}333}=\right)$ 8,481 will be the annual payment, as before.

QUESTION XCVIII.

A, who has a place, or other annual income for life, would borrow a ſum of money of *B*, and (beſides a bond, bearing intereſt) would inſure his life, as a collateral ſecurity; *B* undertakes to be the inſurer, and to remit the payment of the principal, to the heirs of *A*, if he conſtantly pays the ſaid intereſt and inſurance, during his life to him, *B*, or his heirs; what ſum ought *A* to pay, annually for intereſt and inſurance, for 1£. ſo lent?

SOLUTION.

Let *N* be the value of *A*'s life; then (by the ſcholium to queſt 97.) the annual inſurance of 1£. will be $\frac{1-\overline{r-1}\times N}{1+N}$; to which add, $r-1$, the intereſt of 1£.

And the ſum $\left(\frac{1-\overline{r-1}\times N}{1+N}+\overline{r-1}=\right)\frac{r}{1+N}$ will be the annual payment required.

Example. If *A*, the borrower, be aged 66; and intereſt be computed at 4 per cent; then $r=1{,}04$, and $1+N=8{,}333$:

Whence $\left(\frac{1{,}04}{8{,}333}=\right)$ 0,125 will be the ſum required.

SCHOLIUM.

It is neceſſary, here, to obſerve, that the borrower ought not to receive the full ſum of 1£. (here ſuppoſed to be lent) becauſe he is to pay the firſt year's inſurance down; and, conſequently ſhould receive but

$$(1-\frac{1-\overline{r-1}\times N}{1+N}=\frac{1+N-1+\overline{r-1}N}{1+N}$$

$$=\frac{N+\overline{r-1}\times N}{1+N}=)\frac{rN}{1+N}$$: in the above example

$$\left(\frac{1{,}04\times 7{\cdot}333}{8{,}333}=\right)0{,}915.$$

This will appear to be true, by conſidering that the ſum, $\frac{rN}{1+N}$ will purchaſe an annuity of, $\frac{r}{1+N}$, for a life whoſe value is *N*; for if $Na=\frac{rN}{1+N}$; then $a=\frac{r}{1+N}$.

A TABLE of the preſent values of an annuity (ſecured by land) of 1£. per annum, on a ſingle life; ſuppoſing the decrements of life to be equal.

Ages	3 per cent	3½ per cent	4 per cent	4½ per cent	5 per cent	6 per cent
8	19,934	18,336	16,949	15,738	14,674	12,899
9	20.064	18,443	17,038	15,813	14,735	12,947
10	20,064	18,443	17,038	15,813	14,735	12,947
11	19,934	18,336	16.949	15,738	14.674	12,899
12	19,804	18,227	16,858	15,661	14,611	12,852
13	19,671	18,117	16,765	15,583	14,545	12,804
14	19,535	18,005	16,671	15,503	14,477	12,753
15	19.398	17,891	16,575	15.422	14,407	12,702
16	19,258	17,774	16,478	15,340	14,335	12,649
17	19,115	17,655	16,378	15,255	14,263	12,595
18	18,971	17,534	16,276	15,170	14,189	12,539
19	18,824	17,412	16,172	15,080	14,114	12,483
20	18,675	17,287	16,066	14.990	14,036	12,424
21	18,524	17,159	15,958	14,898	13.957	12,364
22	18,369	17,029	15,848	14.804	13,876	12,304
23	18,213	16,896	15,736	14,708	13.793	12,240
24	18,053	16,762	15,621	14,609	13,708	12,176
25	17,892	16,624	15,504	14,510	13.622	12,111
26	17,728	16,484	15,386	14,407	13,533	12,042
27	17,560	16,343	15,265	14,302	13,442	11,974
28	17,390	16,198	15,139	14,195	13,348	11,902
29	17,217	16,050	15,012	14,084	13,252	11,829
30	17,041	15,900	14,882	13,973	13,115	11,753
31	16,863	15,747	14,750	13,857	13,054	11,675

A TABLE of the preſent values of an annuity (ſecured by land) of 1 £. per annum, on a ſingle life; ſuppoſing the decrements of life to be equal.

Ages	3 per cent	$3\frac{1}{2}$ per cent	4 per cent	$4\frac{1}{2}$ per cent	5 per cent	6 per cent
32	16,682	15,590	14,615	13,739	12.952	11,597
33	16,497	15,430	14,476	13,619	12,847	11,515
34	16,300	15,268	14.335	13.496	12,739	11,431
35	16,118	15,103	14,191	13.370	12,629	11,344
36	15.923	14,934	14,044	13,242	12,516	11,256
37	15,725	14,762	13,894	13,110	12,399	11,163
38	15,523	14,586	13,740	12,974	12,279	11,070
39	15,319	14,406	13,582	12,837	12,157	10,973
40	15,111	14,225	13,423	12,695	12,031	10,873
41	14,898	14,039	13,258	12,550	11,902	10 771
42	14,683	13,849	13,091	12,401	11,771	10,665
43	14,464	13,656	12,920	12,248	11,635	10,556
44	14,240	13,459	12,744	12,093	11,495	10,444
45	14,014	13,256	12,566	11,933	11,353	10,329
46	13,782	13,051	12,382	11,770	11,207	10,209
47	13,546	12,842	12,195	11,602	11.055	10,087
48	13,308	12,628	12,002	11,430	10,901	9,960
49	13,064	12,409	11,806	11,253	10,741	9,829
50	12,814	12,186	11,607	11,072	10,578	9.695
51	12,562	11,959	11,402	10,887	10,410	9,556
52	12,305	11,727	11,192	10,697	10,237	9,412
53	12,044	11,489	10.977	10,501	10,059	9,265
54	11,776	11,248	10,757	10,301	9,877	9,111
55	11,506	11,000	10,532	10,096	9,688	8,954

A TABLE of the preſent values of an annuity (ſecured by land) of 1£. per annum, on a ſingle life; ſuppoſing the decrements of life to be equal.

Ages	3 per cent	3½ per cent	4 per cent	4½ per cent	5 per cent	6 per cent
56	11,229	10,750	10,302	9,885	9,495	8,790
57	10,947	10,493	10,066	9,669	9,297	8,621
58	10,660	10,228	9,825	9,447	9,092	8,447
59	10,368	9,960	9,577	9,219	8,882	8,268
60	10,071	9,686	9,324	8,985	8,665	8,081
61	9,767	9,406	9,065	8,746	8,443	7,889
62	9,460	9.121	8,800	8,499	8,214	7,690
63	9,144	8,828	8,528	8,246	7,977	7,488
64	8,824	8,529	8,250	7,985	7,734	7,271
65	8,499	8,225	7,965	7,718	7,484	7,050
66	8,166	7,913	7,673	7,444	7,227	6,822
67	7,827	7,595	7,372	7,162	6,961	6,586
68	7,481	7,268	7,066	6,871	6,686	6,341
69	7,130	6,937	6,752	6,574	6,405	6,087
70	6,771	6,597	6,429	6,269	6,114	5,824
71	6,406	6,249	6,099	5,954	5,814	5,551
72	6,034	5,895	5,760	5,630	5,506	5,269
73	5,655	5,532	5,413	5,298	5,187	4,976
74	5,269	5,162	5,057	4,956	4,858	4,673
75	4,874	4,782	4,691	4,605	4,521	4,358
76	4,473	4,394	4,318	4,243	4,170	4,032
77	4,064	3,998	3,933	3,871	3,810	3,692
78	3,646	3,593	3,540	3,488	3,437	3,341
79	3,221	3,178	3,136	3,094	3,054	2,977

A Table of the differences, between the values of annuities for ſingle lives (ſecured by land) and the quotients ariſing, from the diviſions of the reſpective complements of thoſe lives, by 6 times the rate of intereſt; which differences occur, in moſt of the ſolutions contained in this volume.

Ages	3 per cent	3 ½ per cent	4 per cent	4 ½ per cent	5 per cent	6 per cent
8	7,759	6,259	4,930	3.776	2,770	1,107
9	7,766	6,205	4,858	3,692	2,672	0,997
10	7,766	6,205	4,858	3,692	2,672	0,997
11	7,799	6,259	4,930	3,776	2,770	1,107
12	7,830	6,311	4,999	3,859	2,865	1,217
13	7,859	6,362	5.066	3,940	2,958	1,326
14	7,885	6,411	5,132	4,020	3,049	1,433
15	7,909	6,458	5,196	4,098	3,137	1,539
16	7,931	6,502	5,260	4,176	3,224	1,643
17	7,950	6,544	5,320	4,250	3,311	1,746
18	7,968	6,584	5,378	4,325	3,395	1,848
19	7,983	6,623	5,435	4,395	3,479	1,948
20	7,996	6,659	5,489	4,464	3,560	2.047
21	8,006	6,692	5,541	4,531	3,639	2,144
22	8,013	6,723	5,591	4,597	3,717	2,241
23	8,019	6,751	5,639	4.660	3,793	2,335
24	8,021	6,778	5,686	4,721	3,867	2,428
25	8,022	6,801	5,728	4,781	3.940	2,520
26	8,019	6,822	5,770	4,838	4,009	2,609
27	8,013	6,842	5,810	4,892	4,077	2,698
28	8,005	6,859	5,844	4,945	4,142	2,783
29	7,994	6,871	5,877	4,994	4,204	2,867
30	7,980	6,883	5.908	5,042	4,266	2,948
31	7,964	6,891	5,936	5,085	4,324	3,027

A TABLE of the differences, between the values of annuities for lives (ſecured by land) and the quotients ariſing, from the diviſions of the reſpective complements of thoſe lives, by 6 times the rate of intereſt; which differences occur, in moſt of the ſolutions contained in this volume.

Ages	3 per cent	3½ per cent	4 per cent	4½ per cent	5 per cent	6 per cent
32	7,944	6,895	5,961	5,126	4,381	3,107
33	7,921	6,896	5,982	5,166	4.434	3,182
34	7,895	6,895	6,001	5,202	4,485	3,255
35	7,866	6,891	6,018	5,236	4,534	3,325
36	7.832	6,883	6,031	5,267	4,580	3,394
37	7,796	6,872	6,041	5,295	4,622	3,459
38	7,756	6,857	6,048	5,319	4,660	3,523
39	7,714	6,838	6,050	5,341	4,697	3,583
40	7,668	6,818	6,051	5,358	4,729	3,640
41	7,617	6,793	6,046	5,373	4,759	3,695
42	7,564	6,764	6,039	5,383	4,787	3,747
43	7,506	6,732	6,029	5,390	4,810	3,795
44	7,444	6,696	6,013	5,394	4.829	3,840
45	7,380	6,654	5,995	5,394	4 845	3,883
46	7.310	6,610	5,972	5,390	4,858	3 920
47	7,235	6,562	5,945	5,382	4,865	3,955
48	7,159	6,509	5,912	5,369	4,869	3,985
49	7,077	6,451	5,876	5,352	4,868	4,011
50	6,989	6,389	5,838	5,330	4,864	4,035
51	6,898	6,323	5,793	5,305	4,854	4,053
52	6,804	6,252	5,743	5,274	4,840	4,066
53	6,704	6,175	5,689	5,238	4,821	4,076
54	6,598	6,095	5,629	5,196	4,798	4,079
55	6,490	6,008	5,565	5,152	4,767	4,080

A TABLE of the differences, between the values of annuities for lives (ſecured by land) and the quotients ariſing, from the diviſions of the reſpective complements of thoſe lives, by 6 times the rate of intereſt; which differences occur, in moſt of the ſolutions contained in this volume.

Ages	3 per cent	3½ per cent	4 per cent	4½ per cent	5 per cent	6 per cent
56	6,375	5,919	5,495	5.100	4,733	4,073
57	6,254	5,823	5,419	5,044	4,694	4,061
58	6,129	5,719	5,338	4,981	4,648	4,045
59	5,999	5.613	5,250	4,913	4,596	4,023
60	5,864	5,499	5,157	4,838	4,538	3,993
61	5,722	5,380	5,059	4,759	4,475	3,958
62	5,576	5,256	4,954	4,671	4,404	3,916
63	5,422	5,124	4,942	4,578	4,326	3,872
64	5,264	4,986	4,724	4,476	4,242	3,822
65	5,101	4,843	4,600	4.369	4,151	3,758
66	4,930	4,692	4,468	4,254	4,052	3,677
67	4,753	4,535	4,327	4,132	3,945	3,599
68	4,568	4,369	4,181	4,000	3,829	3 511
69	4,379	4,199	4,028	3,863	3,707	3,414
70	4.182	4,021	3,865	3,717	3,574	3,308
71	3,979	3,833	3,695	3,562	3,433	3,193
72	3,769	3,641	3,517	3,397	3,284	3,068
73	3,551	3,439	3,330	3,225	3,123	2,932
74	3,327	3,230	3.124	3,042	2,953	2,786
75	3,094	3,011	2,928	2,851	2,775	2,628
76	2,855	2,784	2,715	2,648	2,583	2,459
77	2,608	2,549	2,491	2,436	2,381	2,277
78	2,351	2,305	2,258	2,212	2,167	2,083
79	2,089	2,051	2,014	1,978	1,943	1,876

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